Speculative behavior in a housing market: Boom and bust

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\textbf{ABSTRACT}

We study a housing market with household buyers, speculative investors and property developers in a Walrasian scenario. We show that in addition to the factors that affect the real demand of household buyers and the development cost of property developers, investors' speculative behavior is an important factor explaining housing price evolution and dynamics. In particular, investors' extrapolative expectations may drive the housing price to persistently deviate from its benchmark value and even to explode. In contrast, investors' mean-reverting strategy can balance out the position of trend extrapolators, which may stabilize an otherwise explosive housing market. Moreover, the evolutionary process of housing prices driven by investors' speculative behavior is path-dependent in the sense that different initial market conditions may result in different price paths, which corresponds to the localization property empirically documented in the real housing market. In addition, within the stylized model, we provide some policy implications through analyzing the limitation and effectiveness of policy adjustments via down payment and development cost, and find that the decrease of development cost is a better measure to adjust the housing market when it booms or busts.

\textbf{1. Introduction}

In the traditional economic paradigm that underpins the influential rational expectation real estate models of Alonso (1964), Rosen (1979) and Roback (1982), housing prices are determined by fundamental economic factors including national income, monetary policy, population growth, rents and interest rates. Many empirical studies of housing prices, however, have shown that there are often large movements in housing prices that apparently cannot be explained by these fundamental factors (Shiller, 2005, 2008). In a review of the Fed’s forecasting record leading up to the financial crisis, Potter (2011) acknowledges a “misunderstanding of the housing boom ... [which] downplayed the risk of a substantial fall in house prices.”.

A weak explanatory power of economic factors on housing prices is known as the housing price puzzle or anomaly. One of the most important puzzles for housing economists is the strong persistence of price changes from one year to the next. Over the last three decades, an increasing number of anomalies and puzzles have been uncovered in empirical research. They include, for example, (i) price changes are predictable (Case and Shiller, 1989; Clayton, 1998; Schindler, 2013); (ii) price changes and construction levels are quite volatile in many markets; (iii) over longer time periods, the price changes mean revert while quantity changes persist; (iv) most variations in housing prices are local, not national. We refer to Glaeser et al. (2014) for additional discussion of these empirical anomalies. Therefore, the explanations of housing price movements based solely on fundamental economic factors are unsatisfactory. As a result, researchers have shifted their attention from economic factors towards incorporating the microstructure of housing prices, in particular, by considering agents' demand and supply based on their bounded rationality and heterogeneity.

In fact, the idea of heterogeneity and bounded rationality among agents has long been successfully applied to asset pricing in the stock market. Empirical studies (such as Lux (1998), Chiarella et al. (2014), Kabundi et al. (2015)) and theoretical analyses (such as Day and Huang (1990), Brock and Hommes (1997, 1998), Föllmer et al. (2005), He et al. (2009), Wang et al. (2013)) have shown that heterogeneity and bounded rationality among investors are important factors that affect the volatility of asset pricing. They can be used to explain many puzzles and stylized facts in the stock market, including the equity-premium puzzle, interest rate puzzle, volatility clustering, excess volatility and fat tails (see Basak (2005), Boswijk et al. (2007), De Grauwe (2012), He and Li (2015), and therein). We refer to Hommes (2006) and Chiarella et al. (2009) for surveys of the developments in this literature.

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In the past decades, an increasing number of studies provide ample evidences about heterogeneity and bounded rationality of investors in housing markets. For example, Piazzesi and Schneider (2009) use data on expectations from the Michigan Survey of Consumers to study household beliefs during the US housing boom in the early 2000s and show that the data are characterized by heterogeneity and mutual feedback between housing price expectations and actual housing prices. Applying different methodologies, two studies on a broad set of international securitized real estate markets, conducted by Schindler et al. (2010) and Serrano and Hoesli (2010), provide the evidence that persistence and predictability in real estate returns can be used to earn excess returns compared to a passive strategy. Schindler (2013) finds that investors can obtain excess returns from both autocorrelation-based and moving average-based trading strategies compared to a buy-and-hold strategy. Gelain and Lansing (2014) show that under fully-rational expectations, the model significantly underpredicts the volatility of the US price-rent ratio for reasonable levels of risk aversion but the moving-average model predicts a positive correlation such that agents tend to expect high future returns when prices are high relative to fundamentals - a feature that is consistent with a wide variety of survey evidences from real estate markets. Bolt et al. (2014), by analyzing the housing markets of eight different countries, find that the data support heterogeneity in expectations, with temporary endogenous switching between fundamental mean-reverting and trend-following beliefs based on their relative performance. Shiller (2005, 2008) concludes that speculative thinking among investors, the use of heuristics such as extrapolative expectations, market psychology in the form of optimism and pessimism, herd behavior and social contagion of new ideas (new era thinking), and positive feedback dynamics are elements that play an important role in determining housing prices. There is still a lack of theoretical studies about the impact of heterogeneity and bounded rationality on housing prices. Granziola and Kozicki (2015) show only the extrapolative expectation model with time-varying extrapolation coefficient is consistent with the run up in housing prices observed over the 2000–2006 period and subsequent sharp downturn. Gelain and Lansing (2014) demonstrate that the model can approximately match the volatility of the price-rent ratio in the data if agents employ a simple moving-average forecast rule for the price-rent ratio. These studies focus on bounded rationality of agents but ignore potential heterogeneity among them. Malpezzi and Wachter (2005) and Dieci and Westerhoof (2012, 2013) consider both heterogeneity and bounded rationality and show that the models can generate real estate cycles. However, in the models, the demand and supply of agents are given exogenously rather than coming from their own objectives. Bolt et al. (2014) consider the endogenous demand by optimizing agents’ mean-variance utility but the supply is fixed.

This paper revisits the traditional supply-demand mechanism in housing markets in light of the understanding that fundamental economic factors alone are not the sole drivers of housing price movements. We present a discrete-time model of housing prices considering both economic factors and heterogeneous agents. An important difference with the models of Malpezzi and Wachter (2005) and Dieci and Westerhoof (2012) is that they use a price adjustment rule based on excess demand which is exogenously given, while our approach uses a temporary equilibrium price model where all demands and supplies are endogenously determined based on agents’ own objectives and financial constraints. The equilibrium model is similar to Bolt et al. (2014) but with two important differences. Firstly, they fix the housing supply, whereas the supply amount in our model is endogenously decided by property developers through maximizing their profit. Secondly, their fundamental housing price is determined by rents while in our approach, the fundamental value of houses depends on real demand and supply which rely on the fundamental economic factors like national income, development costs and down payment. Within the stylized model, we analytically study the impacts of real demand, development cost and speculative behavior on housing prices using stability and bifurcation theories. An interesting theoretical contribution of our analysis is that from the viewpoints of speculative behavior and heterogeneity among agents, it can explain various stylized facts in housing markets, like persistence and predictability in housing price movements which are connected with the stability of different steady states and the local characteristics of housing price changes which corresponds to the co-existence of two attractors.

The paper proceeds as follows. In Section 2, we set up an equilibrium framework with three types of agents, house buyers, investors and property developers, and study a benchmark market without speculative investors. Compared with the benchmark market, Section 3 studies a one-period case and analyzes the impact of investors’ speculative behavior on housing prices. In Section 4, we examine investors with dynamic beliefs who follow two specific strategies - extrapolation and mean-reversion, by stability and bifurcation theories. In Section 5, within our framework, the paper gives some policy implications on a housing market with speculative investors by analyzing the limitation and effectiveness of policy adjustments through down payment and development cost. In addition, we use real data on the Beijing housing market to demonstrate the applicability of the model and the economic meaning of it. Section 6 concludes the paper. All the proofs of the technical results are given in the Appendix.

2. The model

We consider a housing market with two types of property consumers, including house buyers and investors, and one type of property developers who supply houses. At any given time, house buyers and investors decide their property demands while property developers decide how many units of houses to be constructed, based on their own objectives and budget constraints. In particular, the three types of agents have the following specification of preferences.

