Expert Systems with Applications 40 (2013) 4812–4819

Contents lists available at SciVerse ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

A non dominated ranking Multi Objective Genetic Algorithm and electre method for unequal area facility layout problems

Giuseppe Aiello, Giada La Scalia *, Mario Enea

Dipartimento di Ingegneria Chimica, Gestionale, Informatica, Meccanica, University of Palermo, Italy

ARTICLE INFO

Keywords:
Facility layout problems
Non-dominated Ranking Genetic Algorithm
Slicing structure
Electre method

ABSTRACT

The unequal area facility layout problem (UA-FLP) comprises a class of extremely difficult and widely applicable optimization problems arising in diverse areas and meeting the requirements for real-world applications. Genetic Algorithms (GAs) have recently proven their effectiveness in finding (sub) optimal solutions to many NP-hard problems such as UA-FLP. A main issue in such approach is related to the genetic encoding and to the evolutionary mechanism implemented, which must allow the efficient exploration of a wide solution space, preserving the feasibility of the solutions and ensuring the convergence towards the optimum. In addition, in realistic situations where several design issues must be taken into account, the layout problem falls in the broader framework of multi-objective optimization problems. To date, there are only a few multi-objective FLP approaches, and most of them employ over-simplified optimization techniques which eventually influence the quality of the solutions obtained and the performance of the optimization procedure. In this paper, this difficulty is overcome by approaching the problem in two subsequent steps: in the first step, the Pareto-optimal solutions are determined by employing Multi Objective Genetic Algorithm (MOGA) implementing four separate fitness functions within a Pareto evolutionary procedure, following the general structure of Non-dominated Ranking Genetic Algorithm (NRGA) and the subsequent selection of the optimal solution is carried out by means of the multi-criteria decision-making procedure Electre. This procedure allows the decision maker to express his preferences on the basis of the knowledge of candidate solution set. Quantitative and qualitative objectives are considered referring to the slicing-tree layout representation scheme. The numerical results obtained outperform previous referenced approaches, thus confirming the effectiveness of the procedure proposed.

1. Introduction

The facility layout problem (FLP) is the determination of the most efficient physical arrangement of a number of interacting facilities on the factory floor of a manufacturing system in order to meet one or more objectives. Facilities usually represent the largest and most expensive assets of the organization and are of crucial importance to the organization (Nordin, Zainuddin, Salim, & Ponnusamy, 2009). Tompkins et al. (1996) estimate that between 20% and 50% of operating cost can be attributed to facility planning and material handling., and such costs can be reduced considerably by an effective layout design. Several heuristic approaches have been proposed in the literature in the recent years to find (sub-) optimal solutions to the FLP, including simulated annealing algorithms, tabu search methods, neural networks and genetic algorithms (GAs). According to Sirinaovakul and Thajchayapong (1994), a frequent drawback of such algorithms is that they do not explore enough possibilities while generating their solutions thus being extremely sensitive to the initial solution. Heragu and Alfa (1992) sited these algorithms as local optimization algorithms which, once hit an unattractive region, had no way of backing out and exploring other regions. Glover and Greenberg (1989) noted that reliable heuristic algorithms are not sensitive to their initial solutions and that an exhaustive search of the solution space can be achieved by parallel processing. This should avoid the search procedure to be trapped into inferior solution regions. A GA is a stochastic search technique based on the concept of the survival of the best, emulating the mechanisms of the Darwinian evolution, thus achieving a sub-optimal solution via recursive operations of crossover and mutation (Holland, 1975; Michalewicz, 1992). Most of the studies conducted in FLPs have focused on a single objective, either quantitative or qualitative goodness of the layout (Tuzkaya & Ertay, 2004). In contrast, practical FLPs involve several conflicting objectives. Therefore, both quantitative and qualitative objectives must be considered simultaneously before arriving at any conclusion. A layout that is optimal with respect to a given criterion might be a poor candidate when another criterion is paramount.

* Corresponding author.
E-mail addresses: giada.lascalia@unipa.it, lascalia@dtpm.unipa.it (G. La Scalia).

0957-4174/$ - see front matter © 2013 Published by Elsevier Ltd.
http://dx.doi.org/10.1016/j.eswa.2013.02.026

