On the output and welfare effects of a non-profit firm in a mixed duopoly: A generalization

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\begin{abstract}
We study the output and welfare impacts of a non-profit firm in a mixed duopoly. In particular, we show that technical efficiency at the margin is crucial to determine whether the social responsibility of the non-profit firm increases or reduces welfare, assuming general demand and cost functions. This implies the paradoxical result that more social responsibility may reduce welfare. In addition, we introduce the concept of technical advantage in production and apply it to the study of a mixed duopoly considering convex-quadratic cost functions. Interestingly, a firm may have a technical advantage in production and at the same time be technically less efficient than its rival at the margin. We show that the paradox eventually occurs as the non-profit firm exhibits more social responsibility if firms have quadratic cost functions. This can happen even if the non-profit firm has a substantial technical advantage over its rival.
\end{abstract}

1. Introduction

There is a recent economic literature that analyzes the interaction between private for-profit firms (FPF) and private non-profit organizations (NPO) in mixed oligopoly markets. Part of this literature tries to explain how firms decide to be more profit or consumer oriented. Some examples of this type of work are Goering (2007), Königstein and Muller (2001), Kopel and Brand (2012) and Kopel et al. (2014). Another part of the literature is devoted to studying the effects of the NPO’s concern for consumers on output or welfare. This part of the literature includes the work of Goering (2008), Lien (2002) and Nakamura (2013).

A closely related literature addresses the role of corporate social responsibility in regard to several important issues. For example, Wang et al. (2012) as well as Chang et al. (2014) develop duopoly models to study strategic international trade policy with firms that exhibit concern for consumers. Similarly, Manasakis et al. (2013) and Liu et al. (2015) model the effects of certification, while Manasakis et al. (2014) study corporate governance in imperfectly competitive markets where firms’ social responsibility is related to environmental efforts.

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A standard assumption in the literature is that FPFs maximize profits while NPOs maximize a weighted sum of own profits and consumer surplus (CS). The particular weight that an NPO places on CS represents the degree of social responsibility (or concern for consumers) of this firm. This weight is usually constrained not to exceed the weight that the firm places on own profits. Hence, the extreme cases occur when (a) the weight of CS approaches zero and, consequently, the NPO behaves almost like an FPF and (b) when the NPO places equal weight on profits and CS.

An interesting and paradoxical result in Goering (2008) is that welfare may decrease as the NPO exhibits a higher degree of social responsibility. More precisely, Goering (2008) shows that technical efficiency in production is crucial to determine whether a higher degree of social responsibility of the NPO increases or reduces welfare. If the NPO is technically more efficient than the FPF, then a higher degree of social concern increases welfare. However, if the opposite occurs, then a higher degree of social concern may reduce welfare.

The intuition behind this result is simple. The output of the NPO increases as it cares more for consumers, while the output of the FPF decreases in response to the behavior of the NPO. However, total output increases (Lien, 2002). This has two effects on welfare. On the one hand, the expansion of output itself increases welfare. On the other hand, part of the additional output of the NPO replaces the output of the FPF. If the NPO is technically less (or more) efficient than the FPF, output replacement reduces (or increases, respectively) welfare.

Although the results of Goering (2008) and Lien (2002) are very intuitive, their analysis is limited to linear demand and constant marginal costs. Moreover, most of the articles in the related literature assume that demand is linear and marginal costs are constant (Goering, 2007; Königstein and Muller, 2001; Kopel and Brand, 2012; Kopel et al., 2014; Lambertini and Tampieri, 2015; Nakamura, 2013). Therefore, it is interesting to show – as we do in this article – that the results of Lien (2002) on output and Goering (2008) on welfare can be extended to more general demand and cost functions.

