How are Africa's emerging stock markets related to advanced markets? Evidence from copulas

Jones Odei Mensah, Paul Alagidede*

Wits Business School, University of the Witwatersrand, 2 St Davids Place, Parktown, Johannesburg 2193, South Africa

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ABSTRACT

The finance literature provides ample evidence that diversification benefits hinges on dependence between assets returns. A notable feature of the recent financial crisis is the extent to which assets that had hitherto moved mostly independently suddenly moved together resulting in joint losses in most advanced markets. This provides grounds to uncover the relative potential of African markets to provide diversification benefits by means of their correlation with advanced markets. Therefore, we examine the dependence structure between advanced and emerging African stock markets using copulas. Several findings are documented. First, dependence is time-varying and weak for most African markets, except South Africa. Second, we find evidence of asymmetric dependence, suggesting that stock return comovement varies in bearish and bullish markets. Third, extreme downward stock price movements in the advanced markets do not have significant spillover effects on Africa's emerging stock markets. Our results, implying that African markets, with the exception of South Africa, are immune to risk spillover from advanced markets, improves the extant literature and have implications for portfolio diversification and risk management.

1. Introduction

The nature of dependence across stock returns plays a crucial role in asset pricing, portfolio allocation and policy formulation. Investment practitioners pay close attention to the comovement between equity markets, as a proper grasp of its nature and measurement affects the risk-return trade-off from international diversification; typically, international portfolio diversification becomes less effective when markets are in turmoil. Policy makers, on the other hand, are more interested in how strong linkage across stock markets influences the transmission of shocks, its consequences as well as implications for risk management.

There is vast literature on the dependence between international stock markets, mainly spurred by the seminal contribution of Grubel (1968) who asserted that investors could obtain welfare gains by diversifying their portfolio internationally, where the gains hinges primarily on the correlation between stocks. Linear correlation has been used as the canonical measure of association between stocks due to its convenient properties (see Embrechts et al. 2002). Early works in this area were based on models that jointly price stocks under the assumption of constant correlation (Agmon, 1972; Solnik, 1974). Subsequent contributions present evidence that stock return comovement varies with time (Brooks and Del Negro, 2004; Forbes and Rigobon, 2002; Kizys and Plerdzhioch, 2009). Owing to the drawbacks of linear correlation, multivariate GARCH models have become the typical approach of modelling time-varying stock dependence and there is exponential growth of research in this area (see Syllignakis and Kouretas, 2011; Gjika and Horvath, 2013; Baumöhl and Lyócsa, 2014; Kandu and Sarkar, 2016). However, one major limitation of the multivariate GARCH approach is the assumption that return innovations are characterised by a symmetric multivariate normal or Student-t distribution (Patton, 2006; Garcia and Tsafack, 2011). Evidently, this assumption seems to be at odds with the empirics; the distribution of financial returns possesses heavy tails than those of the normal distribution and dependence between stocks returns are usually nonlinear and asymmetric (Embrechts et al. 2002).

Against this background, researchers have resorted to a relatively new approach, copula, to model the dependence between
stock returns. Copulas are functions that join multivariate distributions to their one-dimensional marginals. For instance, given two random variables \( X \) and \( Y \) with marginal distributions \( F_1(X) \) and \( F_2(Y) \), we can express their joint cumulative distribution function as \( F(x, y) = C[F_1(x), F_2(y)] \) (Sklar, 1959). We can infer from this expression that to obtain the joint distribution, one needs to know how \( X \) and \( Y \) are related, in addition to their individual marginal distributions. In this regard, the copula function \( C \) provides this other information. Thus, copulas by definition provide a realistic description of the dependence structure between random variables over their whole range of variation, including linear and non-linear dependence, symmetric and asymmetric dependence, and extreme or tail dependence. Moreover, copula functions are invariant to non-linear strictly increasing transformations of the data, unlike conventional measures of dependence, such as linear correlations (Embrechts et al. 2002). For example, the dependence between \( X_1 \) and \( X_2 \) will be the same as the dependence between \( \ln(X_1) \) and \( \ln(X_2) \).

Owing to these useful properties, copula models have attracted special attention in recent academic works. For example, Yang et al. (2015) investigates the dependence structure among international stock markets using hierarchical Archimedean copulas and finds strong dependence between Emerging and European stock markets, weak dependence between Frontier and other markets, and evidence of contagion during the global and the EU debt crisis. Similarly, using static copulas, Bashier et al. (2014) studies the dependence pattern across GCC stock returns and concludes that dependence is asymmetric. Bhatti and Nguyen (2012) uses conditional extreme value theory and time-varying copula to capture the tail dependence between the Australian financial market and other selected international stock markets and documents evidence of tail dependence. Mensah and Premaratne (2014) also examine the dependence structure among banking sector stocks from 12 Asian markets using static and time-varying copulas and uncovers evidence of asymmetric dependence. Intriguingly, almost all of these studies are mainly focused on international markets, other than those from Africa.