In general, minimization of the total material handling (MH) cost is often used as the optimization criterion in FLPs. The closeness rating, hazardous movement, safety, and the like are also important criteria in FLPs. In fact, these qualitative factors have significant influence on the final layout and should give consideration. Consequently, the FLP falls into the category of multi-objective optimization problem (MOOP). Multi-objective optimization is a technique to treat several objectives simultaneously without converting them into one. The objective of MOOPs is to find a set of Pareto-optimal solutions, which are the superior solutions when considering all the objectives. In MOOPs, the absolute optimal solution is absent and the designer must select a solution that offers the most profitable trade-off between the objectives as an alternative. Thus, instead of offering a single solution, it is more realistic and appropriate to generate a number of "good" layouts that meet several criteria laid down by the facility designer and let decision makers choose between them based on the current requirement. Presumably, the most comprehensive way to take all these features into consideration in the selection process is to personally involve the decision maker(s) in the selection process, which is the procedure adopted in the Interactive Genetic Algorithms (Brintup, Takagi, Tiwari, & Ramsden, 2006) which have been recently applied to FLP (Hernandez, Morera, & Azofra, 2011). Such procedure, however, may expose the decision maker to a time consuming activity, and may result unpractical in many contexts, where a structured and transparent decision making is required. In such cases a fully automated procedure is preferred to select at least a set of best solution candidates, thus allowing the decision maker to evaluate a limited number of alternatives. For such purpose the different objectives are frequently combined into a single one by means of some aggregation procedures such as in the weighted sum method. The drawbacks of these methodologies are well documented in the multi-objective decision theory, as well as the benefits of a "true" multi-objective exploration of the solution space, resulting from a Pareto based approach. Pareto approaches (Goldberg, 1989) involve the evolution of the Pareto front constituted by the fitness of a generic individual corresponding to each optimality criterion considered. It has been recognized the GAs belonging to this class generally outperform the non-Pareto Based approaches (Tamaki, Kita, & Kobayashi, 1996; Zitzler & Thiele 1999). The methodology here proposed refers to the class of Pareto-based and is developed according to the framework of non-dominated sorting GA (NSGA) proposed by Srinivas and Deb (1995). More specifically, in this paper we propose a novel Multi Objective Genetic Algorithm (MOGA) to solve the facility layout problem considering four separate objectives based on an advanced encoding structure in order to ensure an efficient exploration of the search space. The objectives considered are commonly employed in the literature (Aiello, La Scalia, & Enea, 2012; Harmonosky & Tothero, 1992; Meller & Gau, 1996; Srinivas & Deb 1995), namely the minimization of the total Material Handling Cost the distance and the closeness requirements among the departments, and the desired aspect ratio. Additionally, the presence of feasibility constraints, required to ensure the practicability of the solution determined, may significantly hamper the convergence of the algorithm, which consequently requires a solid and efficient structure. In particular, it is well known that the very basic and most crucial component of a GA is related to the solution representation (i.e. the chromosome encoding scheme), as it significantly affects the overall performance of the algorithm and the quality of the solutions obtained (Datta, Amaral, & Figueira, 2011). In order to be implemented in a genetic algorithm, a layout representation scheme must be encoded into a string form, suitable for being employed within the common genetic operators such as mutation and crossover. The simplifications introduced in the layout representation in order to cope with these requirements, and to ensure that a chromosome can be easily decoded to a unique layout scheme, generally restrict the flexibility of the representation, thus limiting the feasible search space. The two general mechanisms reported in the literature for constructing such layouts are the flexible bay structure (FBS) developed by Goetschalckx (1992), and the more recent slicing tree structure (Arapoglu, Norman, & Smith, 2001; Moghaddam & Shayan, 1998). The slicing structure results from dividing an initial rectangle either in horizontal or vertical direction completely from one side to the other (guillotine cut) and recursively going on with the newly generated rectangles (Scholz, Jaehn, & Junker 2010). The Multi Objective Genetic Algorithm (MOGA) here proposed is hence based on a slicing tree encoding in order to ensure an efficient convergence towards the Pareto frontier, outperforming the current referenced approaches. Finally, the best block layout is determined by employing the well known multi-criteria decision-making procedure Electre. The remainder of this paper is organized as follows. Section 2 describes the genetic algorithm implemented in this study for the facility layout problem and in particular the ranking procedure adopted. To show performance of the suggested algorithm, comparative experiments are done in Section 3. In Section 4 the best solution is determined by means the Electre method and Section 5 concludes the paper with a short summary of the results obtained.

2. Genetic Algorithm

A lot of optimal and heuristic algorithms for solving FLPs have been developed in the past few decades. The majority of these approaches adopt a problem formulation known as the quadratic assignment problem (QAP) that is particularly suitable for equal area facilities. The main drawback of these approaches is that geometric constraints, e.g. unequal sizes of facilities, are not taken into account. In such situations, random search algorithms are the only practicable alternative, although they may just lead to a near-optimal solution. In its classical formulation the UA-FLP involves the minimization of the total material handling cost, however the needs of the real world of dealing with several design criteria such as the space utilization, flexibility, employee satisfaction and safety emerged already in the early stages of research (Muther & Boston, 1973). Consequently, to be more realistic, some researchers have considered more than a single objective in their solution approach to the UA-FLP. The presence of multiple objectives in a single optimization problem, however, significantly affects the optimization procedure since, for example, it gives rise not only to a single optimal solution but to a set of optimal solutions (largely known as Pareto-optimal solutions). In the absence of any additional information, each one of these Pareto-optimal solutions cannot be said to outperform any other. Classical optimization methods (including the multi-criteria decision-making methods) suggest converting the multi-objective optimization problem to a single-objective optimization problem thus emphasizing one particular Pareto optimal solution. According to this concept several authors combine the different objectives into a single one for example by means of Analytic Hierarchy Process (AHP) methodology (Harmonosky & Tothero, 1992; Yang & Kuo, 2003) or using a linear combination of the different objectives (Chen & Sha, 2005). Lee, Roh, and Jeong (2005) propose a genetic algorithm (GA) for multi-floor design considering inner walls and passages, using the weighted method approach to minimize the departmental material handling cost and maximizing closeness rating. A similar approach is proposed by Ye and Zhou (2007), who developed a hybrid GA-Tabu search (TS) algorithm. Over the past two decades, more advanced researches have led to the formulation of multi-objective evolutionary algorithms (MOEAs) (Coello et al., 2007; Day, 2005; Deb, 2001), with the objective to find multiple Pareto-optimal solutions in a single run. In fact, since evolutionary algorithms work with a population of solutions, they can be extended to maintain a diverse set of solutions within the same optimization process. As a consequence in the recent years a number of different GAs were suggested to solve multi-objective optimization problems. These approaches resulted in the development of MOGAs with different structures, namely: MOGA-III (Fonseca & Fleming, 1993), SPEA2 (Zitzler, Laumanns, & Thiele, 2001), NSGA-II (Deb, Pratap, Agarwal, & Meyarivan, 2002), NSGA (Srinivas and Deb, 1995), NPGA (Horn, Nafploitis, & Goldberg, 1994), MOMGA (Van Veldhuizen & Lamont, 2000). In this paper, a Non-dominated Ranking Genetic Algorithm (NRGA) Al Jadaan, Rajamani, and Rao (2008) is proposed, involving a random population *P* to be sorted based on the non-domination of individuals. Each individual is assigned a fitness (or rank) equal to its non-domination level (1 is the best level, 2 is the next-best level, and so on). The usual Ranked based Roulette wheel selection, recombination, and mutation operators are used subsequently to create an offspring population. The NRGA developed operates through the following structure: at each iteration the objective functions are evaluated and the population is ranked, subsequently *N/2* couples are selected for crossover, generating two offspring from each couple of parents. The population thus obtained involves *2N* elements, which are ranked according to their dominance level and the first *N* elements are selected to create the new generation. Subsequently the mutation operator is applied with a specific probability, and a clone control routine is employed. The process is iterative until a specific stopping criterion (e.g. the maximum number of iterations) is reached. The scheme of the genetic search process used in this paper is summarized in figure (Fig. 1).