In this article, we show that technical efficiency at the margin is crucial to determine whether the social responsibility of the NPO increases or reduces welfare in the context of relatively general demand and cost functions. In addition, we introduce the concept of technical advantage in production and apply it to the study of a mixed duopoly with convex cost functions. We say that a firm has a technical advantage over its rival if it can produce the same output that its rival produces at lower marginal and total costs than the rival. Interestingly, a firm may have a technical advantage over the other but be technically less efficient at the margin. Finally, we show that the paradoxical result that more social responsibility reduces welfare eventually occurs as the NPO cares more for consumers if firms have quadratic cost functions. This can happen even if the non-profit firm has a substantial technical advantage over its rival.

2. Model

As in most of the related literature, assume that there are two firms in a mixed market (Chang et al., 2014; Goering, 2007; Kopel and Brand, 2012; Lien, 2002; Nakamura, 2013, 2014; Wang and Wang, 2009; Wang et al., 2012). One of the firms is an NPO and the other an FPF. Firms sell their output \( q_N \geq 0 \) and \( q_f \geq 0 \), respectively, at the market clearing price \( p(Q) \), where \( Q = q_N + q_f \). Suppose that the cost function of firm \( f(=N \text{ or } F) \) is \( c_f(q_f) \). This framework is essentially the same that Goering (2008) and Lien (2002) use to obtain the results that we are discussing. However, they consider linear demand and cost functions.

Instead of specifying the form of demand and cost functions, we will assume that each of them is twice differentiable and satisfies a couple of relatively standard properties. On the one hand, the inverse of the demand function has a negative slope and is not very concave or convex. That is,

\[
p' < 0 \tag{A1}
\]

and

\[
|p''| < -p' \cdot \min \left\{ \frac{1}{q_f}, \frac{1}{q_N} \right\} \tag{A2}
\]

On the other hand, the cost function of firm \( f(=N \text{ or } F) \) increases with output and is either linear or convex. That is,

\[
c'_f > 0 \tag{A3}
\]

and

\[
c''_f \geq 0 \tag{A4}
\]

It should be clear that these conditions are satisfied by many demand and cost functions that are used in economic analysis.

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1 There is an older literature that studies mixed oligopolies in which private and public firms interact (Cremer et al., 1991; De Fraja and Delbono, De Fraja and Delbono, 1989; Harris and Wiens, 1980; Matsumura, 1998). In this literature, public firms are assumed to maximize welfare.
In general, we can start assuming that firm \( i (=N \text{ or } F) \) chooses output \( q_i \geq 0 \) to maximize profits plus a fraction of CS. That is,

\[
p(Q) \cdot q_i - c_i(q_i) + \theta_i \cdot CS(p(Q)) \tag{1}
\]

As in most of the literature, parameter \( \theta_i \in [0, 1] \) is the weight of consumers’ surplus in the objective function of firm \( i \). That is, \( \theta_i \) measures the degree of social responsibility (or concern for consumers) of this firm. The first order condition of the firm is then

\[
p + q_i \cdot p' - c'_i + \theta_i \cdot CS' \cdot p' = 0 \tag{2}
\]

Given that one firm is an NPO and the other an FPF, hereafter we can write \( \theta_N = \theta \) and \( \theta_F = 0 \). In addition, we can use the fact that \( CS' = -Q \) to write the first order conditions of the two firms, respectively, as follows:

\[
p + q_Np' - c'_N - \theta Q p' = 0 \tag{3}
\]

and

\[
p + q_Fp' - c'_F = 0 \tag{4}
\]

Suppose now that we have an interior solution. That is, let the pair \( q_N(\theta) > 0 \) and \( q_F(\theta) > 0 \) solve Eqs. (3) and (4) simultaneously. This pair is the Cournot-Nash equilibrium of the game. Note that \( (A1), (A2) \) and \( (A4) \) imply that the second order conditions for a maximum are satisfied (that is, \( S_N = 2p' + q_Np'' - c'_N - \theta Q p' + p' < 0 \) and \( S_F = 2p' + q_Fp'' - c'_F < 0 ) \).