Although studies abound, there is no empirical evidence on the dependence structure of African stock markets with other international stock markets. Only a smattering of papers has focused on the comovement of Africa’s emerging stock markets with other international markets, despite the region’s growing importance in the economy. It is also instructive to note that the few existing empirical studies on Africa (Adjası and Biekpe, 2006; Alagidede, 2009; Alagidede et al. 2011) focus on comovement using cointegration techniques, which has major weaknesses. For instance, it requires long span of data, which many of the equity markets in Africa, with the exception of a few, do not have, thus rendering a number of the previous studies questionable. Moreover, using linear dependence measures is at odds with the widely acknowledged fact that return distributions are non-normal. It is therefore essential to assess the dependence between African stock markets and other international markets with more accurate measures of dependence.

The contribution of this paper is twofold. Firstly, to the best of our knowledge, this is the first study that applies copula models to investigate the time-varying dependence structure between international and African stock returns. We characterize the bivariate dependence structure between African and other international stock returns through copulas. To model the dynamic dependence, we use the Generalised Autoregressive Score (GAS) model proposed by Creal, Koopman and Lucas (2013), which uses the standardized score of the copula log-likelihood function to update parameters over time. The GAS model performs well in capturing different types of dynamics compared to the lagged and autoregressive specification in Patton (2006) and the DCC specification in Christoffersen et al. (2012) and Christoffersen and Langlois (2013). Thus, our study provides new insight on the dependence structure of African stock markets.

Secondly, this study is novel as it investigates African stock market quantiles conditional on advanced stock price movements, with the aim of uncovering shock spillovers. In this respect, a few studies have applied copula and quantile models in order to capture shock spillovers. For example, Sim and Zhou (2015) used a quantile-on-quantile regression to characterize the effect of oil price shock quantiles on US stock return quantiles. Subsequently, Rebrodeo and Ugolini (2016) used a copula-based approach to investigate the impact of quantile and interquantile oil price movements on different stock return quantiles for a broad set of global indices. They compute unconditional and conditional quantile stock return quantiles based on marginal models for stock returns and copula functions for oil-stock dependence and proof the effectiveness of this approach. In line with Rebrodeo and Ugolini (2016), we capture the dependence structure between stock returns and compute unconditional and conditional quantile using marginal models and copulas.

Foreshadowing the main results, we document that African and advanced market dependence structure is time varying and differs across African stock markets. South Africa’s upside and downside dependence with advanced markets is clearly distinguishable from the remaining African stock markets. Yet, we find no downside spillover effects, even for South Africa whose tail dependence is clearly distinguishable, thus leading to the conclusion that extreme stock market events have a limited impact on African stocks. The paper concludes that there could be a limit to portfolio diversification benefits, from the perspective of international investors, if the South African index and typical African stock indices are held together in a portfolio.

The rest of the paper is structured as follows. Brief discussion of copula theory is undertaken in Section 2. Section 3 presents the empirical application of copula models. Section 4 presents the data while results and analysis are undertaken in Section 5. Section 6 concludes the paper.

2. Copula theory

This section of the paper presents a brief discussion of copula theory. Sklar (1959) first introduced the idea of copula, which states that an \( n \)-dimensional joint distribution can be decomposed into its \( n \) univariate marginal distributions and an \( n \)-dimensional copula: Let \( X = (X_1, ... X_n) \) denote a random vector with distribution function \( F \) and with marginal functions \( F_i, X_i \sim F_i, 1 \leq i \leq n \). There exist a distributional function \( C \), known as the copula of the variable \( X \), that maps \([0,1]^n\) into \([0,1]\) such that

\[
F(x_1, ... x_n) = C[F_1(x_1), ..., F_n(x_n)].
\]

(1)

Thus, the copula \( C \) of the variable \( X \) is the function that maps the univariate marginal distributions \( F_i \) to the joint distribution \( F \). Alternatively, the copula function can be understood by means of the “probability integral transformation” (PIT), \( U_i = F_i(X_i) \) (Patton, 2012). As shown in Patton (2012), conditional on the fact that \( F_i \) is continuous the variable \( U_i \) will have the \( \text{Unif}(0,1) \) distribution irrespective of the original distribution \( F_i \):

\[
U_i = F_i(X_i) - \text{Unif}(0, 1), i = 1, ..., n
\]

(2)

Hence, we can understand the copula \( C \) of the variable \( X \) as the joint distribution of the vector of probability integral transformations, \( U_i = [U_{i1}, ..., U_{in}] \), and thus a joint function whose margins are \( \text{Unif}(0,1) \) (Patton, 2012). Differentiating the above
representation once with respect to all its arguments, we obtain the joint probability density function as follows:

\[
 f(x_0, \ldots, x_n) = c \left( F_1(x_1), \ldots, F_n(x_n) \right) \prod_{i=1}^{n} f_i(x_i)
\]

where \( c(u_0, \ldots, u_n) = \prod_{i=0}^{n} \frac{\partial f_i(u_i)}{\partial u_i} \).

