An Investigation into Width Reduction Effect on the Output of Piezoelectric Cantilever Energy Harvester Using FEM

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***Abstract*—** Piezoelectric materials can be used as mechanisms to transfer mechanical energy, usually ambient vibration, into electrical energy that can be stored and used to power other devices. Recently, increased interest in wearable computer concepts and remote electrical devices has provided motivation for more extensive study of piezoelectric energy harvesting. A typical piezoelectric energy harvester is a unimorph rectangular cantilever located on a vibrating host structure, to generate electrical energy from base excitations. In this paper existing mathematical modeling and finite element approaches using elementary single-degree-of-freedom models are used to investigate the effects of width reduction on the output of unimorph piezoelectric cantilever energy harvester and achieving more power density. It can be seen that by division of a single-piece energy harvester into an array of equal cantilevers and connecting them in parallel, the efficiency of conversion system increases. This research raises a new perspective that, an array of cantilever piezoelectric energy harvesters with a certain total width, can lead to more harvesting energy than a single-piece harvester of similar width.

***Keywords*—** Vibration Energy harvester, Piezoelectric, Wideband operation, Resonant frequency, Finite element simulation.

I. Introduction

Energy harvesting has been around for decades. To feed the world's needs for energy, macro scale energy harvesting technologies have successfully established. On the other hand, for low powered electronics devices, harvesting energy from the ambient vibrations seems to be an ideal solution due to the definite life span and high cost for replacement of the traditional batteries. Three mechanisms are available for vibration energy harvesting; using electrostatic devices, electromagnetic field and utilizing piezoelectric based materials. The performance of piezoelectric vibration energy harvesters is more often than other methods. Compared to other structural forms of beams, a cantilever beam can obtain the maximum deformation and strain under the same conditions. The larger deflection leads to more stress and strain, and consequently a higher output voltage and power. Therefore the vast majority of piezoelectric vibration energy harvesting devices use a cantilever beam structure. A cantilever-type energy harvester has been intensively studied [1-5].

Most of the previous research works focused on designing a linear vibration resonator, which has maximum output power when reaching resonance frequency. Therefore the practical applications of these devices are limited due to narrow bandwidth as well as small power density. If the excitation frequency slightly shifts, the performance of the harvester will dramatically decrease. Since in the majority of practical cases, the vibration in the environment is frequency-varying or totally random with the energy distributed in a wide spectrum, how to broaden the bandwidth of harvesters becomes one of the most challenging issues before their practical deployment [6].

Certain vibration mode can be excited when the driving frequency approaches one natural frequency of the harvester. To date, one of the most important strategies to widen the bandwidth, include using a generator array consisting of small generators with different resonant frequencies. If multiple vibration modes of the harvester structure are utilized, useful power can be harvested over multiple frequency spectra, that is, wider bandwidth can be covered for efficient energy harvesting. Rather than discrete bandwidth due to the multiple modes of a single beam, multiple cantilevers or cantilever array integrated in one energy harvesting device can provide continuous wide bandwidth, if the geometric parameters of the harvester are appropriately selected. Power spectrum of a generator array is a combination of the power spectra of each small generator [6-8].

The geometry of a piezoelectric cantilever beam will greatly affect its vibration energy harvesting ability. The sensitivity of resonant cantilever piezoelectric energy harvesters is directly proportional to the resonant frequency. This useful analytical formula, is confirmed by simulation results in ABAQUS 6.11 software. It is noteworthy that a cantilever beam can have many different modes of vibration, each with a different resonant frequency. The first mode of vibration has the lowest resonant frequency, and typically provides the most deflection and therefore electrical energy. Accordingly, energy harvesters are generally designed to operate in the first resonant mode.

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This research investigates into width reduction effect on the output of rectangular piezoelectric cantilever energy harvester. Piezoelectric composite structures modeling plays an important role in the overall design procedure for smart structures and systems. A reliable model enables optimization in the early development phases as well as simulation of the structural behavior and prediction in different 'what-if" scenario cases. The available tools improvement and the fulfillment of special user requirements are the main motivations for this work. ABAQUS FE software package include finite elements which can be used for modeling piezoelectric properties. The implemented element regards the piezoelectric thin layers polarized in the thickness direction and it is based on the e31 piezoelectric effect. The main focus of this paper is increasing power density and optimum design of cantilever energy harvester with lowest costs by FEM.