3. Analysis

3.1. Output

In order to evaluate the effect of a small change in the degree of social responsibility of the NPO over firms’ output, implicitly differentiate (3) and (4) with respect to \( \theta \). This leads to the following system of equations.

\[
S_N \cdot q'_N + (p' + q_Np'' - \theta (Qp'' + p')) \cdot q'_F = p'Q \tag{5}
\]

and

\[
(p' + q_Fp'') \cdot q'_N + S_F \cdot q'_F = 0 \tag{6}
\]

Let \( D \) denote the determinant of the system. It is not difficult to see that \( (A1), (A2) \) and \( (A4) \) imply that \( D = (p' + q_Fp'')(p' - c'_N) + (p' - c'_F)S_N > 0 \). Now we can solve the system of equations and use assumptions \( (A1), (A2) \) and \( (A4) \) to obtain the following expressions:

\[
q'_N = \frac{p'Q S_F}{D} > 0 \tag{7}
\]

and

\[
q'_F = -\frac{p' Q (p' + q_Fp'')}{D} < 0 \tag{8}
\]

Finally, we can use (7) and (8) to calculate the effect of a marginal change in the degree of social concern on total output

\[
Q' = q'_N + q'_F = \frac{p'Q(p' - c'_F)}{D} > 0 \tag{9}
\]

Expressions (7–9), summarize the effects of social concern on output. Lien (2002) obtains exactly the same results but assumes that demand is linear and marginal costs are constant. Although these results are very intuitive, it is important to emphasize that we are showing that they hold for fairly general demand and cost functions. Social concern has opposite effects on the output of the NPO and the FFP. On the one hand, the NPO increases output as it cares more for consumers, while on the other hand the FFP reduces output in response to the behavior of the NPO. However, total output increases because the effect of social concern on the NPO’s output is larger than its effect on the FPF’s output.

These results are equivalent to the ones obtained by Matsumura (1998) in a slightly different context. Matsumura (1998) models competition between an FPF and a partially privatized firm considering general demand and cost functions. The partially privatized firm and the shares owned by the government in this firm are analogous to the NPO and social concern, respectively. Among other things, Matsumura (1998) assumes that the slopes of the reaction functions of the firms satisfy

\[\text{See the detailed proofs in Appendix A.}\]

\[\text{The objective functions of the partially privatized firm and the NPO are not equal. The partially privatized firm in Matsumura (1998) maximizes a combination of profits and a weighted average of welfare and consumers’ surplus.}\]
the following constraints: \(-1 < \frac{\partial q_f}{\partial q_N} < 0\) and \(-1 \leq \frac{\partial q_N}{\partial q_f}\).\(^4\) We do not make these assumptions explicitly because they follow from (A1), (A2) and (A4).\(^5\)

The fact that the slope of the reaction function of the FPF is negative and less than one in absolute value is crucial to obtain the output results. The intuition is the following. An increase in social concern shifts out the reaction function of the NPO. That is, the NPO has incentives to produce more for any given production level of its rival. The equilibrium point then moves along the reaction function of the FPF. Therefore, the slope of the reaction function of the FPF determines changes in the equilibrium production levels of both firms. Given that the slope of this function is negative, production of the FPF increases while production of the NPF decreases. Moreover, given that the slope is less than one in absolute value, the change in production of the FPF is smaller than the change in production of the NPO. Hence, total output increases.