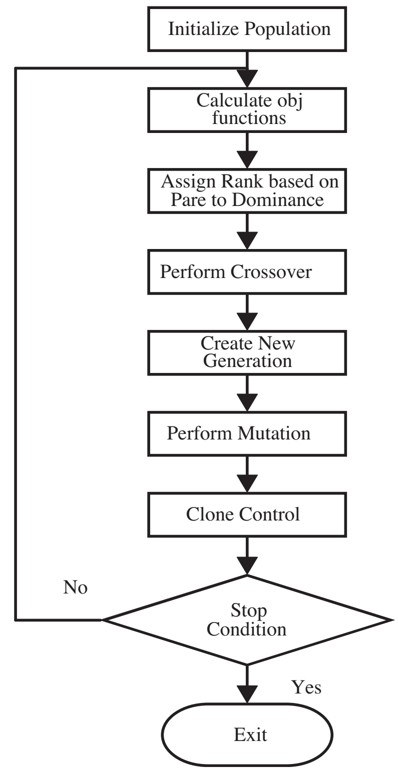

Fig. 1. Flow chart of GA optimization procedure.

2.1. Diversity mechanism and raking procedure

A crucial aspect that drastically affects the convergence of a MOGA is the procedure for the selection of best individuals in the population. Being a Pareto-Based approach, the ranking procedure here employed is referred to the degree of dominance. According to this approach, first non-dominated individuals within the population are identified, they are given the rank 1, and removed from the population. Then, the non-dominated individuals within the reduced population are identified and given the rank 2, followed by their removal from the population. This procedure is repeated until the whole population is ranked The corresponding macro-code is reported in the figure below (Fig. 2).

The least dominated solutions thus determined survive to make the population of the next generation. It must be pointed out that individuals belonging to the non-dominated front cannot be further differentiated (and ranked) unless an additional elitism mechanism is introduced. On one hand this means that the population size must be big enough to involve the whole set of non-dominated solutions, and to maintain the population well differentiated, while on the other hand this suggests the employment of specific operators to maintain a good spread of solutions. For such reason additional elitism mechanism can be introduced, for example in NSGA-II the crowded-comparison approach is used along with the crowded-comparison operator. The algorithm here presented hence includes a distance-based elitist mechanism and a clone control routine which counts the clones in the population and operates a recursive mutation until the mutated element is different from all the others. This is performed until the number of clones is null or lower than a pre-established acceptance threshold.

3. Benchmarking procedure

The facilities layout problem is characterized by an extremely wide solution space, consequently iterative heuristics can inevitably explore a limited number of solutions. Therefore any solution procedure offered so far, is able to cover solution space only partly. Additionally the objectives, the constraints, the data structure and the evolutionary mechanisms vary from one application to the other thus making the benchmarking of the performance a tough issue. In this paper we propose a benchmarking procedure based on the comparison with a previously proposed algorithm, and a performance analysis considering different setup configurations.

```
i =0, rank =0, dom=0; Nranked =0;
while (Nranked<N)
          {Increase rank;
          For each element i in the population
                    {dom=0;
                    For each j element in the population If (j dominates i) then dom =1;
                    If  dom=0
                              {assign rank to element i;
                              increase Nranked;
                              }
                    }
          }
```

Fig. 2. The ranking macrocode.

In order to validate the proposed algorithm, we consider the case from Aromur and Buffa (1963) to undertake experiments and comparisons, and the solutions obtained by Wanga, Hub, and Kub (2005), as benchmark values. Desired departmental areas, product flows between departments and material handling costs are given by the authors and reported below (Tables 1–3).

It must be pointed out that the reference case and the solution proposed are referred to a single objective considering the material handling cost only, with a penalty function on the shape and area ratio. The solutions we determined are on the contrary referred to the multi-object context previously described, with a feasibility constraint on the aspect ratio. The features of the AR functions, the values of the closeness/distance ratings, and the mutation and crossover mechanisms are those previously employed in Aiello et al. (2012), while the most significant genetic parameters are given in the table below (Table 4).