II. THEORY OF MODEL

Figure 1 shows the structure of unimorph piezoelectric rectangular cantilever with length *l*, width *w*, density ρ_s and ρ_p, thickness t_s and t_p, and Young's modulus E_s and E_p for substrate and piezoelectric layers, respectively. Also the total cross-sectional area moment of inertia is $\overline{I}_z$.

As illustrated in [5], the resonance frequency of a rectangular composite cantilever beam can be obtained as:

$$f_r = \frac{\sqrt{1155}}{33\pi l^2}\sqrt{\frac{E_s(t_s-h)^3+E_sh^3+E_p(t_s+t_p-h)^3-E_p(t_s-h)^3}{\rho_st_s+\rho_pt_p}} \quad (1)$$

where *h* for a unimorph section is expressed as:

$$h=\frac{(E_s/E_p)t_s^{\,2}+2t_st_p+t_p^{\,2}}{2(E_s/E_p)t_s+2t_p} \quad (2)$$

External stress to a piezoelectric film in base vibration causes deflection and bending to the cantilever beam. The deflection distorts the internal dipole moments within the piezoelectric film and generates electrical voltages. This results in the generation of charges on the film. From the theory of elasticity of a material, the relationships between the stress and strain of the piezoelectric film are:

$$\sigma_p = E_p(\varepsilon_p - g_{31}D_3) \quad (3)$$

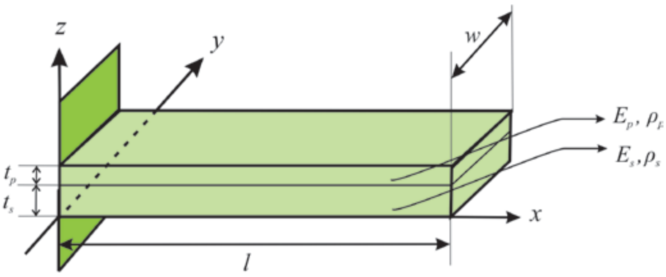

Figure 1 Schematic drawing of a unimorph cantilever beam

$$E_3 = -g_{31}\sigma_p + \frac{D_3}{\varepsilon_r\varepsilon_0} \quad (4)$$

The ε_p and σ_p are the strain and stress of the piezoelectric layer, respectively in *x* direction, g_{31} is the piezoelectric stress constant, D_3 and E_3 are electrical displacement and electric field strength in *z* direction. The ε_0 and ε_r are the permittivity of the vacuum and the relative permittivity of the material, respectively and ε_0=8.85×10^{-12}F/m. When an external force, *F*, is applied to the base of the bender, electric charges, *Q*, will be harvested on the piezoelectric material and they can be obtained by integrating the electric displacement, D_3, to its overlapping area which is

$$Q=\iint D_3dA=\int_0^w\int_0^l D_3dydx \quad (5)$$

where *l* and *w* are the length and width of the piezoelectric cantilever beam, respectively.

Let *n* be the number of cantilevers (when more than one cantilever are used) connected in either series or parallel. The electrical charge harvested from n-parallel and n-series for both the single-active layer and the two-active layer benders are expressed as [9, 10]:

$$Q_{n-parallel}=\frac{-3(2^N)AB(1-A+AB)g_{31}\varepsilon_r\varepsilon_0l^2F}{t^2k} \quad (6)$$

$$Q_{n-series}=\frac{-3AB(1-A+AB)g_{31}\varepsilon_r\varepsilon_0l^2F}{t^2k} \quad (7)$$

where;

$$t=t_s+t_p \quad (8)$$

$$A=\frac{t_s}{t} \quad (9)$$

$$B=\frac{E_s}{E_p} \quad (10)$$

$$h=1+A^4(1-B)^2-2A(2A^2-3A+2)(1-B) \quad (11)$$

$$k=h(1-A+AB)(1+E_pg_{31}^2\varepsilon_r\varepsilon_0) - 39(1-A)A^2B^2E_pg_{31}^2\varepsilon_0 \quad (12)$$

As illustrated in Figure 2, 0-fold (i.e. *N* =0) refers to the unimorph cantilever energy harvester without any splitting. When the unimorph piezoelectric cantilever is split equally into two identical cantilevers, it is referred as 1-fold (*N* =1). The further equal-splitting on each of the two identical unimorph piezoelectric cantilevers is referred as 2-fold (*N* =2).