2.1. Preference of house buyers

A house buyer (denoted by agent B) has real demand for houses and thinks of house as a necessity. At time $n$, there are $q^B_0 (>0)$ house buyers who enter into the housing market. Each house buyer makes a decision about how many units of houses to buy by maximizing his/her consumption utility subject to his/her ability to make down payment, and next period's income. Property developers who supply houses. At any given time, house buyers and investors decide their property demands while property developers decide how many units of houses to be constructed, based on their own objectives and budget constraints. In particular, the three types of agents have the following specification of preferences.

$$
\max_{C^B_n, h^B_n} U^B(C^B_n) + k^B U^B(h^B_n)
\text{ s.t. } C^B_n + \theta^B P_n h^B_n = y_n^B.
$$

(2.1)

where $P_n$ is the housing price (without the rent) per unit of houses at time $n$, $C^B_n$ and $h^B_n$ denote the consumption amounts, respectively, of goods and houses by agent B at time $n$, $U^B(\cdot)$ and $U^B(\cdot)$ are agent B’s utility functions respectively of goods and houses, $k^B$ is the significance factor of properties to agent B compared with his/her goods consumption, $\theta^B \in (0, 1]$ is the down payment ratio on agent B's mortgage and $y_n^B$ is agent B’s income at time $n$. In (2.1), it shows that as long as the house buyer has the financial capacity to pay the down payment, he/she will buy houses to maximize his/her consumption utility at that time and then exit from the housing market. At time $n + 1$, a total of $q^B_{n+1}$ new house buyers will enter into the housing market and make their buying decisions. Without loss of generality, we assume that agent B’s income is constant, that is $y^B_n \equiv y^B$ for every time $n$. In addition, to simplify the analysis, we assume that agent B adopts the logarithmic utilities for both goods and houses, that is $U^B(x) = U^B(x) = \ln(x)$.

In a general case, the solvency of a house buyer should be considered, which can be done within the framework given by Adam et al. (2011). In this paper, we emphasize the impact of speculative behavior from investors and leave the general case for future work.
From the first order condition of (2.1), the optimal volume traded by agent B at time \( n \) is
\[
h^B_n = \frac{y_k \theta^B}{p_d \theta^B (k^B + 1)} > 0, \tag{2.2}
\]
which means that the demand of agent B depends on his/her income \((Y^B)\), his/her down payment per unit of houses \((\theta^BP_d)\) and the significance of houses to him/her \((k^B)\). Thus, to agent B, the higher income, the lower down payment and/or the higher significance, the higher demand of houses.

2.2. Preference of investors

At time \( t \), there are \( \phi^i_t > 0 \) investors in the housing market. An investor (denoted by agent \( I \)) optimizes his/her portfolio by putting his/her money into his/her bank account to obtain gross return \( R_I (> 1) \) or buying houses to make money on capital gain. We denote at time \( n \), his/her portfolio wealth by \( \Pi^I_n \), and the unit amount of houses he/she invests by \( h^I_n \). Thus, the wealth process of agent \( I \) can be formulated as
\[
\Pi^I_{n+1} = (\Pi^I_n - R h^I_n) R_I + \left(P_{t+1} + R_I Q_n\right) h^I_n,
\]
where \( Q_n \) is the price for renting one unit of houses in the period between times \( n \) and \( n + 1 \). Since rents are typically paid up-front (just after buying houses, that is at time \( n^+ \)), to express the rent at time \( n^+ \) in terms of currency at time \( n + 1 \), it should be inflated by the opportunity cost \( R_I \). Assume that agent \( I \) is a myopic mean-variance maximizer with very deep pockets and no budget or short selling constraints. That is, he/she maximizes his/her wealth expectation and minimizes its variance based on his/her beliefs. Therefore, we can write the objective of agent \( I \) as follows
\[
\begin{align*}
\max_{h^I_n} \quad & E^I_t(\Pi^I_{n+1}) - \frac{\sigma^I}{2} V^I_t(\Pi^I_{n+1}), \tag{2.3}
\end{align*}
\]
where \( E^I_t \) and \( V^I_t \) denote agent \( I \)'s 'beliefs' about the expectation and variance based on a publically available information set \( \mathcal{F}_t \) consisting of past prices and rents, that is \( \mathcal{F}_t = \{P_{t-1}, P_{t-2}, \ldots; Q_{t-1}, Q_{t-2}, \ldots\} \), and \( \sigma^I \) is his/her risk aversion coefficient.

By \( E^I_t(\Pi^I_{n+1}) = R_I \Pi^I_t + E^I_t(P_{t+1} + R_I Q_n - R_I P_n) h^I_n \) and \( V^I_t(\Pi^I_{n+1}) = (h^I_n)^2 V^I_t(P_{t+1} + R_I Q_n - R_I P_n) \), we can obtain the optimal trading volume of agent \( I \) at time \( n \) from (2.3) as follows
\[
h^I_n = \frac{E^I_t(P_{t+1} + R_I Q_n - R_I P_n)}{(\sigma^I)^2}, \tag{2.4}
\]
where \((\sigma^I)^2 = V^I_t(P_{t+1} + R_I Q_n - R_I P_n)\) is assumed constant for all times and \( E^I_t(P_{t+1} + R_I Q_n - R_I P_n) \) is time-varying. When \( E^I_t(P_{t+1} + R_I Q_n - R_I P_n) > 0 \), that is when investors expect the future gross return of houses will be higher than the opportunity cost \( R_I \), then they want to buy houses rather than saving money in their bank accounts to increase their wealth. Otherwise, investors want to sell houses.

2.3. Preference of developers

There are \( \phi^F_t (> 0) \) property developers in the housing market at time \( n \). Each developer (denoted by agent \( D \)) decides how many units of houses to be constructed and sold based on his/her profit maximization and financing cost. Without the consideration of delay effect of construction, agent \( D \)'s objective can be described as
\[
\begin{align*}
\max_{u^D_n} \quad & P_{u^D_n} - F^D(h^D_n) - \frac{R - 1}{R^D} D_n \tag{OPD},
\end{align*}
\]
where \( h^D_n \) is agent \( D \)'s construction/sale amount at time \( n \), \( R^D \) and \( D_n \) are, respectively, the amount of self-financing and bank loan, and \( F^D(x) = c x^2/2\) is the developer's cost function\(^5\) with \( c > 0 \). From (2.5), the developer's optimal supply is given by
\[
h^D_n = \frac{P_n}{C}, \tag{2.6}
\]
where \( C = \left(2 - \frac{1}{R^D}\right) > 0 \) is regarded as the development cost per unit of houses and the higher development cost, the lower supply.

2.4. Equilibrium

Equilibrium housing prices are derived by the market clearing condition, that is, the demand of property consumers is equal to the supply of proper developers. This implies,
\[
\phi^B_n h^B_n + \phi^F_n h^F_n = \phi^I_n h^I_n. \tag{2.7}
\]
For simplicity, we assume that \( \phi^I_n (i = B, I, D) \) is constant, denoted by \( \phi^I \). In addition, without loss of generality, we assume \( \phi^D = 1 \). Thus, \( \phi^B \) and \( \phi^F \) can be regarded as the normalized market populations of agents \( B \) and \( I \) by the number of property developers or market forces of a property developer.

2.4.1. Without speculative investors

To highlight the impact of speculative behavior among investors on housing prices, we first consider a benchmark notion without speculative investors. No investors enter into the housing market and then only house buyers and developers can affect the housing price. Therefore, the equilibrium housing price only relies on the real demand of house buyers and the supply of developers, which serves as a benchmark case for the rest of the paper. Combining (2.2) and (2.6) with (2.7), we can easily obtain that at any time \( n \), the equilibrium housing price is a constant, that is \( P_n = P^\ast \), where
\[
P^\ast = \left(C \frac{\phi^B \theta^B y_k b}{\theta^B (k^B + 1)}\right)^{1/2}, \tag{2.8}
\]
which is called a benchmark price and is determined by some fundamental economic factors like the income \((Y^B)\), the development cost \((C)\) and the down payment \((\theta^B)\). The higher income, the higher cost or the lower down payment directly contributes to the higher housing price.

2.4.2. With speculative investors

For a general case, we consider that there are speculative investors in the housing market. They base their investment decision on their own forecast about the future housing price, which can be described as
\[
E^I_t(P_{t+1} + R_I Q_n) = R_I P_{t-1} + R^I_{t+1}, \tag{2.9}
\]
where, considering the existence of the opportunity cost \((R^I)\), \( R_IP_{t-1} \) is a basic level of the future housing price (including the rent) based on the historical information \( \mathcal{F}_t \) and \( R^I_{t+1} \) represents a speculative component, which is adapted with \( \mathcal{F}_t \) and based on investors’ own beliefs.