3.2. Welfare

Welfare is traditionally defined as the sum of firms’ profits and CS. In this case, welfare is ultimately a function of the degree of social concern of the NPO. Given that social concern affects output, we can write the welfare function as follows:

\[
W(\theta) = CS(p(Q(\theta))) + p(Q(\theta)) \cdot Q(\theta) - \gamma N(q_N(\theta)) - \gamma F(q_f(\theta))
\]  

(10)

In order to establish the effect of social concern on welfare, we can differentiate (10) implicitly with respect to \(\theta\) and find

\[
W' = CS' \cdot p' \cdot Q' + p \cdot Q' - \gamma N'q'_N - \gamma F'q'_F
\]

(11)

Given that \(CS' = -Q\) and \(Q' = q'_N + q'_F\), we can rewrite (11) as follows:

\[
W' = (p - \gamma N') \cdot Q' + (\gamma N' - \gamma F') \cdot q'_F
\]

(12)

Eq. (12) decomposes the welfare impact of a small increase in the degree of social concern of the NPO into two parts. We will call these two parts output expansion and output replacement effects, respectively.\(^6\) The first term at the RHS of (12) is positive and represents additional profits of the NPO due to total output expansion. The second term at the RHS of (12) is ambiguous and represents production efficiency gains or losses that occur because the NPO’s output replaces the FPF’s output. If the NPO is more (or less) efficient than the FPF at the margin, then the output replacement effect tends to increase (or, respectively, reduce) welfare.

According to Goering (2008), many studies argue that FPFs are technically more efficient than NPOs. While this seems to be true for nursing homes (Anderson et al., 1999; Fizel and Nunnikhoven, 1992; Nyman and Bricker, 1989), it is not necessarily true in a more general context. For instance, some studies find that public and not-for-profit hospitals are technically more efficient than private for-profit hospitals (Holingsworth et al., 1999; Ozcan et al., 1992). Moreover, Sloan (2000) explains that the empirical literature comparing the efficiency of profit and non-profit hospitals is not conclusive. Therefore, it seems reasonable to assume that both possibilities are equally likely.

We will now show that the interesting results in Goering (2008) also hold under more general demand and cost functions. That is, we will show that the technical efficiency of the NPO in comparison to the FPF at the margin is crucial in determining whether a small increase in the degree of social concern of the NPO increases or reduces welfare. However, we should make a small clarification. Unlike Goering (2008), we emphasize that the following propositions are only valid at the margin because cost functions may not be linear.

**Proposition 1.** If the NPO is technically more efficient than the FPF at the margin (i.e., \(c_N' \leq c_F'\)), then a small increase in the degree of social concern (i.e., \(\theta\)) of the NPO increases welfare.

**Proof.** It follows from \(q_f > 0\), (A1) and (4) that \(p - c_F' > 0\). Hence, \(c_N' \leq c_F'\) implies \(p - c_N' \geq p - c_F' > 0\). Expression (9) then implies that the first term at the RHS of (12) is positive. Expression (8) and \(c_N' \leq c_F'\) imply that the second term at the RHS of (12) is either positive or zero. Hence, we can conclude that \(W' > 0\). Q. E. D. Essentially, this proposition establishes that \(c_N' \leq c_F'\) is a sufficient condition to ensure that the welfare function increases at the margin with social concern. It should be clear then that the welfare function increases monotonically with the NPO’s social concern if the cost functions are linear (that is, if the marginal costs are constant). However, this sufficient condition (i.e., \(c_N' \leq c_F'\)) is not necessarily met for all \(\theta\) in the interval \([0, 1]\) if cost functions are convex. That is, the NPO may be technically more efficient than the FPF (i.e., \(c_N' \leq c_F'\)) at some points, but the FPF may be more efficient than the NPO (i.e., \(c_F' < c_N'\)) at other points.

**Proposition 2.** If the FPF is technically more efficient than the NPO (i.e., \(c_F' < c_N'\)) at the margin, then a small increase in the degree of social concern (i.e., \(\theta\)) of the NPO may reduce social welfare.

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\(^4\) It is not difficult to find that the slopes of the reaction functions are \(\frac{\partial q_f}{\partial q_N} = \frac{-p \cdot q_f}{\gamma N}\) and \(\frac{\partial q_N}{\partial q_f} = \frac{-p \cdot q_f}{\gamma N}\), respectively.