By connecting the split unimorph piezoelectric cantilevers in parallel configuration, the effective capacitance with N-fold to the single piezoelectric cantilever is as:

$$C_N = \frac{(1-A+AB)hw_0 l\varepsilon_r\varepsilon_0}{(1-A)tk} \tag{13}$$

The dynamic model of the unimorph piezoelectric cantilever can be divided into mechanical and electrical models. For the mechanical model, the piezoelectric cantilever can be modeled as a single degree of freedom system (SDOF), which consists of a equivalent spring constant, *K*, of the piezoelectric cantilever, and equivalent mass, *M*, a dashpot with damping coefficient, *C*, and a vibrating base. The equivalent SDOF model is shown in Figure 3 where $x(t)$ is the equivalent mass displacement and $y(t)$ is the vibrating base displacement. It is notable that $z(t)$ is the relative motion between the equivalent mass and the vibrating base.

When a sinusoidal input $y(t)= y_0 \sin(2\pi ft)$ is applied to the base excitation, the peak piezoelectric voltage generated with mechanical-electrical model can be obtained as [11];

$$V_N = \left(\frac{-3AB(2^N)(1-A+AB)g_{31}\varepsilon_r\varepsilon_0 y_0 l^2 K}{t^2 kC_N}\right)\sqrt{\frac{(2\zeta_N r)^2+1}{(2\zeta_N r)^2+(1-r^2)^2}} \tag{14}$$

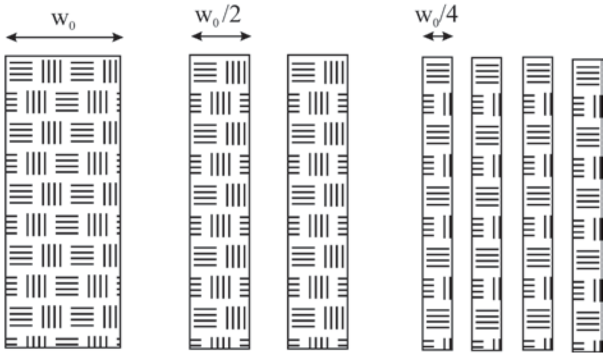

Figure 2 Unimorph piezoelectric cantilevers in 0-fold, 1-fold and 2-fold, respectively

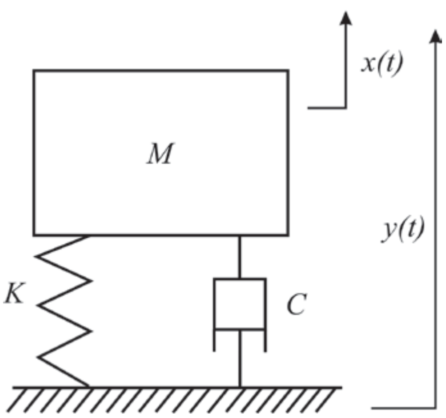

Figure 3 Equivalent SDOF model for the unimorph piezoelectric cantilever with excitation at base

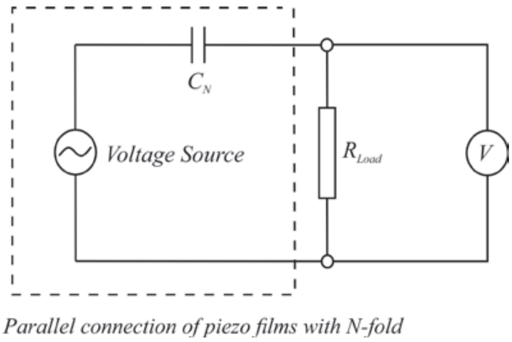

Figure 4 The piezoelectric circuit model connected with a load resistor

where $\zeta = \sqrt{1-\left(\frac{f_{damped}}{f_0}\right)^2}$ is the damping ratio, f_0 is the natural frequency, f_{damped} is damped natural frequency and $r = \frac{f}{f_0}$ is the ratio of the vibrating frequency to the natural frequency.