(footnote continued)

1 Logarithmic utility functions are used to simplify the analysis. If other forms of utility functions were used, some but not all results could hold and some more complex phenomena were expected. Thank one referee for pointing it out.

2 In fact, this gross return \((R_I)\) also can be regarded as the opportunity cost of agent \( I \), including not only a riskfree rate but also the returns generated by other investment choices such as hedge funds and trusts.

3 Here the opportunity cost is only one way to inflate the rent and we also can use another factor like mortgage rate as an inflation factor, which would not change the main results in this paper.

4 Footnote continued) Here the corresponding marginal cost of development equals \( c^2 \), which reflects the possibility that there are scarce inputs into housing production, so that the marginal cost of development rises linearly with the amount of development, see Glaeser et al. (2008).
When \( R_{i+1} > 1 (< 0) \), investors optimistically (pessimistically) believe the housing price should be higher (lower) than its basic level. Thus, the trading volume (2.4) of houses by agent \( I \) is

\[
h^I_t = \frac{R_t P_{t-1} + R^I_{t+1} - R_t P_t}{\alpha'(\sigma')^2}, \tag{2.10}
\]

In this case, equilibrium housing prices not only depend on the real demand of house buyers and the supply of property developers, but also rely on the behavior of investors. To better clarify the role of investors in the housing market, we respectively consider one-period and multi-period cases as follows. In the one-period case, we study the impact of different factors on housing prices like income, down payment ratio, development cost and speculative behavior, while in the multi-period case, we emphasize the dynamic evolution of housing prices based on investors’ updating expectations.

3. One-period model

In this section, we consider a one-period case when investors just invest for one period. Compared with the benchmark case, different roles of three types of agents in the housing market are analyzed. Denote investors’ expectation about the future housing price (including the rent) as \( P'_0 \), that is \( P'_0 = E^I(P_t + R_t Q_t) \), and let

\[
m_0 = \frac{1}{C} + \mathcal{A} R_t, \quad m_1 = \mathcal{A} P'_0 \quad \text{and} \quad m_2 = \frac{q^I \phi_k}{\phi_k(k + 1)} \tag{3.1}
\]

where \( \mathcal{A} = \frac{\varphi^I}{\varphi'} \) is called the risk characteristics of investors. From (3.1), we can see that \( m_0 \) represents developers’ cost factor, \( m_1 \) represents investors’ speculative behavior factor and \( m_2 \) represents buyers’ real demand factor. Combining (2.2), (2.4) and (2.6) with (2.7), we denote the equilibrium housing price as \( \mathcal{P} \) and obtain the following result.

**Proposition 3.1.** The equilibrium housing price \( \mathcal{P} \) satisfies

\[
m_0 \mathcal{P}^2 - m_1 \mathcal{P} - m_2 = 0, \tag{3.2}
\]

that is,

\[
\mathcal{P} = \frac{m_1 + \sqrt{m_1^2 + 4m_0 m_2}}{2m_0} > 0, \tag{3.3}
\]

which negatively depends on the cost factor but positively relies on the factors of speculative behavior and real demand, that is

\[
\frac{\partial \mathcal{P}}{\partial m_0} < 0, \quad \frac{\partial \mathcal{P}}{\partial m_1} > 0 \quad \text{and} \quad \frac{\partial \mathcal{P}}{\partial m_2} > 0.
\]

**Proposition 3.1** shows that the equilibrium price is the result of combined action from \( m_0, m_1 \) and \( m_2 \). Note that \( \frac{\varphi'}{\varphi} = -\frac{1}{\alpha'(\sigma')^2} \). Thus, similar to the benchmark case, the equilibrium price \( \mathcal{P} \) positively depends on development cost and real demand, as shown in Fig. 3.1. But those two factors are not unique impact factors any more and speculative behavior of investors also have the positive effect on the equilibrium price. Especially, investors’ risk characteristics \( \mathcal{A} \) and expectation \( (P'_0) \) have the following impacts on the equilibrium housing price \( \mathcal{P} \).

**Proposition 3.2 (Impact of Investors).** Let \( \mathcal{H}' = \varphi^I h^I \) denote the total optimal volume traded by investors. If \( P'_0 \geq \mathcal{P} \), then \( \mathcal{H}' = 0 \) and furthermore, \( \mathcal{P} = \mathcal{P} \) and \( \frac{\partial \mathcal{P}}{\partial m_0} = 0 \).

**Proposition 3.2** shows how the equilibrium housing price with speculative behavior compares to the benchmark price. If \( P'_0 = R_t \mathcal{P} \), we are back to the benchmark case with \( \mathcal{P} = \mathcal{P} \). For \( P'_0 \neq R_t \mathcal{P} \), investors’ speculative behavior becomes an important factor for housing prices.

The liquidity supplied to and taken away from the market by investors changes with their expectations about future prices. If investors’ expectation \( P'_0 \) is higher than the future value \( R_t \mathcal{P} \) of the benchmark price, investors are in the position to buy, that is \( \mathcal{H}' > 0 \), which pushes the price beyond the benchmark level, as illustrated in the first quadrants of the top panel of Fig. 3.2. Meanwhile, the equilibrium housing price increases with their risk characteristics. This is because the higher risk characteristics \( (\mathcal{A}) \) of investors, the lower risk aversion coefficient \( (\sigma') \) or expectation about the volatility \( (\sigma') \) of the housing market and/or the more investors in the market, which increases their demand to buy houses. Furthermore, the housing price is pushed up, see the bottom panel of Fig. 3.2.

Similarly, when investors are in the position to sell, that is \( P'_0 < R_t \mathcal{P} \), their role in the housing market is similar to a developer and they can be regarded as suppliers of houses. Therefore, it can drive the equilibrium price below the benchmark level and decreasing with the risk characteristics of investors, as shown in Fig. 3.2.

Therefore, changes in investors’ expectations will result in changes in the market clearing price. If investors updated their expectations based on the past market information, then investors’ expectations were time-varying, which would give us more interesting price evolution and dynamics, like the persistence and predictability of housing prices and even the localization property, shown in the empirical literature. We will explore this in the following section.

4. Multi-period models

To analyze the impact of investors’ dynamic expectations, we follow the idea of Dieci and Westerhoff (2012) and consider that investors can update their beliefs about the trend of the housing price, which are based on the historical information about housing prices but not the contemporaneous realizations. In addition, considering heterogeneity in beliefs, in the paper, we assume that investors can adopt two ways to update their beliefs - extrapolation and mean-reversion. To examine the role of different types of investors, we first assume that there is only one type of investors who use an extrapolative method to update their expectations about future housing prices. Second, we consider investors update their expectations about future housing prices based not only on an extrapolative component but also on a mean-reverting one. We contrast the two cases to explain different impacts of the extrapolative and mean-reverting beliefs on the dynamic evolution of housing prices.

4.1. The model with extrapolation

We consider investors who use an extrapolative method to update their beliefs, meaning they are confident in the continuation of the price trend in the next period. Investors’ belief under the extrapolative method can be formalized as

\[
R_{i+1} = R_{i+1}^E = f(P_{i-1} - \mathcal{P}), \tag{4.1}
\]

where \( f(>0) \) is the extrapolative intensity to the housing price trend. Then (4.1) implies that when the housing price is above (below) its benchmark value, trend followers optimistically (pessimistically) believe in a further price increase (decrease).

Note that each investor is a mean-variance maximizer. The demand of each investor is therefore given by

\[
h^I_t = h^I_t = \frac{R_t P_{t-1} + R^I_{t+1} - R_t P_t}{\alpha'(\sigma')^2}. \tag{4.2}
\]

Then combined with (2.2) and (2.6), the equilibrium price is deter-
mined by the market clearing condition given by (2.7), satisfying
\[ m_P m_P m_P = 0, \]
\[ n m_n n m_n (4.3) \]
where
\[ \bar{m}_{1,a} = \bar{m}_{1,a}(P_{n-1}) = \mathcal{A}(R_f P_{n-1} + R_{*n+1}). \] (4.4)
This gives us a one-dimensional discrete dynamic system which can be described as
\[ P_n = \bar{G}(P_{n-1}) = \frac{\bar{m}_{1,a}(P_{n-1}) + \sqrt{\bar{m}_{1,a}^2(P_{n-1}) + 4m_0 m_2}}{2m_0}, \] (4.5)

The following proposition characterizes the price dynamics of the above dynamic system.