\(^5\) We actually show that \(-1 < \frac{\partial q_f}{\partial q_N} < 0\) and \(-1 < \frac{\partial q_N}{\partial q_f}\). See the detailed proofs in Appendix B.

\(^6\) While Goering (2008) explains the intuition of this result based on the two effects, he does not arrive at an expression that decomposes them explicitly, as (12) does.
Proof. Expression (8) and \( c'_{N} > c'_{F} \) imply that the second term at the RHS of (12) is negative. If \( p - c'_{N} \leq 0 \), then (9) implies that the first term at the RHS of (12) is either negative or zero. In this case, we are done. Therefore, assume that \( p - c'_{N} > 0 \). Expression (9) then implies that the first term at the RHS of (12) is positive. Although the sign of \( W' \) is ambiguous in general, we can pick \( c'_{N} \) sufficiently large to make \( p - c'_{N} \) arbitrarily close to zero. Therefore, we can pick \( c'_{N} \) sufficiently large to make \( W' < 0 \). Q. E. D. The fact that \( c'_{N} > c'_{F} \) is a necessary but not sufficient condition for the welfare function to decrease at the margin with \( \theta \) makes the paradox appear as an extreme result in the context of linear cost functions. However, we will show that the paradox eventually occurs for sufficiently large \( \theta \) in the interval [0, 1] if firms have convex (quadratic) cost functions.

3.3. Linear demand and quadratic cost functions

It is natural to extend the analysis in Goering (2008) to the case of quadratic cost functions. If we make this assumption, one of the firms may have a technical advantage over the other but still be technically less efficient under certain circumstances. In particular, this can happen if the firm that has the technical advantage produces more output than the other. Of course, this is irrelevant if cost functions are linear (that is, if firms have constant marginal costs). If firms have constant marginal costs and one firm is technically more efficient than the other, then this firm will be more efficient than the other regardless of the firms’ output levels.

Following De Fraja and Delbono (1989), Wang and Wang (2009), Wang et al. (2012) and Chang et al. (2014), we will assume that firm \( i = (N, F) \) has a quadratic cost function. In particular, suppose that

\[
c_i(q_i) = \frac{1}{2}k_iq_i^2.
\]

(13)

Parameter \( k_i > 0 \) is a measure of the convexity of the cost function and the efficiency of the firm in a certain sense. We will use this parameter to define the concept of technical advantage. First, note that the marginal cost of firm \( i \) is \( c'_i = k_iq_i \). Therefore, we can say that firm \( i \) has a technical advantage over firm \( j \) if \( k_i < k_j \) because firm \( i \) has lower marginal costs than firm \( j \) for any given level of output (that is, for any \( q_i > q_j > 0 \)). In other words, firm \( i \) can produce the same output as firm \( j \) at lower marginal cost and total cost.

Normalize parameters \( k_N \) and \( k_F \) in order to satisfy \( k_N + k_F = 1 \). Let \( k_N = k \) and \( k_F = 1 - k \). Note that the NPO has a technical advantage over the FFP if \( k < \frac{1}{2} \), none of the firms has a technical advantage over the other if \( k = \frac{1}{2} \) and the FFP has a technical advantage over the NPO if \( k > \frac{1}{2} \). However, even if the NPO has a technical advantage over the FFP because \( k < \frac{1}{2} \), the FFP can be technically more efficient than the NPO at the margin if \( q_i > q_F(1 - k) \).