When an external resistive load is added to the output of the unimorph piezoelectric energy harvester in parallel with N-fold as shown in Figure 4, the load voltage can obtained from potential divider as;

$$V_{Load,N}(t) = \frac{R_{load}V_N}{\sqrt{R_{load}^2+Z_p^2}}\sin(2\pi ft) \tag{15}$$

where

$$\phi = \tan^{-1}\left(\frac{1}{2\pi fC_N R_{load}}\right) \tag{16}$$

The root-mean-square (RMS) voltage across the resistive load can expressed as;

$$V_{Load,N,RMS} = \psi\sqrt{\frac{(2\zeta_N r)^2+1}{\left(R_{load}^2+Z_p^2\right)\left((2\zeta_N r)^2+(1-r^2)^2\right)}} \tag{17}$$

where

$$\psi = \left(\frac{-3AB(2^N)(1-A+AB)g_{31}\varepsilon_r\varepsilon_0 y_0 l^2 KR_{Load}}{\sqrt{2}t^2 kC_N}\right) \tag{18}$$

The average power output in the resistive load is obtained as;

$$P = \frac{V_{Load,N,RMS}}{R_{Load}} \tag{19}$$

III. FINITE ELEMENT SIMULATION

ABAQUS 6.11 which has the capabilities of analysis of piezoelectric materials, was used to simulate the performance of a unimorph cantilever. The model is a polypropylene reinforced beam which has 45mm length and

20mm width. The thicknesses of the substrate and piezoelectric layers are 124µm and 52µm, respectively. The relative permittivity, ε_r, and the piezoelectric stress constant, g_{31}, are given as 13 and 216×10^{-3} m^2C^{-1}, respectively. The effective mass of the initial structure is 0.15g and the modulus of elasticity in substrate and piezoelectric layers is 9×10^{9} and 3×10^{10}, respectively. Complete transversely isotropic properties are assigned to the piezoelectric strip for elastic, piezoelectric and dielectric properties. The inner surface of the piezoelectric strip is tied to the substrate to form the assembly. A constraint is deployed to ensure uniform potential on the surfaces of the piezoelectric strip which simulates the presence of electrodes. Mechanical boundary conditions are specified to ensure a clamped condition while electrical boundary conditions are specified to render the beam inactive by maintaining the piezoelectric surface bonded to the substrate material at zero electric potential throughout the analysis. The electric potential in ABAQUS is stored in the variable EPOT in the field output. The substrate is modeled with Quadratic Hexagonal (C3D20R) solid continuum elements with reduced integration.

It is assumed that damping is attributed only for the brass material and not the piezoelectric material. Damping is defined by proportional (Rayleigh) damping given by the equation [12, 13]:

$$x_i = \frac{1}{2}\left(\frac{a}{w_i} + w_i b\right) \tag{20}$$

where ξ is the modal damping ratio, 'α' and 'β' are proportional coefficients and 'ω' is the angular frequency of a corresponding i^{th} vibration mode. For the current analysis, a damping ratio of 0.119 was maintained for the first mode and $N=0$ and hence the constants α and β are chosen to be 34.247 and 2.369E-4 respectively.

Both the brass and piezoelectric surfaces are clamped to ensure full transfer of strain energy to the piezoelectric material. An electrical boundary condition is also imposed such that the electric potential at the contact surfaces between the piezoelectric layers and the brass substrate is zero throughout the analysis rendering the beam inactive.

A direct steady state dynamic analysis is performed on the system. A frequency step is initialized prior to the dynamic step so as to enable subdivision of each frequency range using Eigen- frequencies. The method chosen as the Eigen solver was Subspace method, with 18 vectors used per iteration with the maximum iterations being 30. A total of 10 Eigen values were requested with the maximum value of frequency interest being 1000 Hz. Symmetric method was chosen for matrix storage in the frequency step. Eigen vectors were chosen to be normalized by mass. Non-linear effects due to large deformation or displacements were not considered in this analysis.

Figure 5, Figure 6 and Figure 7 show the mode shapes and resonance frequencies of piezoelectric cantilever energy harvesters in absence of damping for N=0, N=1 and N=2, respectively. As mentioned in [5] it can be seen that the resonance frequency in absence of damping, is almost constant and independent of the beam width.