The comparison clearly refers to the material handling cost function only, calculated by using the flow information of unit loads and the manhattan distances between the centroids of the departments. The results, given in Table 5, have been determined in unconstrained conditions initially, and subsequently the AR constraint has been enforced. In both cases the proposed algorithm outperforms the reference results, with 16.5% cost reduction in the first case and 33.6% in the second, and these values are reached in approximately 650 iterations. Computation has been performed employing a general purpose workstation, with a computational

Table 1
Desired departmental areas for the case considered.

Facility	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Area	27	18	27	18	18	18	9	9	9	24	60	42	18	24	27	75	64	41	27	45

Table 2
Departments flows.

Dep	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	120	80	0	0	0	0	0	0	40	80	0	0	80	0	0	0	0	0	0
2	120	0	80	1630	30	0	930	0	80	90	0	0	0	0	0	0	0	0	460	0
3	80	80	0	0	0	130	0	0	210	260	0	0	0	870	0	0	0	0	910	0
4	0	1630	0	0	60	380	500	0	130	0	0	70	0	0	0	0	0	100	1050	0
5	0	30	0	60	0	0	150	90	0	60	0	0	0	0	90	0	0	0	0	0
6	0	0	130	380	0	0	410	0	0	0	0	30	0	0	0	0	0	70	0	0
7	0	930	0	500	150	410	0	1600	0	110	0	0	0	60	0	0	0	110	0	250
8	0	0	0	0	90	0	1600	0	0	0	0	0	40	0	0	0	0	0	500	2230
9	0	80	210	130	0	0	0	0	0	0	0	0	0	500	0	0	500	0	0	0
10	40	90	260	0	60	0	110	0	0	0	30	800	0	1240	160	0	0	0	350	0
11	80	0	0	0	0	0	0	0	0	30	0	150	0	200	80	1500	350	90	0	0
12	0	0	0	70	0	30	0	0	0	800	150	0	0	0	110	0	1000	0	560	0
13	0	0	0	0	0	0	0	40	0	0	0	0	0	500	40	500	0	40	0	0
14	80	0	870	0	0	0	60	0	500	1240	200	0	500	0	650	0	0	60	0	0
15	0	0	0	0	90	0	0	0	0	160	80	110	40	650	0	0	350	0	0	0
16	0	0	0	0	0	0	0	0	0	0	1500	0	500	0	0	0	1000	0	0	0
17	0	0	0	0	0	0	0	0	500	0	350	1000	0	0	350	1000	0	0	500	0
18	0	0	0	100	0	70	110	0	0	0	90	0	0	60	0	0	0	0	310	0
19	0	460	910	1050	0	0	0	500	0	350	0	560	0	0	0	0	500	310	0	0
20	0	0	0	0	0	0	250	2230	0	0	0	0	0	0	0	0	0	0	0	0

Table 3
Departments material handling cost.

Dep	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0.015	0.015	0	0	0	0	0	0	0.026	0.014	0	0	0.015	0	0	0	0	0	0
2	0.015	0	0.012	0.015	0.026	0	0.015	0	0.015	0.015	0	0	0	0	0	0	0	0	0.015	0
3	0.015	0.012	0	0	0	0.017	0	0	0.015	0.015	0	0	0	0.015	0	0	0	0	0.015	0
4	0	0.015	0	0	0.018	0.015	0.015	0	0.018	0	0	0.020	0	0	0	0	0	0.015	0.015	0
5	0	0.026	0	0.018	0	0	0.015	0.015	0	0.026	0	0	0	0	0.015	0	0	0	0	0
6	0	0	0.017	0.015	0	0	0.015	0	0	0	0	0.015	0	0	0	0	0	0.015	0	0
7	0	0.015	0	0.015	0.015	0.015	0	0.015	0	0.017	0	0	0	0.016	0	0	0	0.015	0	0.015
8	0	0	0	0	0.015	0	0.015	0	0	0	0	0	0.015	0	0	0	0	0	0.015	0.015
9	0	0.015	0.015	0.018	0	0	0	0	0	0	0	0	0	0.015	0	0	0.015	0	0	0
10	0.026	0.015	0.015	0	0.026	0	0.017	0	0	0	0.012	0.015	0	0.015	0.012	0	0	0	0.015	0
11	0.014	0	0	0	0	0	0	0	0	0.012	0	0.015	0	0.015	0.012	0.015	0	0	0.015	0
12	0	0	0	0.020	0	0.015	0	0	0	0.015	0.015	0	0	0	0.015	0	0.015	0	0.015	0
13	0	0	0	0	0	0	0	0.015	0	0	0	0	0	0.016	0.026	0.012	0	0	0	0
14	0.015	0	0.015	0	0	0	0.016	0	0.015	0.015	0.015	0	0.016	0	0.015	0	0	0.015	0	0
15	0	0	0	0	0.015	0	0	0	0	0.012	0.012	0.015	0.026	0.015	0	0	0.015	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0.012	0	0	0	0.012	0	0	0
17	0	0	0	0	0	0	0	0	0.015	0	0	0.015	0	0	0.015	0.012	0	0	0.015	0
18	0	0	0	0.015	0	0.015	0.015	0	0	0	0	0	0	0.015	0	0	0	0	0.015	0
19	0	0.015	0.015	0.015	0	0	0	0.015	0	0.015	0.015	0.015	0	0	0	0	0.015	0.015	0	0
20	0	0	0	0	0	0	0.015	0.015	0	0	0	0	0	0	0	0	0	0	0	0

Table 4
Comparisons of the results.

Most significant genetic parameters	
Mutation probability	40%
Population size	50
Number of generations	2500
Number of clones	2

Table 5
Comparisons of the results.