The game has a closed form solution if the demand function is given by \( p(Q) = a - bQ \) and the cost functions are given by (13). In particular, the Cournot-Nash equilibrium of this game is the pair

\[
q_N(\theta) = \frac{a(1 + b - k + b\theta)}{k(1 - k) + b(2 + 3b) - b\theta(1 + b - k)}
\]

(14)

and

\[
q_F(\theta) = \frac{a(b - k - b\theta)}{k(1 - k) + b(2 + 3b) - b\theta(1 + b - k)}
\]

(15)

It is not difficult to calculate the effect of the NPO’s concern for consumers on output. That, is

\[
q'_{N} = \frac{ab(1 + 2b)(1 - k + 2b)}{(k(1 - k) + b(2 + 3b) - b\theta(1 + b - k))^2} > 0
\]

(16)

\[
q'_{F} = -\frac{ab^2(1 + 2b)}{(k(1 - k) + b(2 + 3b) - b\theta(1 + b - k))^2} < 0
\]

(17)

and

\[
Q' = \frac{ab(1 + 2b)(1 - k + b)}{(k(1 - k) + b(2 + 3b) - b\theta(1 + b - k))^2} > 0
\]

(18)

Note that (16–18) have the expected signs. That is, these signs are consistent with the signs of (7–9), respectively.

Let \( g(\theta) = (1 - k)^2 - \frac{(1 - k)^2 - b(1 - b)}{1 + 2b} \). We can use (3) and (4), as well as (14), (15), (17) and (18), to rewrite (12) as follows

\[
W' = p'(\theta Q - q_N)Q' + p'(Q - 2q_F - \theta Q)q'_F = p'Qq'_F \left(1 - \frac{Q'}{q'_F}\right) + \frac{Q'}{Q} + \left(\frac{Q'}{q'_F} - 2\right)
\]

(19)
We will use Eq. (19) to relate the degree of social responsibility of the NPO (i.e., \( \theta \)) with the shape of the welfare function.

**Proposition 3.** For any given \( b \geq 1 \) and \( 0 < k < 1 \), there exists a unique \( \theta' \in (0, 1) \) such that the welfare function increases with \( \theta \) if \( 0 < \theta < \theta' \), achieves a maximum at \( \theta = \theta' \) and decreases with \( \theta \) if \( \theta' < \theta < 1 \).

**Proof.** Note that \( b \geq 1 \) and \( 0 < k < 1 \) imply that \( g(0) = \frac{(1-k)^2b(4k-3b)}{1+b} > 0 \), \( g'(\theta) = -\frac{k}{1+b} < 0 \) and \( g(1) = \frac{k(k-3b)}{1+b} < 0 \). The fact that \( g(0) > 0 \) and \( g(1) < 0 \) implies the existence of \( \theta' \in (0, 1) \) such that \( g(\theta') = 0 \).

Uniqueness follows from \( g'(\theta) < 0 \). Finally, it follows from (19), \( Q > 0 \) and \( q_F < 0 \) that the sign of \( W' \) equals the sign of \( g(\theta) \). Therefore, \( W' > 0 \) if \( 0 < \theta < \theta' \), \( W' = 0 \) if \( \theta = \theta' \) and \( W' < 0 \) if \( \theta' < \theta < 1 \). Q. E. D.

This proposition says that the welfare function is concave at the critical point if firms have quadratic cost functions and demand is not very sensitive to the price (i.e., \( b \geq 1 \)). Therefore, some degree of social responsibility of the NPO is desirable. However, at some point more concern for consumers reduces welfare. More precisely, regardless of whether the NPO has a technical advantage over the FPF or vice versa, welfare decreases if the NPO’s concern for consumers is sufficiently large.

It turns out that the welfare function is still concave at the critical point if demand is sensitive to price changes (i.e., \( 0 < b < 1 \)). However, the FPF should not have a large technical advantage over the NPO for the welfare function to achieve a maximum at some \( \theta' \in (0, 1) \). In particular, we can define \( k'(b) = 1 + 2b - \sqrt{b + 3b^2} \) as the technical advantage threshold. Note that the function \( k'(b) \) is convex in \( 0 < b < 1 \) and achieves a minimum at \( b = \frac{1}{4} \). Therefore, it is not difficult to see that \( k'(b) \in (\frac{1}{4}, 1) \) as long as \( 0 < b < 1 \).