**Proposition 4.1 (Stability and Bifurcation).** Denote \( \bar{f}^* = (\mathcal{A}C)^{-1}, \)
\[ f^{**} = 2\bar{f}^* \] and \( \bar{F} = \frac{\bar{F}}{\mathcal{A}C^{1/2}}. \)

1. When \( 0 < f < f^{**} \), the benchmark price \( \bar{F} \) is a unique steady state of (4.5), which is stable.
2. When \( \bar{f}^* < f < f^{**} \), there are two steady states in (4.5), the benchmark price \( \bar{F} \) and a non-benchmark steady state \( \bar{F}' \), where \( \bar{F} \) is stable and \( \bar{F}' \) is unstable.
3. When \( f = f^{**} \), (4.5) undergoes a transcritical bifurcation.

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Fig. 3.1. The relationships between the equilibrium housing price and different factors. Here \( R_f = 1.05, \ k_B = 2, \ \theta = 0.3, \ \sigma = 5, \ \sigma' = 0.1, \ \phi = 0.1, \ \psi = 1 \) with (a) \( \bar{p}_0 = 5, \ Y = 10 \) and (b) \( \bar{p}_0 = 5, \ C = 0.1 \).

Fig. 3.2. The relationships between the equilibrium housing price and investors' expectation \( P_{QI} \) or risk characteristics \( I \). Here \( Y = 10, \ R_f = 1.05, \ k_B = 2, \ \theta = 0.3 \) and \( \phi = 1 \). (a–b) For the impact of investors' expectation, take \( \sigma = 5, \ \sigma' = 0.1, \ \psi = 1 \) and \( C = 0.1 \). (c–d) For the impact of investors' risk characteristics, take \( \bar{p}_0 = 4 \) and let \( C = 1 \) for the selling case and \( C = 0.1 \) for the buying case.
When \( f > \star \), the benchmark price \( P \) loses its stability and becomes unstable while \( \P \) becomes stable. From Proposition 4.1, we see that when investors’ extrapolative intensity increases, system (4.5) has different steady states, as illustrated in Fig. 4.1(a) and their stabilities can change, as shown in Fig. 4.1(b). In detail, when investors’ extrapolative intensity is low, that is \( 0 < f < \star \), then (4.5) maintains the characteristics of the benchmark case. The benchmark price is a unique steady state and is stable in the sense that price deviations from the benchmark one will eventually vanish in the long run. However, if the extrapolative intensity is fairly strong so that \( \star < f < \star \), then the small upward deviations from the benchmark level \( \star \) do not persist while the big upward deviations \( \star > \star \) may evolve into permanent deviations which will eventually blow up the housing price. The fairly strong extrapolative behavior causes large upward deviations from the benchmark level to be further reinforced by the increasing housing prices and demand, which is consistent with the survey result of Piazzesi and Schneider (2009). Nevertheless, deviations below the benchmark level will converge to the benchmark price. This is because there exists a real demand from house buyers in the market and the extrapolative behavior of investors is only fairly strong, which results the house price cannot deviate downward from the benchmark price very far and it eventually converges back to the benchmark price \( P \) (see Fig. 4.1(b)).

Finally, when the extrapolative intensity is very strong, that is \( f > \star \), then the benchmark price loses its stability and deviations of housing prices from the benchmark price are inevitable. Whether the deviation is upward or downward depends on the market situation. If the initial price is higher than the benchmark level, then the upward price trend will persist and lead to an eventual blow up, as illustrated in the top panel of Fig. 4.1(c). On the other hand, if the initial price is lower than the benchmark value, then the downward price trend will continue until it hits the level \( \P \) which is strictly positive because of the existence of the real demand from house buyers at any time, as illustrated in the bottom panel of Fig. 4.1(c). In this case, investors act as suppliers of properties, which increase the supply of houses and drive the housing price down.

Note that except the unique steady state case of the benchmark equilibrium as the extrapolative intensity is small, that is \( 0 < f < \star \), the evolution of housing prices is path-dependent. That is to say, different initial prices will result in different price dynamics, either explosion or convergence to some bounded level. If the determinants of initial prices were some local factors, like local income, cultures, commodity prices and policy environment, then the price changes would display the localization rather than national property, which phenomenon has been shown in the empirical literature. In addition, from Fig. 4.1, we can see that the price trends are continuous, which corresponds to the persistence and predictability of price changes existing in the real housing market. Therefore, our model has the good

Fig. 4.1. \( \star = 0.5 \) and \( \star = 1 \) at \( T = 0.1, k = 0.2, \theta = 0.3, C = 0.1, R = 1.05, r' = 5, \sigma = 0.1 \) and \( \phi = 1 \). Here in (b), the dash-dot lines correspond to unstable steady states while stable steady states are given by solid lines.
ability to explain some stylized facts in housing markets.

4.2. The model with extrapolation and mean reversion

In Section 4.1, we see that extrapolative investors who believe in the persistence of price trend may cause the housing market to explode because of their increasing demand and reinforced price expectations. In this section, we show that the interaction among different types of investors increases the complexity of the system, nevertheless, stabilizing the system.

Similar to Dieci and Westerhoff (2012), we consider two types of speculative behavior in the housing market. In addition to the extrapolative component shown in Section 4.1, there is a mean-reverting component used in the market. The mean-reverting component can be written as

\[ R_{n+1}^{MR} = g(P - P_{n-1}), \]  

where \( g \in (0, 1) \) measures the mean-reverting speed adopted by investors. Eq. (4.6) implies that if the housing price is above (below) its benchmark price, mean-reverting investors believe that the price is overestimated (underestimated) and the future price will decrease (increase). In other words, they believe that the housing price cannot deviate far away from its benchmark price in the long run. Thus, the mean-reverting strategy should play a part in stabilizing the housing price.\(^8\)

Thus, the total speculative demand from investors is

\[ \mathcal{H}_n^I = \omega_a g^\alpha h_a^F + (1 - \omega_a)g^\beta h_a^{MR}, \]  

where

\[ h_a^I = \frac{R_f R_{n-1}^F + R_a^MR_{n-1}^F - R_f R_a^F}{a'(\alpha')^2}, \]  

and \( \omega_a \) and \( 1 - \omega_a \) stand for the market fractions respectively of extrapolative and mean-reverting investors. Furthermore, the speculative demand \( \mathcal{H}_n^I \) of agent \( I \) is

\[ h_n^I = \frac{\mathcal{H}_n^I}{g^\beta} = \omega_a h_n^F + (1 - \omega_a)g^\beta h_n^{MR} = \frac{R_f R_{n-1}^F + R_a^MR_{n-1}^F - R_f R_a^F}{a'(\alpha')^2}, \]  

where \( R_{n+1}^{MR} \) is a forecast variable with a nonlinear law of motion given by

\[ R_{n+1}^{MR} = \omega_a R_{n+1}^F + (1 - \omega_a)R_{n+1}^{MR}, \]  

which is the average of different investors’ beliefs weighted by their market fractions. We consider social interactions between different investors and assume investors can switch their forecasting rules with respect to market circumstances such that in each step the impacts of those two components are time-varying. The switching mechanism is following a formulation by He and Westerhoff (2005) and Bauer et al. (2009). Investors seek to exploit price trends (that is, bull and bear markets). However, the more the price deviates from its benchmark value, the more agents come to the conclusion that a benchmark market correction is about to set in. As a result, an increasing number of investors opt for the mean-reverting strategy. Furthermore, similar to Dieci and Westerhoff (2012), the relative impact of extrapolators is formalized as

\[ \omega_a = \frac{1}{1 + \mu (P_{n-1} - \bar{P})^2}. \]  

then it is reduced to the case in Section 4.1. Hereafter, if not specified, we assume that \( \mu > 0 \).