	Wanga et al. (2005)	Proposed GA	Proposed GA (constrained)
Material handling cost (MHC)	5926.6	3092.34 (−47.82%)	3900 (−33.6%)
Aspect ratio (AR)	–	–	0.74
Closeness request	–	1077.26	1336
Distance request	–	15.29	13.1

time less than one minute (the determination of a reliable value for the computational time would require specific machine-time analysis which is outside the scope of this paper).

The block layouts of the optimal solutions generated by means the proposed algorithm are reported in the figures below. The first layout (Fig. 3(a)) refers to the case in which only the total material cost is considered and it shows the presence of three departments, 7, 8 and 9 with a bad aspect ratio. On the other hand, adding the objective function related to the aspect ratio (Fig. 3(b)) the corresponding layout has changed substantially and the MHC, still shows an improvement compared to the reference case.

In Fig. 3, the departments having an adjacency and distance requests are highlighted in light grey and dark grey areas, respectively.

Additionally, the evolution of Pareto-front obtained taking into account the objective functions related to the material handling cost and to the aspect ratio has been determined. In this case, the set of all the non-dominated elements in the population (i.e. the Pareto front) has been extracted in each step, and four representative cases at different stages of evolution progress (namely 500, 1000, 1500 and 2500 iterations) are reported in Fig. 4. The results show that in the initial steps of the evolution, the Pareto fronts are not clearly defined, and they may even overlap with each other. As the population evolves, however, better solutions are generated and the frontier moves to the upper right corner, in a clear distinguishable way.

4. Selection with electre

Electre is a multi-criteria decision-making procedure that can be applied when a set of alternatives must be ranked according

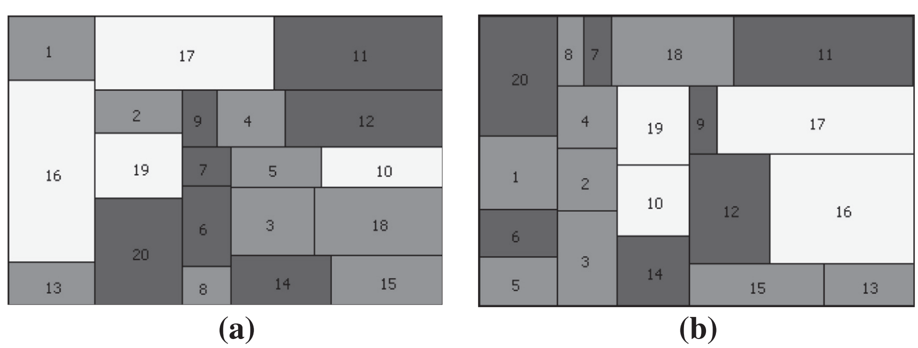

Fig. 3. Block layouts corresponding to the optimal solutions in constrained (a) and un-constrained (b) conditions.

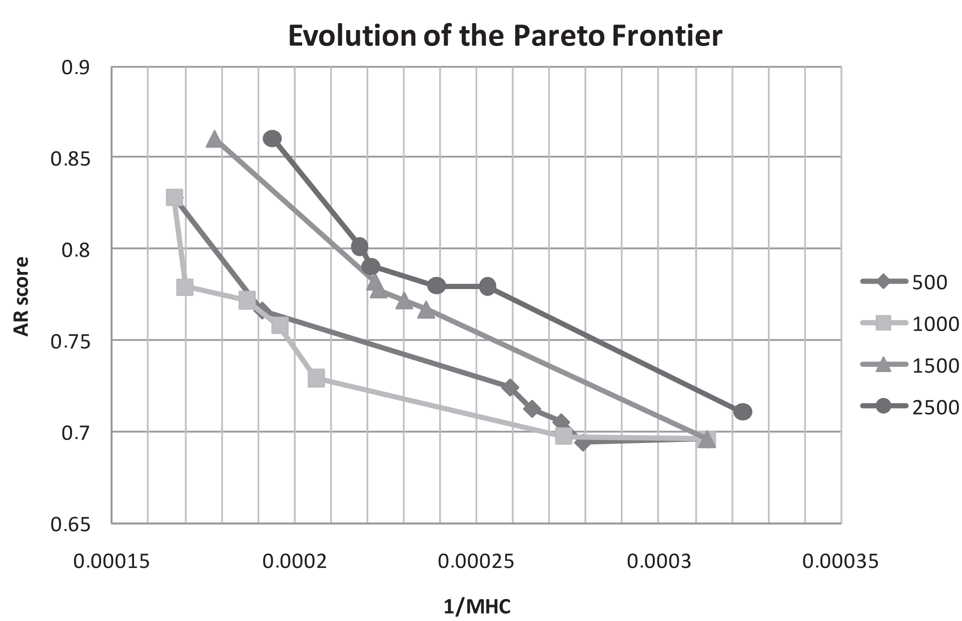

Fig. 4. Evolution progress of the Pareto front.