**Proposition 4.** For any given \( 0 < b < 1 \) and \( 0 < k < k'(b) < 1 \), there exists a unique \( \theta' \in (0, 1) \) such that the welfare function increases with \( \theta \) if \( 0 < \theta < \theta' \), achieves a maximum at \( \theta = \theta' \) and decreases with \( \theta \) if \( \theta' < \theta < 1 \).

**Proof.** Note that \( 0 < b < 1 \) and \( 0 < k < 1 \) still imply that \( g'(\theta) = -\frac{k}{1+b} < 0 \) and \( g(1) = \frac{k(k-3b)}{1+b} < 0 \). However, the sign of \( g(0) = \frac{(1-k)^2b(4k-3b)}{1+b} \) is ambiguous. On the one hand, \( g(0) > 0 \) if \( k \) is relatively small. On the other hand, \( g(0) \) decreases with \( k \) and becomes negative as \( k \) approaches 1. The numerator of \( g(0) \) is zero at \( k'(b) = 1 + 2b - \sqrt{b + 3b^2} \). Therefore, \( g(0) > 0 \) if \( 0 < k < k' \), \( g(0) = 0 \) if \( k = k' \) and \( g(0) < 0 \) if \( k' < k < 1 \). Using the same argument as in the previous proof, if \( 0 < k < k' \) then \( g(0) > 0 \), \( g'(\theta) < 0 \) and \( g(1) < 0 \). This implies the existence of a unique \( \theta' \in (0, 1) \) such that \( g(\theta') = 0 \). Hence, \( W' > 0 \) if \( 0 < \theta < \theta' \), \( W' = 0 \) if \( \theta = \theta' \) and \( W' < 0 \) if \( \theta' < \theta < 1 \) as long as \( 0 < k < k' \). Q.E.D.

Finally, we can show that the welfare function decreases monotonically with \( \theta \) if demand is sensitive to price changes (i.e., \( 0 < b < 1 \)) and the FPF has a large technical advantage over the NPO.

**Proposition 5.** For any given \( 0 < b < 1 \) and \( k^2 < k < 1 \), the welfare function decreases with \( \theta \in [0, 1] \). Therefore, welfare is maximized at \( \theta = 0 \).

**Proof.** Note that \( 0 < b < 1 \) and \( 0 < k < 1 \) imply that \( g'(\theta) = -\frac{k}{1+b} < 0 \) and \( g(1) = \frac{k(k-3b)}{1+b} < 0 \). Although the sign of \( g(0) = \frac{(1-k)^2b(4k-3b)}{1+b} \) is ambiguous in general, \( g(0) < 0 \) if \( k' 

In summary, the paradoxical result that more social responsibility of the NPO reduces welfare at some point eventually occurs as the NPO shows greater concern for consumers if firms have quadratic cost functions. The intuition is straightforward. The NPO increases output as it cares more for consumers. In response to this action, the FPF reduces output. Given that marginal costs increase with output, the marginal cost of the NPO increases, while the marginal cost of the FPF decreases as the NPO exhibits more social responsibility. Eventually, when the marginal cost of the NPO becomes sufficiently large in comparison to the marginal cost of the FPF, the output replacement effect dominates the output expansion effect and welfare falls.

**4. Conclusions**

In this article, we study the impacts of a non-profit firm on output and welfare in a mixed duopoly. We show that technical efficiency at the margin is crucial in determining whether more social responsibility of the NPO increases or reduces welfare assuming relatively general demand and cost functions. This leads to the paradoxical result that more social responsibility of the NPO may eventually reduce welfare.

We introduce the concept of technical advantage in production in order to study a mixed duopoly where firms have convex cost functions. In particular, we say that a firm has a technical advantage over its rival if it can produce the same output that its rival produces at lower marginal and total costs than the rival. This concept is useful if firms have convex instead of linear cost functions. The point is that a firm may have a technical advantage over its rival but higher marginal costs at the margin because it produces more output. Interestingly, we can show that the paradoxical result that more social
responsibility reduces welfare is very likely to occur if firms have quadratic cost functions. This happens even if the NPO has a substantial technical advantage over the FPF.