Estimation of the joint cdf \( F \) consist the following two steps:

(i) We identify and estimate the marginal (ii) we identify and estimate the copula functions. The flexibility of separating the marginal distributions and the copula means that one can combine different family of distributions, which will be valid. For example, combining a skewed distributed variable with a symmetrically distributed variable via \( t \)-copula will be a valid joint distribution, although strange. Hence, the abundant research on modelling univariate distributions becomes useful to the researcher, leaving just the task of modelling the dependence structure (Patton, 2012). A further account of unconditional copulas can be found in Cherubini Luciano and Vecchiato (2004), Nelsen (2007) and Heinen and Valdesogo (2012).

The literature due to Patton (2006) extends Sklar's theorem to conditional distributions, making it useful for serial series applications. Since the marginal distribution for returns of financial series exhibit time-varying means and volatility, the conditional copula, serves as a useful tool in capturing the dependence in that regard. Let \( X_i \sim (X_0, \ldots, X_n) \) denote a stochastic process and \( F_t \) denote an information set available at time \( t \), and let the conditional distribution of \( (X_0, \ldots, X_n) \) \( \mid F_{t-1} \) be \( F_t \). Then

\[
 F_t(x_0, \ldots, x_n | F_{t-1}) = C \left[ F_{t1}(x_0 | F_{t-1}), \ldots, F_{tn}(x_n | F_{t-1}) \right]
\]

An important step required in applying Sklar's theorem to conditional distribution is to ensure that the conditioning information be same for all marginal distributions and the copula. The common practice is to assume that the marginal models depend only on their respective past information whereas the copula can be conditioned on past information of all series. An account of the case when differing information sets are used can be found in Fermanian and Wegkamp (2012).

3. Empirical methods

3.1. Marginal models

Prior to fitting the bivariate copula models, we must first specify appropriate models for the conditional marginal distributions. Financial time series exhibit some well-documented characteristics such as long-memory, fat-tails, and conditional heteroscedasticity. Thus, it suffices to apply autoregressive-moving average (ARMA(p,q)) models to the conditional means (where \( p \) is the order of the autoregressive part and \( q \) is the order to the moving average part) as well as generalised autoregressive conditional heteroskedasticity (GARCH(p,q)) models to the conditional variances (where \( p \) and \( q \) are the order of the GARCH and ARCH terms, respectively) as follows:

\[
 Y_t = \varepsilon_t + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}
\]

\[
 \varepsilon_t = \sigma_t z_t, \; z_t \sim \text{NIID}(0,1)
\]

\[
 \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2
\]

where \( Y_t \) is the log-difference of stock market price at time \( t \); \( c \) is the constant term in the mean equation; \( \varepsilon_t \) is the real-valued discrete time stochastic process at time \( t \); \( z_t \) is an unobservable random variable belonging to an i.i.d. process; \( \sigma_t^2 \) is the conditional variance of \( \varepsilon_t \); \( \alpha_i \) and \( \beta_i \) are the constant, ARCH parameter, and GARCH parameters respectively. In the case of GARCH(1,1) model, the following inequality restrictions must be satisfied to ensure that the model is rightly specified: (i) \( \omega \geq 0 \), (ii) \( \alpha_1 \geq 0 \), (iii) \( \beta_1 \geq 0 \), and (iv) \( \alpha_1 + \beta_1 < 1 \). When \( \alpha_1 + \beta_1 = 1 \) then the conditional variance will not converge on a constant unconditional variance in the long run (Bollerslev, 1986). We estimate the GARCH models by maximum likelihood.

3.2. Copula models

Eq. (1) outlined the copula distribution for \( n \) number of assets. For simplicity, this study only considers bivariate copulas. Hence, the bivariate distribution \( F \) with margins \( F_1, F_2 \) can be written as:

\[
 F(x_1, x_2) = C \left[ F_1(x_1), F_2(x_2) \right]
\]

The copula of the joint distribution function for a random vector \( X = (X_1, X_2) \) can be written as

\[
 C(u, v) = F \left[ F_{1}^{-1}(u), F_{2}^{-1}(v) \right]
\]

where the quantile functions of margins is \( F^{-1}(u) = \inf \{ x : F(x) \geq u \}, u \in [0,1] \).

An important observation from Eq. (8) is that the joint distribution is split into marginal parts and the dependence structure (copula) without losing any information. As mentioned earlier, the marginal parts, \( F_1 \) and \( F_2 \) are not required to be from the same distribution family.