to a set of criteria reflecting the decision maker's preferences. Relationships between alternatives and criteria are described using attributes referred to the aspects of alternatives that are relevant according to the established criteria. In multi-criteria decision problems, although logical and mathematical conditions required to determine an optimum do not exist, a solution representing a good compromise according to the conflicting criteria established can be individuated. Electre method is based upon pseudo-criteria. A pseudo-criterion allows, by using proper thresholds, to take into account the uncertainty and ambiguity that can affect the evaluation of the performance, so that, if the difference in the performance of two alternatives is minimal, according to a certain criterion, such alternatives can be considered indifferent according to that criterion. Another peculiarity which differentiates Electre from other methodologies is that it is not compensative, which means that a very bad score in one objective function is not compensated by good scores in other objectives. In other words, the decision maker will not choose an alternative if it is very bad compared to another one, even on a single criterion. This occurs if the difference between the values of an attribute of two alternatives is greater than a fixed veto threshold. Electre is based upon outranking relations: an alternative a outranks another alternative b if sufficient reasons exist to assert that a is as good as b and good reasons to reject such assertion do not exist. Outranking is therefore based upon concordance/discordance principle, which consists in the verification of the existence of a concordance of criteria in favor of the assertion that an alternative is as good as another one, and upon the verification that strong discordance among the score values that may reject the previous assertion does not exist. For each criterion, the following thresholds are introduced:

q_j – indifference threshold,
p_j – preference threshold,
v_j – veto threshold,

where $q_j \leqslant p_j \leqslant v_j$.

By means of these thresholds, the following six preference relations between alternatives *a* and *b* may be established, referring to the values $g_j(a)$ and $g_j(b)$ of the attribute *j*:

(1) $(a\ I\ b)_j$ – a is indifferent to *b* with respect to the criterion *j* if $|g_j(a) - g_i(b)| \leqslant q_j$;
(2) $(a\ WP\ b)_j$ – a is weakly preferred to *b* with respect to the criterion *j* if;$q_j \leqslant g_j(a) - g_j(b) \leqslant p_j$;
(3) $(a\ SP\ b)_j$ – *a* is strongly preferred to b with respect to the criterion *j* if $g_j(a) - g_j(b) \geqslant p_j$;
(4) $(a\ NR\ b)_j$ – the assertion that a outranks b cannot be refused with respect to the criterion *j* if$g_j(b) - g_j(a) \leqslant p_j$;
(5) $(a\ WR\ b)_j$ – the assertion that a outranks b is weakly refused with respect to the criterion *j* if $p_j < g_j(b) - g_j(a) \leqslant v_j$;
(6) $(a\ SR\ b)_j$ – the assertion that a outranks b is strongly refused with respect to the criterion *j* if $g_j(b) - g_j(a) > v_{jj}$.

For each criterion, thresholds (q_j, p_j and v_j) can either be fixed values or functions of the performance, according to the expression

$$S_j(a) = \alpha g_i(a) + \beta p_j \tag{1}$$

The relations related to the minimization of the material handling cost and the maximization of the adjacency and the distance between the departments are named "concordance" relations and are used to evaluate the reasons favorable to the assertion that alternative a outranks alternative *b*, according to criterion *j*. Expressions related to the aspect ratio are named "discordance" expressions and are used to measure the strong reasons that lead to reject the assertion that a outranks *b* with respect to criterion *j*. Concordance, indicated by $c_j(a,b)$, is equal to 1 if $g_j(a)$ is greater than $g_j(b)$ or, in any case, expression (1) is verified, is equal to 0 if $g_j(b) - g_j(a) \geqslant p_j$;, while it is evaluated by the following equation when $q_j \leqslant g_j(b) - g_j(a) \leqslant p_j$:

$$C_j(a,b) = \frac{p_j + g_j(a) - g_j(b)}{p_j - q_j} \tag{2}$$

Discordance, indicated with $d_j(a,b)$, is 0 when expression related to the aspect ratio is verified, 1 when expression aspect ratio = $\prod ars_i$ is verified (ars_i, represents the aspect ratio satisfaction function) while it is expressed by the following equation when expression *aspect ratio* = *min*(ars_i) is verified:

$$d_j(a,b) = \frac{g_j(b) - g_j(a) - p_j}{v_j - p_j} \tag{3}$$

For each pair of alternatives *a* and *b*, the values of concordance $c_j(a,b)$ with respect to each criterion *j*, are aggregated in the global concordance matrix, by means of a weight k_j assigned to each criterion. The generic element of such a matrix is expressed by:

$$C(a,b) = \sum_j k_j c_j(a,b) \tag{4}$$

A further step consists in the definition of the credibility of "a outranks b", that summarizes the information expressed by concordance and discordance:

$$S(a,b) = \begin{cases} C(a,b) \\ \textit{If } d_j(a,b) \leqslant c(a,b)\ \forall j \\ C(a,b) \prod\limits_{\forall j | d_j(a,b) > c(a,b)} \frac{1-dj(a,b)}{1-c(a,b)} \end{cases} \tag{5}$$

The next step of the method is the so-called descending distillation: on the basis of the credibility parameter, the alternatives are ranked in descending order. A further threshold is considered:

$$\lambda = \max_{\forall a,b} Q_j \tag{6}$$

A credibility level λ′, less but close to λ, is established so that the interval (λ–λ′) can be considered as an indifference interval of credibility. A Boolean matrix is then calculated as follows:

$$B(a,b) = \begin{cases} 1 \forall a,b | s(a,b) > & \lambda' \\ 0 & \textit{otherwise} \end{cases} \tag{7}$$

Finally, for each alternative *i*, the difference *Q*(*i*) between the number of alternatives *j* that are outranked by alternative *i* at level λ′ or higher (i.e. the alternatives *j* having *B*(*i*,*j*) = 1) and the number of alternatives *k* that outrank the alternative *i*, at level λ′ or higher (i.e. the alternatives *k* having *B*(*k*,*i*) = 1), is calculated. The first distillates are the alternatives *i* having

$$Q(i) = \max_{\forall a,b} Q_j. \tag{8}$$

If the set containing all the alternatives, for which the previous relation is verified, has a cardinality higher than 1, the described procedure is applied recursively until the set contains only one alternative or a group of alternatives that cannot be differentiated further. In this last case, an ascending distillation can be applied, ranking the alternatives in ascending order. This new ranking, coupled with that obtained by descending distillation, leads to a unique final ranking. Among the different versions of the Electre method, Electre III (Roy, 1996.) has been employed.