Therefore, the equilibrium price is determined by the market clearing condition given by (2.7), satisfying

\[ m_1 P_n^F - m_2 P_n - m_3 = 0, \]  

where

\[ m_n = m_1 (P_{n-1}) = \mathcal{A}(R_f R_{n-1}^F + R_a^F), \]  

which is also a one-dimensional discrete dynamic system as

\[ P_n = G(P_{n-1}) = \frac{m_n (P_{n-1}) + \sqrt{m_n^2 (P_{n-1}) + 4m_n m_2}}{2m_0}. \]  

To get the steady states of system (4.14), we let \( P_{n-1} = P_\ast \) and obtain

\[ (P_\ast - P)T(P_\ast) = 0, \]  

where

\[ T(P_\ast) = a(P_\ast)^3 + b(P_\ast)^2 + cP_\ast + d = 0, \]  

and

\[ a = \left[ \mathcal{A}g + \frac{1}{C} \right], \]  

\[ b = -\left[ 2\mathcal{A}g + \frac{1}{C} \right]P_\ast, \]  

\[ c = -\left[ 1 - \mu P_\ast^2 \right] - \mathcal{A}(1 - \mu P_\ast^2), \]  

\[ d = \frac{P_\ast}{C}(1 + \mu P_\ast^2). \]

Then the discriminant \( \Delta \) of (4.16) is

\[ \Delta = 27a'^2d^2 - 18abcd + 4ac^3 + 4b'd' - b'e^2. \]

Furthermore, we can get the following proposition.

**Proposition 4.2 (Existence of Steady States).** The benchmark price \( \bar{P} \) is always a steady state of (4.14). When \( \Delta < 0 \), (4.14) has another two steady states \( (P_1^\ast, P_2^\ast) \) where \( P_2^\ast \geq P_1^\ast > 0 \).

Proposition 4.2 demonstrates the structure change of (4.14) because of the existence of two new steady states when \( \Delta < 0 \) as shown in Fig. 4.2(a). At \( \Delta = 0 \), two new equal steady states \( (P_1^\ast, P_2^\ast) \) appear, which corresponds to a saddle-node bifurcation. So we call \( \Delta = 0 \) a saddle-node bifurcation boundary. In addition, similar to Proposition 4.1, we can show that the benchmark price \( \bar{P} \) may lose its stability at \( f = f^{\ast\ast} \) which corresponds to a transcritical bifurcation and is therefore called a transcritical bifurcation boundary. If we use the extrapolative intensity \( (f) \) and mean-reverting speed \( (g) \) as a pair of parameters to plot the bifurcation boundaries, as illustrated in Fig. 4.2(b), we show that for any fixed \( g \), with the increase of \( f \), system (4.14) can undergo two types of bifurcations. However, when \( f \) is fixed and \( g \) increases, the benchmark price \( \bar{P} \) may remain stable or unstable and at most system (4.14) may just undergo a saddle-node bifurcation. Therefore, we choose \( f \) as a bifurcation parameter, on the one hand to illustrate those two types of bifurcations and on the other hand to compare with the case in Section 4.1.

**Proposition 4.3 (Stabilities and Bifurcations).** Let \( f^\ast \) denote a unique, positive solution of \( \Delta = 0 \) and \( f^{\ast\ast} = 2(\mathcal{A}/C)^{-1} \). Assume \( \partial P_\ast/\partial P_\ast^F \Big|_{P_\ast = f^{\ast\ast}} < 0 \), then

1. When \( 0 < f < f^\ast \), the benchmark price \( \bar{P} \) is always stable.
2. At \( f = f^\ast \), a saddle-node bifurcation occurs and two non-benchmark steady states \( (P_1^\ast, P_2^\ast) \) appear.
3. When \( f^\ast < f < f^{\ast\ast} \), \( P_1^\ast \) is unstable and \( P_2^\ast \) is stable while the stability of the benchmark price \( \bar{P} \) keeps invariant.
4. At \( f = f^{\ast\ast} \), a transcritical bifurcation occurs.
5. When \( f > f^{\ast\ast} \), the benchmark price \( \bar{P} \) becomes unstable while both \( P_1^\ast \) and \( P_2^\ast \) are stable.
Proposition 4.3 shows that the benchmark price is not always stable and it may undergo two types of bifurcations. When investors’ extrapolative intensity $f$ is small, that is $0 < f < f^*$, the benchmark price is stable. It goes from stable to unstable as the extrapolative intensity becomes increasingly strong. In particular, when this intensity $f$ increases to $f^*$, the system undergoes a saddle-node bifurcation. That is, there exist two non-benchmark steady states $(P_1^*, P_2^*)$ at $f^* < f < f^{**}$, with one ($P_2^*$) being stable and the other ($P_1^*$) unstable. At the same time, the benchmark price $(P)$ remains stable. This implies that there are two attractors in the system - $P_2^*$ and $P_1^*$, as shown in Fig. 4.3(a). Thus, the two attractors split the price space into two parts separated by $P_2^*$, where prices exhibit different evolutionary processes. The coexistence of a stable benchmark price and a stable non-benchmark price in our simple evolutionary model can be explained by the following simple economic intuition. When investors’ extrapolative intensity is relatively low (that is $f^* < f < f^{**}$), similar to Proposition 4.1, for prices near the benchmark level, investors’ extrapolative behavior is not strong enough to cause permanent deviations from the benchmark price. In addition, because of the existence of real demand and relatively low extrapolation, the prices cannot deviate downward from the benchmark value very far and eventually converge back to it. For the upward price trend that has deviated far away from the benchmark level, the extrapolative trend-following behavior strengthens the upward deviation of prices with stronger demands that feed back into price increases, which, similar to the case in Section 4.1, lets the price trend persistent and predictable. Unlike that case, however, this up trend cannot be sustained endlessly because of mean-reverting investors in the market. As the prices deviate further from the benchmark level, more and more investors form strong beliefs that the prices will mean revert. As a result, an increasing number of investors opt for the mean-reverting strategy and begin to sell houses, which increases the total house supply in the market. This creates a balance between the buying and selling sides, which stabilizes the equilibrium price at $P_2^*$, as illustrated in Fig. 4.3(b). From this, we can see that the introduction of mean-reverting investors into the model, on one side, increases the complexity of the system; on the other side, has the role of stabilizing an otherwise explosive market because of the interaction between the opposite trading strategies of different investors. Therefore, the rich types of investors are useful to stabilize the system bounded in a certain range rather than exploding because of their counteracting behavior.

As the extrapolative intensity $f$ increases beyond $f^{**}$, the bench-

\[ f = 0.4 \]
\[ f = 0.95 \]
\[ f = 1.2 \]

(a) Steady states at $g = 0.5$

\[ \text{Transcritical bifurcation boundary (} f = f^{**} \) \]
\[ \text{Saddle-node bifurcation boundary (} \Delta = 0 \) \]

(b) Bifurcation boundaries

Fig. 4.2. $\gamma^B = 10, k^B = 2, \theta^B = 0.3, \mu = 0.1, \sigma^B = 0.1, \alpha = 5, \varphi = 1, R_f = 1.05$.
losses its stability while the originally unstable steady state 
\( \alpha = 0.1 \), \( \varphi = 0.1 \). Otherwise, the
market price relative to the benchmark level. Finally, mean-reverting
extrapolation from investors is strong enough for prices to move away
reasons make the path-dependent phenomenon occur. First, the
extrapolation from investors is strong enough for prices to move away
from the benchmark price, which leads to the instability of the
benchmark price at
\( \theta = 0.5 \), \( \beta = 0 \). However, whether the adjustment of the down
payment ratio has the desired effect depends on the timing of the policy
adjustment. We take \( \theta^B = 0.3 \) and \( \theta^P = 0.7 \) to illustrate our point.
In the long run, we know that because of a higher real demand
corresponding to a lower down payment, the benchmark price at
\( \theta = 0.3 \) is higher than that at \( \theta = 0.7 \) and so are the corresponding
non-benchmark prices, as shown in Fig. 5.1. Therefore, in the long run,
an increase(decrease) in the down payment ratio may cool(warm) the
housing market to some extent. However, this may not be true in the
short run.