An advantage of copula models is that for many forms we can easily obtain tail dependence which measures captures the probability that the two random variables are in their lower (upper) joint tails. The tail dependence captures the behaviour of the random variables during extreme events. For example, given two stock market returns, \( X_1 \) and \( X_2 \), tail dependence measures the probability that we will observe an extremely large fall (rise) of stock market \( X_t \) given that the stock market \( X_t \) has experienced an extreme fall (rise). The tail dependence determines whether the two markets crash or boom together, thus investors holding long portfolios are mainly concerned with the downward movement, whereas the risk of large upward movement is the concern of investors holding short positions. We can define the lower and upper tail dependence between \( X_1 \) and \( X_2 \) as:

\[
 r^L \lim_{u \to 0} \Pr \{ F(X_1) \leq uF(X_2) \leq u \} = \lim_{u \to 0} \frac{C(u, u)}{u}
\]

\[
 r^U \lim_{u \to 0} \Pr \{ F(X_1) \geq uF(X_2) \geq u \} = \lim_{u \to 1} \frac{1 - 2u + C(u, u)}{1 - u}
\]

where \( r^L \) and \( r^U \in (0,1) \). If the above limits exist and if \( r^L \) and \( r^U > 0 \), \( X_1 \) and \( X_2 \) tend to be left (lower) or right (upper) tail dependent.

For the purpose of capturing different patterns of tail dependence, we estimate Eq. (8) for four different copula specifications as shown in Table 1. The functional forms considered are the Gaussian (Normal), Student’s \( t \), Gumbel, and rotated Gumbel copulas. The Gaussian copula is the most widely used in finance due to its convenient properties. However, it is unable to capture tail dependence. The Student’s \( t \) copula on the other hand assumes symmetric dependence for both lower and upper tails of the joint distribution. The rotated Gumbel copula is useful only when examining dependence during market crashes; conversely, the Gumbel copula is able to capture only upper tail dependence, thus making it useful during periods of market boom.
3.3. Generalized Autoregressive Score (GAS) model

The time-varying copulas are estimated based on the Generalized Autoregressive Score (GAS) model of Creal et al (2013). We assume the copula parameter evolves as a function of its own lagged value and a “forcing variable” related to the scaled score of the copula log-likelihood. This approach uses strictly increasing transformation (e.g. log) to copula parameters in order to ensure that parameters are constrained to lie in a particular range (e.g. \( \rho \in (-1, 1) \)). Following Patton (2012), the evolution of the transformed parameter is denoted by:

\[
\begin{align*}
    f_t &= h(\delta_t) \iff \delta_t = h^{-1}(f_t) \\
    f_{t+1} &= \theta + \beta f_t + \alpha t^{-1/2} s_t \\
    S_t &= \frac{d}{dp} \log c(u_t, v_t; \delta_t) \\
    I_t &= E_{t-1}[S_t] = I(\delta_t)
\end{align*}
\]

By these expressions, the future value of the copula parameter depends on a constant, the present value, and the score of the copula log-likelihood \( t^{-1/2} s_t \). We apply the GAS model to the time-varying Gaussian, Gumbel and rotated Gumbel copulas.¹ We use \( \delta_t = (1 - \exp[-f_t])/(1 + \exp[f_t]) \) to ensure that the Gaussian copula parameter lie in \((-1, 1)\). We use the function \( \delta_t = 1 + \exp(f_t) \) to ensure that the Gumbel and rotated copula parameter is greater than one.

We can estimate the copula parameters using two alternative frameworks: Maximum Likelihood Estimation (MLE) method and the Inference functions for the Margins (IFM). We estimate the copulas in this study by the latter method due to its advantages over the MLE. First, unlike the MLE, the IFM requires fewer computations; second, it is highly efficient; thirdly, the goodness of the margins can be assessed separately from that of the copula; lastly, the series of random variables are not required to be of equal length (Bhatti and Nguyen, 2012).

3.4. Advanced stock return quantile effects on African stock return quantiles

Apart from the dependence structure, we also examine whether extreme price movements in the advanced stock markets have any spillover effects on African stocks. In this regard, we examine the impact of lower quantile advanced stock price movements on African stock price quantile. In line with Reboredo and Ugolini (2016), the \( \alpha \)-quantile of stock return distribution at time \( t \), given by:

\[
p(y_t \leq q^{\alpha}_{y,t}) = \alpha, \quad \alpha \text{ can be computed as:}
\]

\[
q^{\alpha}_{y,t} = F^{-1}_{y,t}(\alpha) \tag{16}
\]

where \( y_t \) is the stock return, \( F^{-1}_{y,t}(\alpha) \) is the inverse of the distribution function of \( y_t \). The \( \alpha \)-quantile for low values of alpha is typically referred to as value-at-risk (VaR). Furthermore, we can obtain the conditional \( \alpha \)-quantile of African stock return distribution at time \( t \) for a given \( \rho \)-quantile of advanced market stock return given by \( p(y_t \leq q^{\rho}_{y,t}, t \leq q^{\alpha}_{y,t}) = \alpha \) as:

\[
q^{\rho,\alpha}_{y,t} = F^{-1}_{y_t \leq q^{\rho}_{y,t}}(\alpha) \tag{17}
\]

where \( F^{-1}_{y_t \leq q^{\rho}_{y,t}}(\alpha) \) denotes the inverse distribution of \( y_t \) conditional on the fact that \( x_t \leq q^{\rho}_{x,t} \).