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Appendix A.

We show first that \( S_F < 0 \). Note that \( S_F = (p' + q_Fp'') + (p' - c_F') \). The sign of \( p'' \) is ambiguous. However, \( (A1) \) and \( (A4) \) imply that \( S_F < 0 \). Hence, suppose that \( p'' > 0 \). In this case, \( (A2) \) implies that \( q_Fp'' < -p'' \). It follows that \( p' + q_F p'' < 0 \). This fact as well as \( (A1) \) and \( (A4) \) imply that \( S_F < 0 \).

Now, we show that \( S_N < 0 \). Note that \( (A4) \) implies that \( S_N \leq 2p' + q_N p'' - \theta(Qp'' + p') \). Therefore, it suffices to show that \( 2p' + q_N p'' - \theta(Qp'' + p') = (1 - \theta)(p' + q_N p'') + p' - q_N p'' \theta < 0 \). It is easy to note that \( (A1) \) and \( 0 < \theta \leq 1 \) imply that \( (1 - \theta)(p' + q_N p'') + p' - q_N p'' \theta < 0 \). Assume first that \( p'' < 0 \). This assumption, \( (A1) \) and \( 0 < \theta \leq 1 \) imply that \( (1 - \theta)(p' + q_N p'') < 0 \). Furthermore, \( (A2) \) and \( p'' < 0 \) imply that \( p' < q_N p'' \) while \( p'' < 0 \) and \( 0 < \theta \leq 1 \) imply that \( q_N p'' < q_N p'' \theta \). It follows that \( p' - q_N p'' \theta < 0 \). This shows that \( S_N < 0 \) if \( p'' < 0 \). Assume now that \( p'' > 0 \). This assumption, \( (A1) \) and \( 0 < \theta \leq 1 \) imply that \( p' - q_N p'' \theta < 0 \). Similarly, \( p'' > 0 \) and \( (A2) \) imply that \( p' + q_N p'' > 0 \). This shows that \( S_N < 0 \) if \( p'' > 0 \).

Appendix B.

We have to show that \(-1 < \frac{\partial \theta}{\partial \mu} < 0 \). We first show that \( \frac{\partial S}{\partial \mu} = -\frac{p' + q_F p''}{S_F} < 0 \). Given that \( S_F < 0 \), we only have to show that \( p' + q_F p'' < 0 \). Note that \( (A1) \) implies that \( \theta p'' + q_F p'' < 0 \) if \( p'' < 0 \). Hence, assume that \( p'' > 0 \). In this case, \( (A2) \) implies that \( q_F p'' < -p'' \). It follows that \( p' + q_F p'' < 0 \). We now show that \(-1 < \frac{p' + q_F p''}{S_F} \) or, equivalently, that \( \frac{p' + q_F p''}{S_F} < 1 \). Given that \( S_F < 0 \), this expression is also \( S_F < p' + q_F p'' \). We can use \( S_F = (p' + q_F p'') + (p' - c_F') \) to find that this expression becomes \( p' - c_F' < 0 \). Finally, \( A1 \) and \( A4 \) imply that this condition holds.

We have to show also that \(-1 < \frac{\partial \theta}{\partial \mu} < 0 \). We first show that \( \frac{\partial S}{\partial \mu} = \frac{p' + q_F p'' - \theta(Qp'' + p')}{S_F} \) or, equivalently, that \( 1 > \frac{p' + q_F p'' - \theta(Qp'' + p')}{S_F} \). Given that \( S_F < 0 \), this expression is \( S_F < p' + q_F p'' - \theta(Qp'' + p') \). Using the definition of \( S_N \), this expression is \( p' - C_N' < 0 \). It follows from \( (A1) \) and \( (A4) \) that this condition holds.

References