For instance, if investors have very strong extrapolative intensity
like \( \theta = 1.1 \) which makes the benchmark price unstable, then when the
market is booming, the extrapolative belief of investors will push the
housing price up and up. At that time, the policy maker might want to
increase the down payment ratio in order to avoid the housing market
overheating. However, the effectiveness of this adjustment is state-
dependent. If the price is still in its up-trend (for example, at around
\( n = 70 \)), then the market will continue its trend after the increase in the
down payment ratio from \( \theta^B = 0.3 \) to \( \theta^B = 0.7 \) as illustrated in
Fig. 5.2(a). Therefore, in this case, the adjustment of the down payment
ratio does not have a significant impact on the housing market. Only
when the market has been booming for a while (for example, at
\( n = 200 \)), can the increase in the down payment ratio push the housing
price down, as shown in Fig. 5.2(c). The amount of the drop in prices,
however, is limited. The intuition is as follows. Although the down
payment ratio plays a role in setting the benchmark value, it has the
limited impact on investors’ beliefs about the housing price trend.
Increasing the down payment ratio, therefore, mainly drives some real
demand out of the market but not the speculative demand, and
sometimes even increases investors’ demand. Similarly, when the
market goes down, decreasing the down payment ratio provides the
limited price support no matter when this policy adjustment is put into
effect, as illustrated in the right panel of Fig. 5.2.
In summary, the effectiveness of adjusting the down payment ratio
depends on the timing of the policy implementation and policy
regulations based on the down payment ratio have limited power.

5. Policy implications and application

Based on the analysis in Section 4, the structural knowledge of the
system may yield important policy insights that can reduce the
volatility of housing prices. For example, by (2.8) and Proposition
4.3, we know that the evolution of the housing market depends on the
market environment because the benchmark price \( P \) and the bifurca-
tion points \( f^* \) and \( f^{**} \) all depend on certain fundamental factors, for
example, the down payment ratio and development cost. Thus, housing
prices can be regulated through the adjustment of fundamental factors
that change the market supply and/or demand. In the following, we
take the down payment ratio and development cost as examples to
illustrate the relationship between housing prices and the policy
implementations within the framework given in Section 4.2. At last,
with real data on Beijing housing prices, practical relevance of the
paper is illustrated.

5.1. Down payment ratio

From (2.2) and (2.8), we know that the down payment ratio can
affect the real demand of house buyers and the benchmark level of
housing prices, which is the reference price adopted by investors.
Therefore, policy makers can regulate housing markets by changing the
down payment ratio. However, whether the adjustment of the down
payment ratio has the desired effect depends on the timing of the policy
adjustment. We take \( \theta^B = 0.3 \) and \( \theta^P = 0.7 \) to illustrate our point.

In the long run, we know that because of a higher real demand
corresponding to a lower down payment, the benchmark price at
\( \theta = 0.3 \) is higher than that at \( \theta = 0.7 \) and so are the corresponding
non-benchmark prices, as shown in Fig. 5.1. Therefore, in the long run,
an increase(decrease) in the down payment ratio may cool(warm) the
housing market to some extent. However, this may not be true in the
short run.

For instance, if investors have very strong extrapolative intensity
like \( \theta = 1.1 \) which makes the benchmark price unstable, then when the
market is booming, the extrapolative belief of investors will push the
housing price up and up. At that time, the policy maker might want to
increase the down payment ratio in order to avoid the housing market
overheating. However, the effectiveness of this adjustment is state-
dependent. If the price is still in its up-trend (for example, at around
\( n = 70 \)), then the market will continue its trend after the increase in the
down payment ratio from \( \theta^B = 0.3 \) to \( \theta^B = 0.7 \) as illustrated in
Fig. 5.2(a). Therefore, in this case, the adjustment of the down payment
ratio does not have a significant impact on the housing market. Only
when the market has been booming for a while (for example, at
\( n = 200 \)), can the increase in the down payment ratio push the housing
price down, as shown in Fig. 5.2(c). The amount of the drop in prices,
however, is limited. The intuition is as follows. Although the down
payment ratio plays a role in setting the benchmark value, it has the
limited impact on investors’ beliefs about the housing price trend.
Increasing the down payment ratio, therefore, mainly drives some real
demand out of the market but not the speculative demand, and
sometimes even increases investors’ demand. Similarly, when the
market goes down, decreasing the down payment ratio provides the
limited price support no matter when this policy adjustment is put into
effect, as illustrated in the right panel of Fig. 5.2.

In summary, the effectiveness of adjusting the down payment ratio
depends on the timing of the policy implementation and policy
regulations based on the down payment ratio have limited power.
and $\theta$ as an example to illustrate the effectiveness of the model, we use monthly Beijing housing prices from the WIND Financial Terminal\footnote{WIND is a leading financial data provider in China, providing accurate and complete data on financial markets and the macroeconomy.} to test the explanation power of our model on the real housing market. For the data from February, 2002 to February, 2016, shown in Fig. 5.5, although during the Financial crisis in 2008, there are some adjustments in the housing prices, we can see that the real time series of the Beijing housing market has a significantly increasing pattern. In particular, after the financial crisis, the housing prices of Beijing bounce up quickly, beyond the increases of the fundamental economic factors. Therefore, the behavior of speculative investors would exist in the market, similar to the phenomena illustrated in (4.14). Note that system (4.14) is a deterministic nonlinear dynamic system while the real data is a stochastic time series. To match this gap, based on the idea of He and Li (2015), we introduce a noise term in (4.14) as follows.

5.2. Development cost

Different from the case of the down payment ratio, decreases the benchmark level of the housing price and increases the rationality of the market because of the increasing market fraction of mean-reverting investors. On the other side, it weakens the sensitivity of extrapolative investors corresponding to the increase of $f^{**}$, which leads the new benchmark price stable at $C = 0.075$. Therefore, the housing market can be cooled down to converge to the new benchmark level, see Fig. 5.4(a). Similarly, when the market goes down, decreasing the development cost is also a better measure to adjust the housing market because in this case, the new benchmark price is stable, which attracts the market back to its fundamental level, as illustrated in Fig. 5.4(b).

On the contrary, no matter whether the market is overheated with bubbles or stuck in depression, an increase of the development cost is not a good way to adjust it. In some cases, this method even can deteriorate the market. This is because the increase of the development cost can raise the benchmark price and strengthen the sensitivity of extrapolative investors (that is, the decrease of $f^{**}$), which makes the whole housing market more unstable and catalyzes the further boom or bust of the market, as shown in the bottom panel of Fig. 5.4.

Summing up, decreasing the development cost can increase the rationality of the market and decrease the sensitivity of extrapolative investors, which helps the stability of the benchmark price, reviving the market and stabilizing housing prices.

5.3. Application

To illustrate the effectiveness of the model, we use monthly Beijing housing prices from the WIND Financial Terminal\footnote{WIND is a leading financial data provider in China, providing accurate and complete data on financial markets and the macroeconomy.} to test the explanation power of our model on the real housing market. For the data from February, 2002 to February, 2016, shown in Fig. 5.5, although during the Financial crisis in 2008, there are some adjustments in the housing prices, we can see that the real time series of the Beijing housing market has a significantly increasing pattern. In particular, after the financial crisis, the housing prices of Beijing bounce up quickly, beyond the increases of the fundamental economic factors. Therefore, the behavior of speculative investors would exist in the market, similar to the phenomena illustrated in (4.14). Note that system (4.14) is a deterministic nonlinear dynamic system while the real data is a stochastic time series. To match this gap, based on the idea of He and Li (2015), we introduce a noise term in (4.14) as follows.
from 0.075 and to match the characteristics of the real housing prices. The real

1

f

=1.

θ

θ

be the structural parameters of interests in our housing pricing model that only. Therefore, in this paper, following Li et al. (2010), we select

structural parameters of the model (5.1) through a systematic procedure. In particular, we pay attention to the range of changes in the weights.12 A genetic algorithm is then exploited to find the optimal solution of the parameters, a genetic algorithm is adopted to solve the problem.