Given the conditional mean and variance information (Eqs. (4)–(6)), we compute the unconditional \( \alpha \)-quantile of the stock return as:

\[
q^{\alpha}_{y,t} = \mu_t + F^{-1}(\alpha) \sigma_t \tag{18}
\]

Moreover, since Sklar’s theorem allows us to express the joint distribution of \( y \) and \( x \) as \( F_{y,x}(x,y) = C(F_x(x), F_y(y)) \), we can compute the conditional quantiles using copula as:

\[
q^{\rho,\alpha}_{y,t} = F^{-1}_{y_t \leq q^{\rho}_{y,t}}(\alpha) \tag{19}
\]

4. Data

We use the following stock indices: FTSE/JSE All Share (JSEOVER) for South Africa, Hermes Financial (EGHFINC) for Egypt, Nigeria All Share (NIGALSH), Nairobi SE (NSE20) for Kenya, FTSE 100 for United Kingdom and the S&P 500 COMPOSITE for the United States. These are tradeable indices readily available to market participants; hence, the returns are a true reflection of the gains an investor could make by holding them in a portfolio. The four African markets are the largest, in terms of listed companies, in their respective sub regions, that is Southern Africa, North Africa, East Africa, and West Africa. Another reason for this selection is that all the markets have daily data for a relatively long sample period. The data is gleaned from Datstream and covers the period January 2000 to April 2014. The returns are calculated as 100 times the difference in the log of prices.

Table 2 shows the descriptive statistics. The mean percentage returns are close to zero in all cases and small compared to the standard deviations indicating high volatility in all the markets.

---

¹ For the student-t copulas, we considered the ARMA(1,1)-type process Patton (2006) for the linear dependence parameter as follows:

\[
\rho_t = \rho_0 + \rho_1 (y_{t-1} + y_{t-2} + \cdots + y_{t-m}) + \epsilon_t \]

where \( \epsilon_t = (1 - e^{-\gamma t})(1 + e^{-\gamma t})^{-1} \) is modified logistic transformation that which forces the value of within the interval (-1,1).
Comparing the means, we notice Nigeria is the highest, followed by South Africa, whereas USA shows the lowest performance over the sample period. Furthermore, with the exception of Kenya, all the stock returns are negatively skewed and have excess kurtosis, suggesting a relatively higher probability of extreme negative returns compared to extreme positive returns. The Ljung-Box test confirms the presence of strong autocorrelation. The Jarque-Bera statistic (Jarque and Bera, 1980, 1981) strongly rejects the null hypothesis of normality in the return distributions. Finally, the ARCH-LM test (Engle, 1982) strongly confirms the presence of ARCH-effects in the individual series, thus, it suffices to model the return distributions with GARCH models.

Table 3 shows linear correlation among the six markets. Of importance is the correlation between the United States and African markets on one hand, and the correlation between the United Kingdom and African markets on the other hand. The ranking for the USA-related pairs from lowest to highest is USA-Nigeria, USA-Egypt and USA-South Africa. Similarly, the linear correlation from lowest to highest for the UK-related pairs is UK-Nigeria, UK-Kenya, UK-Egypt and UK-South Africa. With the exception of the USA-South Africa (0.5881) and UK-South Africa (0.3448), correlation is generally low among the remaining pairs. At the markets on one hand, and the correlation between the United Kingdom and African markets on the other hand. The ranking for the USA-related pairs from lowest to highest is USA-Nigeria, USA-Kenya, USA-Egypt and USA-South Africa. Similarly, the linear correlation from lowest to highest for the UK-related pairs is UK-Nigeria, UK-Kenya, UK-Egypt and UK-South Africa. With the exception of the USA-South Africa (0.5881) and UK-South Africa (0.3448), correlation is generally low among the remaining pairs. At the
Table 6 Estimates of static and time-varying copulas: US-related Pairs.