Experimental study about width slitting method is done in [9, 10]. Estimated damping ratio for each case and related data are expressed in TABLE 1.

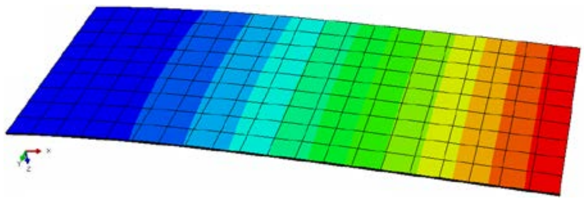

Figure 5 first mode shape for N=0 (f_r=27.787)

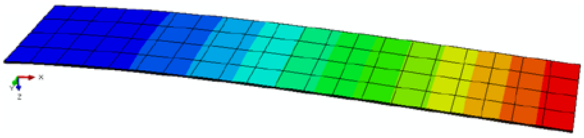

Figure 6 first mode shape for N=1 (f_r=27.533)

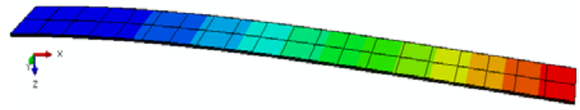

Figure 7 first mode shape for N=2 (f_r=27.342)

TABLE 1
UNITS FOR MAGNETIC PROPERTIES

Folding Number(N) / width(mm) / number of splits	Damping ratio, ξ_N	ξ_0 / ξ_N
N=0 / 20 mm / 1	0.119	1
N=1 / 10 mm / 2	0.097	1.223
N=2 / 5 mm / 4	0.079	1.506

IV. RESULTS AND DISCUSSION

The theoretical model is verified as compared with experimental and finite element simulation results. Based on the experimental results in [10], increasing number of fold causes decreasing damping ratio. Therefore, an exponential equation is proposed to relate the damping ratio of the N-fold to 0-fold as;

$$\frac{x_0}{x_N} = e^{gN} \tag{21}$$

The proposed relation is shown in Figure 8 and γ is found to be 0.2047.

A shaker powered by frequency generator is used in the experiment to provide base vibration to the piezoelectric cantilever beam with vibration frequency, f, in the range of 21 Hz to 32 Hz and the base displacement, d_0, is set as 2 mm. Figure 9 shows the analytical, simulation and experimental results in different states, when a 2MΩ resistor is selected as a load. It can be seen that a good agreement is obtained between the experimental, analytical and simulation results. As mentioned in [10], TABLE 2 summarizes the significant findings from the experimental study. The results show that by increasing the number of folds, the harvested power increases and the bandwidth reduces. For wideband application, the resonant frequency can be adjusted for different beams of the same fold number by adding appropriate tip masses. Therefore multiple vibration modes of the harvester structure can be utilized and useful power can be harvested over multiple frequency spectra.

In parallel connection of the split piezoelectric films, the combined capacitance remains constant, but charge production increases. So the open circuit voltage, V, and the load voltage, V_{Load}, increase accordingly.

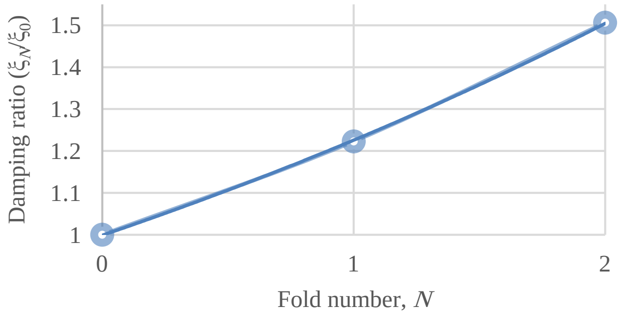

Figure 8 relationship between fold number and damping ratio

TABLE 2
SUMMARY OF EXPERIMENTAL RESULTS

Folding number (N)	Bandwidth (Hz)	Output power to the external load of 2 ΩM (μW)
N=0	6.5	19.2
N=1	5.3	47.0
N=2	4.3	53.0

V. CONCLUSION

This paper comprehensively investigates the width reduction effect on power harvested from a three dimensional FE model of a clamped-free type unimorph cantilever piezoelectric energy harvester. A high level of detail in FE modeling in ABAQUS 6.11 is provided. The results, importantly the electric potential, voltage and maximum power developed are found using the software and a set of basic electrical equations.