A genetic algorithm is derived from the theory of natural selection. Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by relying on bio-inspired operators such as mutation, crossover and selection. In this paper, the six parameters of the model are encoded into chromosome. The differences between numerical features of the time series of the real housing prices, and those generated by the model are functioned into the objective function. More precisely, denote θ the parameter space including the above six parameters. Let θ = (μ, m2, μ, g, C, f) ∈ Θ be the vector of the six parameters, XBJ be the simulated time series of the model, XBJ be the real data of the Beijing housing prices, thus we solve

\[ \theta^* \in \arg \min_{\theta \in \Theta} \| \tilde{\beta}_M - \beta_M \| + \omega_1 \max(X_M) - \max(X_M) + \omega_2 \min(X_M) \] (5.2)

for the standard Euclidean norm \( \| \cdot \| \) and \( \omega_1, \omega_2 > 0 \) are adjustment weights.12 A genetic algorithm is then exploited to find the optimal parameters (\( \theta^* \)) to match the characteristics of the real housing prices. From Fig. 5.6, we can see that the selection method of model parameters is very effective. The selected housing pricing model is able to characterize successfully not only the strong autocorrelations, but also the persistently increasing pattern in the Beijing housing prices as well. From Table 1, we know that this continuously increasing pattern is generated by the extrapolative behavior of investors. In fact, we can see that during the period from February, 2002 to February, 2016, the extrapolative intensity (f) of investors is very strong and is much bigger than the saddle-node and transcritical bifurcation points (f* and f**). The adjustment weights are used to let the deviations of the autocorrelations and those of the price maximum and minimum at the same level.

12
both, which means that the benchmark price is unstable and the price will deviate from the benchmark price upwards because its initial price is bigger than \( P \). In other words, in the real housing market, compared with some fundamental factors, including national income, down payment ratio and development cost, the behavior of investors, including their bounded rationality and heterogeneity, is more important to push the housing price up.

For the continuously increasing trend of the Beijing housing prices, the Beijing government has taken many measures to cool down the Beijing housing market, for example, by increasing the down payment ratio and adding restrictions to home-purchase qualification. However, the consequence of these policies is not obvious, that is, the increasing trend has not changed. From (5.1), we can get some insights on the failure of these policies. In particular, we know that from April, 2010, the government took steps to cool the housing market by increasing the down payment ratio to at least 30% and especially, to 50% for the second house, which was later increased to 60% in 2011 and then 70% in 2013. However, we can see that these measures have little impact on the Beijing housing prices. In fact, using the model (5.1), we can calibrate the parameters corresponding to the situations before and after the housing policy adjustments. That is, we take the data before April, 2010 as the market without increasing the down payment ratio and the data after May, 2010 as the market adjusted by the down payment policies. From Table 1, before the policy change, the extrapolative behavior is very salient, which makes the benchmark price unstable because of \( f > f^{**} \). By contrast, after increasing the down payment ratio, the extrapolative intensity is decreased while the mean-reverting speed is increased. It seems that it would cool the market down because the mean-reverting behavior has the role of stabilizing the housing price and the extrapolative behavior is weakened. However, we know that all the six parameters, not just one, play a part in the housing market together. Hence, on the whole, the extrapolative and mean-reverting behavior cannot cool the market down because we can see that the extrapolative intensity \( f \) is still bigger than \( f^{**} \), which means that the housing price is still kept far away from the benchmark price. Similar to the illustration in Section 5.1, the effectiveness of the down payment policy is not obvious.

Therefore, the housing price cannot regress back to the benchmark level and may even deviate further away.

The overall analysis in this subsection shows that the selection method of model parameters is very effective. By calibrating the structure parameters, we can know the market situation and get some insights of the policy adjustments. This is probably due to the simplicity of the housing pricing model, which makes it possible to identify potential sources and mechanisms of the model in matching some important characteristics in the Beijing housing prices. However, this simplicity also makes the model unable to match too many characteristics, for example, the time series of the real data, illustrated in Fig. 5.6(c), which is beyond the scope of the paper and we leave it as our future work.

### 6. Conclusion

We study a housing pricing model with heterogeneous agents including house buyers, investors and property developers. Each agent maximizes his/her own objective subject to financial constraints. House buyers want to maximize their consumption utility based on consumption goods and real demands of houses when they are able to make down payments of the houses they buy. Investors make investment decisions in order to maximize their wealth’s mean-variance

![Autocorrelations of the model](a.png)

![Autocorrelations of the real data](b.png)

![Time series of the model](c.png)

![Time series of the real data](d.png)

**Fig. 5.6.** Autocorrelations and time series of the real data and the simulated data.

<table>
<thead>
<tr>
<th>Time period</th>
<th>( \beta )</th>
<th>( m_1 )</th>
<th>( \mu )</th>
<th>( g )</th>
<th>( C )</th>
<th>( f )</th>
<th>( f^{**} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002–2006</td>
<td>1.2362</td>
<td>0.1562</td>
<td>0.1002</td>
<td>0.0884</td>
<td>1.9378</td>
<td>1.1778</td>
<td>0.6709</td>
</tr>
<tr>
<td>2002–2010</td>
<td>1.0512</td>
<td>0.1937</td>
<td>0.3497</td>
<td>0.0526</td>
<td>1.5714</td>
<td>1.5364</td>
<td>1.0081</td>
</tr>
<tr>
<td>2010–2016</td>
<td>1.6921</td>
<td>1.8027</td>
<td>0.1925</td>
<td>0.4011</td>
<td>1.7454</td>
<td>0.5328</td>
<td>0.2898</td>
</tr>
</tbody>
</table>
utility based on their speculative expectations. Property developers supply houses to maximize their profit. Without speculative investors, the equilibrium housing price is determined by real cost and demand, which serves as a benchmark price. In contrast, because of the existence of speculative behavior, the equilibrium housing price may persistently deviate from the benchmark level and even explode.

Furthermore, the model produces several bifurcation routes to the deviations of housing prices from the benchmark value, as investors’ extrapolative intensity increases. The bifurcations depend on different types of investors’ behavior. Investors with only extrapolative strategy give rise to a transcritical bifurcation of the benchmark price, in which the benchmark price loses its stability and one non-benchmark price becomes a stable attractor. In this case, when the extrapolative intensity is large, an upward deviation of housing prices away from the benchmark level can lead to an explosion of the house market. In the whole evolutionary process, the housing price trends are persistent and predictable, which is observed empirically. However, investors with both extrapolative and mean-reverting strategies lead to a saddle-node bifurcation. Although the interaction between heterogeneous investors increases the complexity of the whole system such as the appearance of two attractors, investors’ different strategies balance each other and help stabilizing the market to keep the system bounded. The appearance of two attractors makes the housing price evolution depend on its local value. This corresponds to the local characteristics of housing price changes observed in the real housing market. Finally, from the viewpoint of policy implications, the model indicates that the down payment ratio is not a very effective means to adjust the housing market whenever the market is booming or busting, which is illustrated by the real data of the Beijing housing market. But, decreasing the development cost of developers can increase the rationality of the market and decrease the sensitivity of extrapolative investors, which helps the stability of the benchmark price, reviving the market and stabilizing housing prices.

Therefore, by constructing some theoretical models, this paper demonstrates that speculative behavior of investors is an important factor that affects the housing price dynamics and evolution. The results obtained in this paper can provide interesting insights into the generation mechanism on some stylized facts observed in the real market and policy implications. However, in order to make the model parsimonious and to focus on the speculative behavior of investors, we consider a very simple housing market with heterogeneous agents in this paper. The house buyers’ real demand is assumed to be based on the current consumption utility rather than on the whole life utility with solvency constraints. Justification and variation of the real demand with financial constraints are of interest. It is also interesting to extend the analysis to a non-smooth model in which investors have short selling constraints. For property developers, it is better that the delay effect of construction is regarded as their decision factor. Those improvements could contribute to a more realistic housing price model. We leave these issues for future research.

Acknowledgments

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Appendix A: Proofs of propositions

Proof of Proposition 3.1. By (3.3), it is easy to obtain
\[
\frac{\partial P}{\partial m_1} = -m_0(m_1 + \sqrt{m_1^2 + 4m_0m_2}) - 2m_0m_2 < 0,
\]
and
\[
\frac{\partial P}{\partial m_2} = \frac{1}{\sqrt{m_1^2 + 4m_0m_2}} > 0.
\]

Proof of Proposition 3.2. By \(\frac{\partial P}{\partial m_1} > 0\), we know \(P = P^f\) only at \(P^f = R_fP^*\). In addition, if \(P^f \leq R_f\), then \(P^f \leq P^*\) and furthermore, \(P^f \geq 0\). From (3.3), we can obtain
\[
\frac{\partial P}{\partial A} = \frac{P}{(2m_0P - m_1)A^3}A^2.
\]
By \(2m_0 - m_1 > 0\), the other result can be obtained.