### Panel A: Parameter estimates for time-invariant copulas

<table>
<thead>
<tr>
<th>South Africa</th>
<th>Egypt</th>
<th>Kenya</th>
<th>Nigeria</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gaussian copula</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5692 *</td>
<td>0.1260 *</td>
<td>0.0084</td>
</tr>
<tr>
<td>(0.0108)</td>
<td>(0.0163)</td>
<td>(0.0178)</td>
<td>(0.0169)</td>
</tr>
<tr>
<td><strong>AIC</strong></td>
<td>1455.641</td>
<td>61.3942</td>
<td>2.2598</td>
</tr>
<tr>
<td><strong>Student-t copula</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5740</td>
<td>0.1265 *</td>
<td>0.0076</td>
</tr>
<tr>
<td>(0.0097)</td>
<td>(0.0162)</td>
<td>(0.0179)</td>
<td>(0.0171)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1015 *</td>
<td>0.0287 *</td>
<td>0.0100</td>
</tr>
<tr>
<td>(0.0165)</td>
<td>(0.0137)</td>
<td>(0.0080)</td>
<td>(0.0100)</td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>1501.034</td>
<td>66.1012</td>
<td>4.7556</td>
</tr>
<tr>
<td><strong>Gumbel copula</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\kappa}$</td>
<td>1.5571 *</td>
<td>1.0700 *</td>
<td>1.0086 *</td>
</tr>
<tr>
<td>(95%CI)</td>
<td>[1.5203 1.5939]</td>
<td>[1.0478 1.0921]</td>
<td>[0.9950 1.0222]</td>
</tr>
<tr>
<td><strong>AIC</strong></td>
<td>1308.717</td>
<td>44.357</td>
<td>−85.1466</td>
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<tr>
<td><strong>Rotated Gumbel copula</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\kappa}$</td>
<td>1.5805 *</td>
<td>1.0726 *</td>
<td>1.0096 *</td>
</tr>
<tr>
<td>(95%CI)</td>
<td>[1.5472 1.6248]</td>
<td>[1.0502 1.0949]</td>
<td>[0.9944 1.0248]</td>
</tr>
<tr>
<td><strong>AIC</strong></td>
<td>1435.2998</td>
<td>48.302</td>
<td>−96.283</td>
</tr>
</tbody>
</table>

### Panel B: Parameter estimates for time-varying copulas

<table>
<thead>
<tr>
<th>South Africa</th>
<th>Egypt</th>
<th>Kenya</th>
<th>Nigeria</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TVP-Gaussian</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi_0$</td>
<td>1.2956 *</td>
<td>0.2199 *</td>
<td>0.0063</td>
</tr>
<tr>
<td>(0.0099)</td>
<td>(0.0092)</td>
<td>(0.0043)</td>
<td>(0.0053)</td>
</tr>
<tr>
<td>$\psi_1$</td>
<td>0.0336 *</td>
<td>0.0073</td>
<td>0.0293</td>
</tr>
<tr>
<td>(0.0077)</td>
<td>(0.0094)</td>
<td>(0.0080)</td>
<td>(0.0080)</td>
</tr>
<tr>
<td>$\psi_2$</td>
<td>0.9484 *</td>
<td>0.9626 *</td>
<td>0.9000 *</td>
</tr>
<tr>
<td>(0.0075)</td>
<td>(0.0052)</td>
<td>(0.0043)</td>
<td>(0.0009)</td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>−1588.8831</td>
<td>−81.5262</td>
<td>−11.7411</td>
</tr>
<tr>
<td><strong>TVP-Student-t</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>15.0156</td>
<td>42.2699</td>
<td>168.7239 *</td>
</tr>
<tr>
<td>(3.8400)</td>
<td>(84.475)</td>
<td>(3.9330)</td>
<td>(218.8590)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.0285</td>
<td>0.0082</td>
<td>0.0280</td>
</tr>
<tr>
<td>(0.0090)</td>
<td>(0.0094)</td>
<td>(0.0080)</td>
<td>(0.0080)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9618 *</td>
<td>0.9882 *</td>
<td>0.8746 *</td>
</tr>
<tr>
<td>(0.0140)</td>
<td>(0.0170)</td>
<td>(0.0360)</td>
<td>(0.0011)</td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>−1615.879</td>
<td>−85.6932</td>
<td>−10.9631</td>
</tr>
<tr>
<td><strong>TVP-Gumbel</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>−0.0090 *</td>
<td>−0.0080 *</td>
<td>−0.1873</td>
</tr>
<tr>
<td>(0.0019)</td>
<td>(0.0024)</td>
<td>(0.3425)</td>
<td>(0.0174)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.0754 *</td>
<td>0.0736 *</td>
<td>−0.0666</td>
</tr>
<tr>
<td>(0.0178)</td>
<td>(0.0223)</td>
<td>(0.0089)</td>
<td>(0.0171)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9849 *</td>
<td>0.9972 *</td>
<td>0.9581 *</td>
</tr>
<tr>
<td>(0.0041)</td>
<td>(0.0009)</td>
<td>(0.0016)</td>
<td>(0.0048)</td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>−1401.9678</td>
<td>−67.4109</td>
<td>6.6844</td>
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<tr>
<td><strong>TVP-rotated Gumbel</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>−0.0082</td>
<td>−0.0084</td>
<td>−0.0144</td>
</tr>
<tr>
<td>(0.0014)</td>
<td>(0.0013)</td>
<td>(0.0357)</td>
<td>(0.0138)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.0736</td>
<td>0.0588</td>
<td>0.0231</td>
</tr>
<tr>
<td>(0.0070)</td>
<td>(0.0380)</td>
<td>(0.0463)</td>
<td>(0.0138)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9852 *</td>
<td>0.9968 a</td>
<td>0.9963 a</td>
</tr>
<tr>
<td>(0.0029)</td>
<td>(0.0041)</td>
<td>(0.0009)</td>
<td>(0.0040)</td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>−1530.5475</td>
<td>−72.8748</td>
<td>15.2325</td>
</tr>
</tbody>
</table>

Notes: The table reports the maximum likelihood estimates for the different pair-copulas. 95% confidence intervals are given in brackets. Standard errors are given in parenthesis.