By reducing the width of the cantilever energy harvester, the natural frequency of the overall vibrating system remains constant, but damping decreases. So the cantilever vibrates at higher amplitudes, especially at the resonant frequency. This finding can be used to increase the output from a piezoelectric energy harvester by splitting the piezoelectric cantilever beam into cantilevers of smaller width and then connecting all of them in parallel to form an array of smaller piezoelectric energy harvesters. A substantial increase in harvested energy can be observed from an array of smaller width cantilevers compared with a single piezoelectric cantilever of similar total width. Implementing multimodal energy harvester system, by exploiting a cantilever array integrated in one device, where the first mode of each cantilever beam is one of the vibration modes of the device.

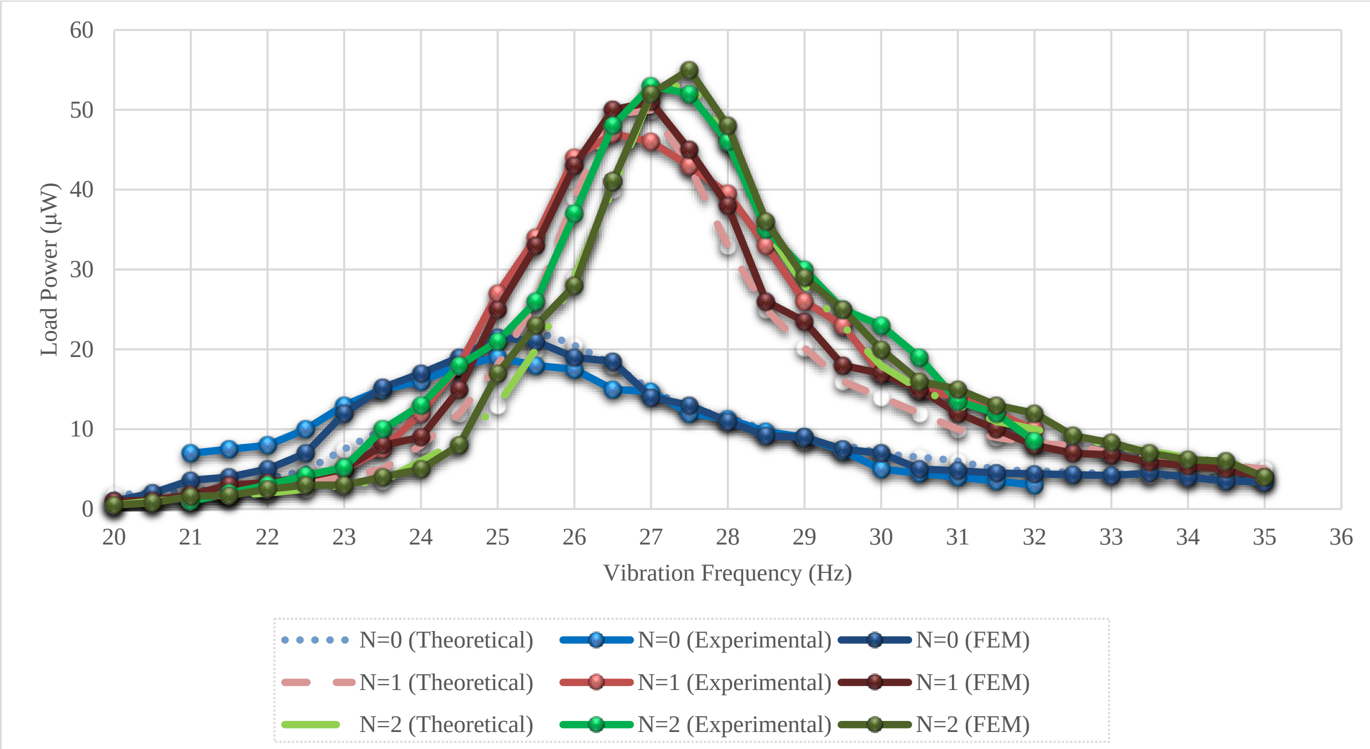

Figure 9 Comparison of harvested power by cantilevers with different fold numbers using different methods

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