Proof of Proposition 4.1. (1) By (4.3) and letting \(P_{n-1} = P_n = P^*\), we can know
\[
(P^* - P^f)(\tilde{\varepsilon}P^* + \tilde{\chi}) = 0,
\]
where
\[
\tilde{\varepsilon} = \frac{1}{C} - A^2f \quad \text{and} \quad \tilde{\chi} = \frac{P^f}{C}.
\]
Therefore, when \(f < \tilde{f}^*\), (A.1) has a unique positive solution, which is the benchmark price \(P^f\). When \(f > \tilde{f}^*\), (A.1) has another positive solution
\[
\tilde{P} = \frac{P^f}{\tilde{f}^* - 1}.
\]
(2) By (4.5), we know,
\[
P_n = \tilde{G}(P_{n-1}, f) = \frac{m_{2,n} + m_{1,n}}{2m_1}.
\]
Then\footnote{We use subscripts for partial derivatives:}
\[
\frac{\partial P}{\partial R_{n-1}}\bigg|_{R_{n-1}=\tilde{P}} = \frac{\partial G'(\tilde{P}, f)}{\partial R_{n-1}} = \frac{\bar{A}(CR_f + f)}{\bar{A}(CR_f + 2)} \tag{A.3}
\]
and
\[
\frac{\partial P}{\partial R_{n-1}}\bigg|_{R_{n-1}=\tilde{P}} = \frac{\partial G'(\tilde{P}, f)}{\partial R_{n-1}} = \frac{\bar{A}(CR_f + f)}{\bar{A}(CR_f - 1)^2 + \bar{A}(CR_f + 1)} \tag{A.4}
\]
Therefore, $\bar{P}$ is stable as $f < f^*$ but unstable as $f > f^*$ while $\tilde{P}$ is unstable as $\tilde{f} < f^* < \tilde{f}^*$ but stable as $f > \tilde{f}^*$. (3) At $f = f^*$, we know that $\bar{P} = G(\bar{P}, f^{**})$, $\bar{G}_\theta(\bar{P}, f^{**}) = 1$ and $\bar{G}_\theta(\bar{P}, f^{**}) = 0$. In addition, it is easy to test

1. Nondegeneracy condition:
\[
\bar{G}_\theta(\bar{P}, f^{**}) \neq 0.
\]
2. Transversality condition:
\[
\bar{G}_\theta(\bar{P}, f^{**}) \neq 0.
\]
Therefore, by the bifurcation theory,\footnote{We use subscripts for partial derivatives:}
we know that at $(R_{n-1}, f) = (\bar{P}, f^{**})$, (A.2) undergoes a transcritical bifurcation, such that when $f > f^*$, $\bar{P}$ loses its stability and $\tilde{P}$ changes from unstable to stable.

**Proof of Proposition 4.2.** Denoting $P_1^*$, $P_2^*$ and $P_3^*$ as three solutions of (4.16) and by Vieta’s formulas, then we know $P_1^*P_2^*P_3^* = -\frac{d}{\alpha} < 0$ which means (4.16) has a negative real root, denoted as $P_3^*$. For another two roots, $P_1^*$ and $P_2^*$, they depend on the discriminant (4) of (4.16).

1. When $\Delta > 0$, $P_1^*$ and $P_3^*$ are nonreal complex conjugate roots.
2. When $\Delta = 0$, $P_1^*$ and $P_2^*$ are equal real roots.
3. When $\Delta < 0$, $P_1^*$ and $P_2^*$ are different real roots.

In addition, by $P_1^* + P_2^* + P_3^* = -\frac{b}{a} > 0$, we know when $\Delta \leq 0$, $P_1^*$ and $P_2^*$ are positive.

**Proof of Proposition 4.3.** (1) By (4.14), we know
\[
P_\alpha = G(R_{n-1}, f) = \frac{m_\alpha + \sqrt{m_\alpha^2 + 4m_\alpha m_\beta}}{2m_\alpha}.
\]
Then similar to Proposition 4.1, we can prove that when $f < f^*$, $\bar{P}$ is a stable steady state.

(2) Now we first prove that $0 < f^* < f^{**}$. Note that $f^*$ is a solution of $\Delta = 0$ where $\Delta$ can be written as the following format
\[
\Delta = \Delta(f) = \delta f^3 + \delta f^2 + \delta f + \delta_1.
\]
By the symbolic computation in Matlab R2010a, we can know the discriminant of (A.6) is positive and $\delta_1 < 0$. Thus, (A.6) has two nonreal complex conjugate roots and one real positive root which is $f^*$. Because of $\Delta(0) > 0$ and $\Delta(f^{**}) < 0$, then $0 < f^* < f^{**}$.

(3) When $f < f^*$, the discriminant of (4.16) is positive, that is $\Delta > 0$, and then $\bar{P}$ is a unique positive steady state of (A.5) but at $f = f^*$, two new steady states of (A.5) appear which are $P^* = P_1^* = P_2^* > \bar{P}$ and
\[
\frac{\partial P}{\partial R_{n-1}}\bigg|_{R_{n-1}=P_i,f^*} = G_R(P_i*, f^*) = \frac{N^*}{D^*}. \tag{A.7}
\]
where
\[
N^* = \bar{A}\bar{P}_1^*(R_1) + \omega^2(2\omega^3 - 1)f^* - (1 - \omega^2)(2\omega^3 + 1)\gamma, \quad D^* = \sqrt{m_\alpha^2 + 4m_\alpha m_\beta}|_{n-1=\bar{P}^*}. \tag{A.8}
\]
and $\alpha = (1 + \mu(\bar{P}^* - \bar{P}^*))^{-1}$. By the symbolic computation in Matlab R2010a, we can get $G_R(P_i*, f^*) = 1$.

In addition, by the assumption,

\footnote{We use subscripts for partial derivatives:}
\[
\bar{G}_\theta(P_i*, f^*) = \frac{\partial \bar{G}_\theta(P_i*, f^*)}{\partial R_{n-1}}\bigg|_{R_{n-1}=P_i*,f^*}.
\]
Similarly, $\bar{G}_\theta = \frac{\partial \bar{G}_\theta}{\partial P}$, $\bar{G}_\theta = \frac{\partial \bar{G}_\theta}{\partial P}$ and $\bar{G}_\theta = \frac{\partial \bar{G}_\theta}{\partial P}$.

\footnote{We use subscripts for partial derivatives:}
\[
\tilde{G}_\theta(P_i*, f^*) = \frac{\partial \tilde{G}_\theta(P_i*, f^*)}{\partial R_{n-1}}\bigg|_{R_{n-1}=P_i*,f^*}.
\]
Similarly, $\tilde{G}_\theta = \frac{\partial \tilde{G}_\theta}{\partial P}$, $\tilde{G}_\theta = \frac{\partial \tilde{G}_\theta}{\partial P}$ and $\tilde{G}_\theta = \frac{\partial \tilde{G}_\theta}{\partial P}$.

\footnote{We use subscripts for partial derivatives:}
(Nondegeneracy Condition): \( G_{pp}(P^*_f, f^*) < 0 \), and moreover, by \( P^*_f = P^*_s > P \) at \( f = f^* \), it is easy to get

(Transversality Condition): \( G_1(P^*_f, f^*) > 0 \).

Therefore, we know that at \((P_{-1}, f) = (P^*_f, f^*)\), (A.5) undergoes a saddle-node bifurcation, such that when \( f > f^* \), \( P^*_f \) is stable and \( P^*_s \) is unstable.

(4) Similar to Proposition 4.1, we can prove that at \((P_{-2}, f) = (P^*_f, f^*)\), (A.5) satisfies the nondegeneracy and transversality conditions of a transcritical bifurcation, that is

\[ G_{pp}(P^*_f, f^*) \neq 0 \quad \text{and} \quad G_1(P^*_f, f^*) \neq 0. \]

Therefore, we know that at \((P_{-1}, f) = (P^*_f, f^*)\), (A.5) undergoes a transcritical bifurcation.

References