* denotes statistical significance at 1% level.

if one uses it to make inferences about diversification opportunities. Besides other shortcomings, correlation is a linear measure and is unable to capture the nonlinear dependence among the markets, hence the need for the copula technique, which is more robust.
5. Empirical results

5.1. Estimates of marginal models

Prior to estimating the copula models, we apply an ARMA filtration to the stock return series to ensure the residuals have an expected return of zero and free from autocorrelation. We then test the fitted series for ARCH-effects using the ARCH-LM test and the results indicate that each of the series shows evidence of heteroscedasticity. We therefore determine the optimal lag length for each univariate GARCH and fit various specifications to the second moments. Table 4 shows the estimates of the ARMA-GARCH models for the stock returns. The best fitting models based on the Akaike information criterion (AIC) are AR(1)-GARCH(1,1) for South Africa, ARMA(1,1)-GJR-GARCH(1,1) for Egypt, ARMA(1,1)-GARCH(1,1) for Kenya, AR(2)-GARCH(1,1) for USA and AR(2)-GARCH(1,1) for Nigeria and U.K. Table 4 shows that the estimated conditional variance is impacted by past squared shocks (around 0.5886 to 0.9078) as well as past conditional variance (around 0.7334 to 0.2624) as well as past conditional variance as indicated by the p-values shown in Table 5.

Subsequent to the marginal specifications, we use the empirical distribution function to transform the standardized iid residuals into uniform margins, thus making our model semiparametric. Semiparametric models have much empirical appeal compared with the fully parametric models (Patton, 2012). We then carry out the goodness-of-fit for the marginal models by applying the Breusch-Godfrey serial correlation LM (BGLM) (Breusch, 1978; Godfrey, 1978) test to the probability integral transformations (PITs) of the underlying error terms from each of the ARMA(p,q)-GARCH(p,q) processes. We carry out the BGLM test for the first four moments of the probability integral transforms (u and v) of the standardized residuals from the marginal models; that is, we regress \((u - \bar{u})^{k}\) and \((v - \bar{v})^{k}\) on 10 lags of both variables lags for \(k = 1, 2, 3, 4\). The p-values shown in Table 5 gives no indication of serial correlation, thus justifying the appropriateness of the marginal models.

5.2. Copula estimates

Table 6 reports estimates of static and time-varying copula dependence between the US and African stock markets. The results for the UK-related pairs are shown in Table 7. Since the parameter estimates for the Gaussian and student-t copula captures the dependence between the markets, we can state that the higher the value of \(\hat{\rho}\), the higher the dependence between the stock markets.

The \(\hat{\rho}\) estimates in Table 6 are statistically significant for all African markets, with the exception of Kenya. The \(\hat{\rho}\) estimates for South Africa shows a moderate positive relationship with the US and it is clearly distinguishable from Egypt and Nigeria, which show weak positive linear relationship with the US stock market. Moreover, the time-varying Gaussian and student-t copula parameters both show the existence of time-varying dependence between the markets.

For the U.K stock market, the Gaussian and student-t copula parameter estimates in Table 7 show the existence of weak uphill linear relationship with South Africa and Egypt only; \(\hat{\rho}\) is not statistically significant for Kenya and Nigeria. Both the time-varying Gaussian and student-t copula parameters corroborate the existence of dynamic dependence for South Africa and Egypt.

Fig. 1 depicts the temporal evolution based on the Gaussian copula GAS specification between the US and African stock markets (grey lines) on one hand, as well as the UK and African stock markets (black lines), on the other hand. Clearly, there is no...
similarity in the temporal evolution of dependence for the bivariate relationships. An upward trend can be found for the US-Nigeria pair, while Egypt shows significant peaks, coinciding with the sub-prime and Euro debt crises. The dynamic path for the Kenyan and UK-Nigeria pair is akin to a white noise process, while the US-South African pair exhibits mild clustering. These points to the fact that African markets do not respond uniformly to events in the advanced markets.