4.1. Electre parameters

The values of the thresholds required by Electre method have been calculated as percentage of the value of the respective

Table 6
The best solution obtained by means Electre III.

Solution	Chromosome	Aspect ratio	Handling cost	Adjacency	Distance
1	0-0-0-0-0-15-8-7-6-4-1-20-11-19-2-16-9-5-14-17-10-12-3-18-13	0.71	5128	150	1227
2	0-0-0-0-0-1-11-20-15-8-7-6-19-9-4-2-17-16-12-5-3-14-10-18-13	0.69	6211	91.5	2218

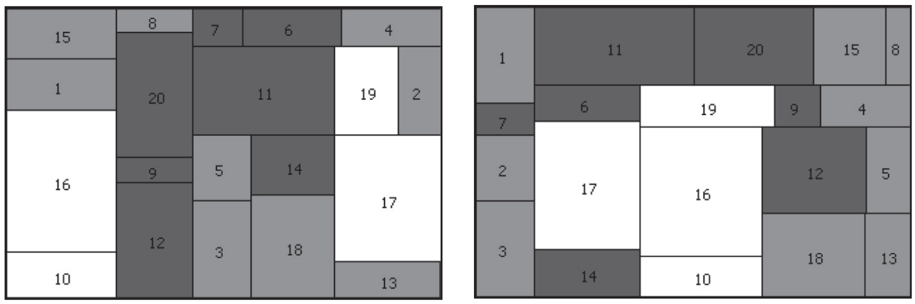

Fig. 5. Block layout corresponding to the optimal solutions.

objective function. In particular, a difference less or equal than 15% has been considered indifferent, while preference and veto thresholds have been fixed equal to 30% and 50%, respectively. The weights of the objectives are assumed all equal to 1.

4.2. Results

The feasible and non-dominated solutions with rank 1, obtained by the genetic algorithm, are in number of 6 on a population of 50 individuals. These solutions constitute the Electre input, which, by ascending and descending distillation procedure, has given the best solution. The structure of the solution obtained is reported in Table 6 and the corresponding layouts are shown in Fig. 5. The departments having an adjacency and distance requests are highlighted in the same grey scale used in Fig. 3.

5. Conclusions

The unequal area FLP has been an emerging topic in the recent years. A large volume of current research in unequal area FLPs has been conducted to satisfy both quantitative and qualitative aspects in the layout. In particular the topic of the Multi Objective optimization problems approached by Genetic Algorithms is nowadays one of the most promising and investigated research field. The proposed approach is capable of finding in a first phase a set of Pareto-optimal layouts that optimizes the objective functions simultaneously throughout the entire evolutionary process, giving the decision maker a restricted number of solutions among which he can chose those that he considers the best. This choice is simplified by the employment of a structured multi-criteria decision procedure. In this phase only, the decision maker must provide further information on the problem also on the basis of the results obtained in the previous step. The proposed approach falls within the search before multi-critera decision category. Such methodology is generally preferable compared with traditional multi-objective optimization algorithms named multi-criteria decision-making before search, which rely upon the establishment of a normalized weight vector.

The benefits of the proposed method have emerged in the comparison with referenced results. Further improvements of the proposed methodology will include the development of a more comprehensive procedure to approach the decision process, including the aspects related to the intrinsic uncertainty and referring to the typical methodologies of the approximate reasoning, such as the fuzzy theory.

References

Aiello, G., La Scalia, M., & Enea, A. (2012). Multi Objective Genetic Algorithm for the facility layout problem based upon slicing structure encoding. *Expert Systems with Applications, 39*(12), 10352–10358.

Al Jadaan, O., Rajamani, L., & Rao, R. (2008), Non-dominated ranked genetic algorithm for solving multi-objective optimization problems: Nrga, *Journal of Theoretical and Applied Information Technology*, 4(1), 60–67.

Arapoglu, R. A., Norman, B. A., & Smith, A. E. (2001). Locating input and output points in facilities design – A comparison of constructive, evolutionary, and exact methods. *IEEE Transactions on Evolutionary Computation, 3*, 192–203.

Aromur, G. C., & Buffa, E. S. (1963). A heuristic algorithm and simulation approach to the relative location of facilities. *Management Science, 9*, 294–309.

Brintup, A. M., Takagi, H., Tiwari, A., & Ramsden, J. (2006). Evaluation of sequential, multi-objective, and parallel interactive genetic algorithms for multi-objective optimization problems. *Journal of Biological Physics and Chemistry, 6*, 137–146.

Chen, C. W., & Sha, D. Y. (2005). Heuristic approach for solving the multiobjective facility layout problem. *International Journal of Production Research, 43*(21), 4493–4507.

Coello C. A., Lamont G. B., & Van Veldhuizen D.A. (2007). Evolutionary algorithms for solving multi-objective problems, 2nd ed., Springer.

Datta, D., Amaral, A. R. S., & Figueira, J. R. (2011). Single row facility location problem using a permutation-based genetic algorithm. *European Journal of Operational Research, 213*, 388–394.