5.3. Tail dependence between advanced and African stock returns

The Gumbel (rotated Gumbel) captures upper (lower) tail dependence structure between the markets. Given that the implied tail dependence is defined as $2 - 2^{1/k}$, we can say that a higher value of $k$ from the Gumbel (rotated Gumbel) indicates higher upper (lower) tail dependence between the stock markets. The static Gumbel (rotated Gumbel) parameter $k$ in Tables 6 and 7 is statistically significant for both the US and UK related pairs. Comparing the values reveals moderate dependence for US-South Africa and weak dependence for all other pairs, except UK-Nigeria whose Gumbel copula parameter ($k = 1.000$) implies no upper tail dependence. Thus, we can say that, with the exception of South Africa, the remaining three African markets are generally less sensitive to the advanced markets.

Fig. 2 illustrates the dynamic upper (lower) tail dependence based on the TVP Gumbel (rotated Gumbel) copula GAS specification. Dependence in the tails closely evolve and lower tail seems to be mostly greater than upper tail, suggesting the presence of asymmetry in some bivariate relationships. South Africa shows a more volatile tail dependence with the US compared to other African countries. Kenya’s tail dependence with UK and US seems to be the least volatile among all the pairs. Although there is no clear similarity in temporal evolution of tail dependence for the bivariate pairs, most of them seemed to have responded to the Global Financial Crises and Euro Crisis with peaks of turbulence (e.g. US-Egypt, UK-Egypt, UK-South Africa, and US-Nigeria), which is in line with studies that point to an increase in financial market dependence during crisis (Kenourgios et al. 2011; Righi and Ceretta, 2013; Mensah and Premaratne, 2014). Yet, with the exception of South Africa, there is weak tail dependence for the remaining African markets, suggesting a low probability of contagion or shock spillovers. The lack of strong association at the tails for Egypt, Kenya and Nigeria, points to the mild segmentation of these markets from the advanced stock markets and this could be due to barriers such as the quality of information on most African markets. As noted, South Africa’s tail dependence with the advanced markets is relatively stronger compared to the other African countries. South Africa is a member of BRICS, and it comes as no surprise that its stock index moves closely with the developed markets. Moreover, there is a dual listing agreement between South Africa’s stock exchange and that of UK, so most of South Africa’s large companies have exposure to UK. On the other hand, the USA is South Africa’s third largest trading partner, both in terms of exports and imports. Again, the economic and financial status of the USA coupled with the fact that it has the largest stock market in the world makes it very influential on both advanced and emerging markets, including South Africa.
5.4. Conditional quantile spillover effects

The weak dependence, particularly at the lower tails for Egypt, Kenya and Nigeria, is an indication that these African markets, with the exception of South Africa, are reasonably immune to risk spillovers from the advanced markets. In other words, the degree of comovement is too low to warrant the easy spread of contagious shocks from the advanced stock markets along with its broad systemic implications. To shed more light on the spillover implications of the weak tail dependence uncovered in the previous paragraphs, we examine the impact of USA and UK quantile stock return movements on African stock return quantiles. We use information from the marginal and copula models to compute the unconditional and conditional stock return quantiles following Equation (18, 19). In the interest of space, we consider only extreme downwards (0.05) stock price movements.

Fig. 3 depicts the dynamics of both unconditional and conditional stock return quantiles over the entire sample period. As can be visually perceptible by plots in Fig. 3, we found that unconditional stock return quantiles were below the conditional quantiles for all African countries, suggesting the absence of any significant spillover effects from the US and UK markets. This corroborates the weak lower tail dependence reported for the copula models. We can therefore say that extreme downward stock price movements in the US and UK do not have significant spillover effects on Africa’s emerging stock markets. Although the South African index is distinguishable from the other three African indices, in terms of its tail dependence with the advanced markets, we do not find compelling evidence of spillover effects from extreme events, as would be expected.

6. Conclusion

This paper examines the dependence structure among African and advanced stock markets using daily stock prices from January 2000 to April 2014 and copulas. The empirical results show that dependence is time-varying and weak for most African markets, except South Africa. Further, we find evidence of asymmetric dependence, suggesting that stock return comovement varies in bearish and bullish markets. The results indicate a relatively strong downside and upside dependence in South Africa compared to other African markets. Finally, extreme downward stock price movements in the advanced markets do not have significant spillover effects on Africa’s emerging stock markets.

In general, the evidence presented has important implications for market participants and policy makers in diverse ways. First, the presence of weak dependence between the African stock markets (excluding South Africa) and advanced stock markets points to the potential gains for international investors holding African stocks. Moreover, policymakers in quest of drawing greater portfolio investment to the continent may find the results useful. However, there could be limits to international portfolio diversification benefits if the South African index is held together with stock indices of typical African nations. Our finding should regenerate interest amongst practitioners to reassess how assets are allocated for effective diversification. Second, our results imply that African markets, with the exception of South Africa, are immune to risk spillovers from the more advanced markets and the tendency to boom or crash together is minimal. In light of recent volatility in global stock markets with the associated spread of...
contagious shocks from advanced to emerging markets, as well as the broad macroeconomic implications, our findings might be useful to policy makers and regulators, particularly in African countries, in designing and implementing appropriate intervention policies.

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