Day R. O. (2005). Explicit building block multiobjective evolutionary computation: methods and application. PhD thesis, Air Force Institute of Technology, USA, June.

Deb, K. (2001). *Multiobjective optimization using evolutionary algorithms*. Chichester, UK: Wiley.

Fonseca C. M. & Fleming P. J. (1993). Genetic algorithms for multiobjective optimization: Formulation, discussion and generalization. In *Fifth international conference on genetic algorithms* (pp. 416–423) San Mateo, CA.

Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation, V6*(2), 182–197.

Glover, F., & Greenberg, H. J. (1989). New approaches for heuristic search: A bilateral linkage with artificial intelligence. *European Journal of Operational Research, 39*, 119–130.

Goetschalckx, M. (1992). An interactive layout heuristic based on hexagonal adjacency graphs. *European Journal of Operational Research, 63*, 304–321.

Goldberg, D. E. (1989). *Genetic algorithms. Search, optimization and machine learning*. Addison-Wesley.

Harmonosky, C. M., & Tothero, G. K. (1992). A multi-factor plant layout methodology. *International Journal of Production Research, 30*(8), 1773–1789.

Heragu, S. S., & Alfa, A. S. (1992). Experimental analysis of simulated annealing based algorithms for the layout problem. *European Journal of Operational Research, 57*, 190–202.

Hernandez, L. G., Morera, L. S., & Azofra, A. A. (2011). An interactive genetic algorithm for the unequal area facility layout problem. *Advances in Intelligent and Soft Computing., 87*, 253–262.

Holland, J. H. (1975). *Adaption in natural and artificial system*. Ann Arbor, MI: University of Michigan Press.
Horn, J., Nafploitis, N., & Goldberg, D. E. (1994). A niched Pareto genetic algorithm for multiobjective optimization. In *First IEEE conference on evolutionary computation* (pp. 82–87) IEEE Press.
Lee, K. Y., Roh, M. I., & Jeong, H. S. (2005). An improved genetic algorithm for multi-floor facility layout problems having inner structure walls and passages. *Computers & Operations Research, 32*(4), 879–899.
Meller, R. D., & Gau, K. Y. (1996). Facility layout objective function and robust layouts. *International Journal of Production Research, 34*(10), 2727–2742. International Journal of Production Research, 36, 1549–1569.
Michalewicz, M. (1992). *Genetic algorithm + data structure = evolution program*. Berlin, Heidelberg, New York: Springer.
Moghaddam, R. T., & Shayan, E. (1998). Facility layout design by genetic algorithm. *International Journal of Computers and Industrial Engineering, 35*(3–4), 527–530.
Muther, R. (1973). *Systematic layout planning*. Boston: Cahners Books.
Nordin, N. N., Zainuddin, Z. M., Salim, S., & Ponnusamy, R. R. (2009). Mathematical modeling and hybrid heuristic for unequal size facility layout problem. *Journal of Fundamental Sciences, 5*(1), 87–89.
Roy, B. (1996). *Multicriteria methodology for decision aiding*. Dordrecht, The Netherlands: Kluwer.
Scholz, D., Jaehn, F., & Junker, A. (2010). Extensions to STaTS for practical applications of the facility layout problem. *European Journal of Operational Research, 204*, 463–472.
Sirinaovakul, B., & Thajchayapong, P. (1994). A knowledge base to assist a heuristic search approach to facility layout. *International Journal of Production Research, 32*, 141–160.
Srinivas, N., & Deb, K. (1995). Multiobjective optimization using nondominated sorting in genetic algorithms. *Evolutionary Computation, 2*, 221–248.
Tamaki, H., Kita, H., & Kobayashi, S. (1996). Multiobjective optimization by genetic algorithms: A review. In *IEEE international conference on evolutionary computation ICEC'96*, Nagoya, Japan.
Tompkins, J. A., White, J. A., Bozer, Y. A., Frazelle, E. H., Tanchoco, J. M. A., & Trevino, J. (1996). *Facilities planning*. Wiley.
Tuzkaya, U., & Ertay, T. (2004). An application of fuzzy ahp/dea methodology for the facility layout design in the presence of both quantitative and qualitative data. In *Applied computational intelligence – Proceedings of the 6th international FLINS conference* (pp. 507–512).
Van Veldhuizen, D. A. & Lamont, G. B. (2000). Multiobjective optimization with messy genetic algorithms. In *Proceedings of the 2000 Symposium on Applied Computing* (pp. 470–476) ACM.
Wanga, M. J., Hub, M. H., & Kub, M. Y. (2005). A solution to the unequal area facilities layout problem by genetic algorithm. *Computers in Industry, 56*, 207–220.
Yang, T., & Kuo, C. (2003). A hierarchical AHP/DEA methodology for the facilities layout design problem. *European Journal of Operational Research, 147*, 128–136.
Ye, M., & Zhou, G. (2007). A local genetic approach to multiobjective, facility layout problems with fixed aisles. *International Journal of Production Research, 45*, 5243–5264.
Zitzler, E., Laumanns, M., Thiele, L. (2001): SPEA2: Improving the Strength Pareto Evolutionary Algorithm for Multiobjective Optimization. In K.C. Giannakoglou and others, eds, Evolutionary Methods for Design, Optimisation and Control with Application to Industrial Problems (EUROGEN 2001), (pp 95–100) 2002. International Center for Numerical Methods in Engineering (CIMNE).
Zitzler, E., & Thiele, L. (1999). Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach. *IEEE Transaction on Evolutionary Computation, 3*, 257–271.