Crack swarm inspection for estimation of crack location in carbon fiber reinforced plastics: A numerical study

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A B S T R A C T

The present study proposes crack swarm inspection (CSI) for estimating crack location and size in carbon composite laminates from the surface voltage distribution. This technique generates a large number of virtual microscopic cracks, and calculates the surface voltage distribution of the composites using anisotropic electric potential functions and doublet strings. Using genetic algorithms, the virtual microscopic cracks formed a swarm to coincide with the measured surface voltage; thereby, the crack sizes and locations are estimated from the position of the crack swarm. The CSI was applied to crack detection in carbon laminated composite plates; it was confirmed that the existence of cracks in each partitioned section was detected with >80% probability, in reference to the crack location and size information. Furthermore, we also confirmed that the estimation accuracy was affected by the electric current density in the thickness direction, and addressed the recommended electrode interval based on the minimum size of the estimated crack.

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1. Introduction

The applications of carbon fiber reinforced plastics (CFRPs) have expanded, particularly in the field of aeronautics, because of its outstanding specific strength and stiffness as compared with metals [1,2]. However, when CFRP laminated plates are used as a structural component, even small impacts can easily lead to delamination, considerably reducing strength [3]. To ensure the structural integrity (and to reduce the high costs incurred by periodic inspections), there is an urgent need for a health monitoring system for CFRP laminated plates. Previously, the following nondestructive inspection techniques have been used: ultrasonic inspection [4–6], X-ray inspection [7,8], acoustic emission (AE) [9,10], optical fiber inspection [11–13], and visual inspection. However, these methods are costly and time-consuming, or may decrease the structural strength by requiring an embedded sensor. Against this backdrop, there is a demand for a simple, non-destructive health monitoring process that detects delamination during periodic inspections or during operation. The monitoring method must be suitable for existing structures, and must not cause a reduction in strength by requiring sensors to be embedded.

As a useful health monitoring technique that can be applied during operation, the electrical resistance change method [14,15] has been proposed for monitoring fiber fractures [16,17], strain [18], and fatigue [19]. In this technique, the electrical resistances that arise because of delamination are measured using electrodes attached to the structural surface, so as not to cause a reduction in strength. The suggested technique was applied to monitoring the occurrence of delamination in CFRP laminated plates based on changes in the electrical resistance between a number of electrodes on the CFRP surface [20–23]. However, this method does present complex challenges for measurement systems because it is necessary to measure the electrical resistance of every current in all of the spaces between adjacent electrodes to measure the delamination-dependent changes in electrical resistance between electrodes. To address this problem, the electric potential difference technique is used; it involves installing a pair of electric current electrodes and a large number of voltage electrodes, making it possible to identify cracks using a single electric current [24–27]. However, these techniques require multiple tests and expensive numerical analyses; thus, there is a need to develop a technique that is simpler to implement.

A simple technique has been proposed for analyzing electric current density in CFRP, in which changes in voltage due to delamination are analyzed by installing a string of anisotropic doublets at separation points [28,29]. This technique makes it possible to...
analyze the electric current density of a CFRP laminated plate using a mirror image relationship. However, even though it is possible to calculate voltage changes with this technique in situations where delamination is known to have occurred, it is difficult to estimate the location and size of delamination from the voltage change.

Therefore, in this research, crack swarm inspection (CSI) is proposed, which estimates crack size and location based on a large number of virtual microscopic cracks in the inner layers of the CFRP using anisotropic electric potential functions and doublet strings. A genetic algorithm (GA) is used to minimize the difference with the voltage true value distribution and voltage distribution with virtual microscopic cracks in CFRP by changing the virtual crack position; thereby, the virtual cracks form a swarm, and the crack location and size are estimated from the crack swarm. CSI is applied to crack detection in unidirectional laminated plates, and its validity is investigated.

2. Crack estimation with crack swarm inspection

2.1. Overview of CSI

Fig. 1(a) shows a schematic diagram of the CSI method. CSI is a technique for estimating cracks by simulating many virtual cracks, which approximate the voltage distribution of damaged CFRP. One benefit is that this method allows estimation of internal cracks using only the surface voltage distribution data. The analysis process involves calculating the electric current density distribution in CFRP using an anisotropic potential function, and producing a large number of virtual microscopic cracks (unit cracks) by installing doublets [30] to oppose the electric current density in the thickness direction at the crack locations. This technique allows the position and size of cracks to be estimated using a genetic algorithm, which treats the position of cracks as a design variable, to minimize the difference between the surface voltage distribution of a CFRP that has crack swarm, and the real-world value of the voltage distribution measured in a cracked CFRP. This approach is shown in Fig. 1(b). In the present study, the aim is to investigate the delaminations in a unidirectional CFRP; as such, the virtual cracks modeled by doublets are placed in parallel to the specimen surface. To extend this to the estimation of transverse cracks in the multidirectional laminates, the directions of the doublets should be arbitrary and treated as variables.

2.2. Analysis of electrical current density using anisotropic electrical potential function

The electric potential function $\phi$ is calculated using the Cartesian coordinate system: the direction that the fibers are arranged into is labeled $x$, the transverse direction is labeled $y$, the thickness direction is labeled $z$; the current densities $i_x$, $i_y$, $i_z$ are calculated using the following equations [31].

\[
\begin{align*}
    i_x &= -\sigma_x \frac{\partial \phi}{\partial x} \\
    i_y &= -\sigma_y \frac{\partial \phi}{\partial y} \\
    i_z &= -\sigma_z \frac{\partial \phi}{\partial z}
\end{align*}
\]

(1)

Here, $\sigma_x$, $\sigma_y$, $\sigma_z$ refer to the electric conductivity of the CFRP in the direction of the fiber, the transverse direction, and the thickness direction, respectively. If there is no electrical current source, the following equation can be derived from the equation of continuity.

\[
\frac{\partial i_x}{\partial x} + \frac{\partial i_y}{\partial y} + \frac{\partial i_z}{\partial z} = 0
\]

(2)

If $\sigma_x$, $\sigma_y$, $\sigma_z$ are constant regardless of location, then the following equation can be derived by substituting equation (2) into Eq. (1).

\[
\sigma_x \frac{\partial^2 \phi}{\partial x^2} + \sigma_y \frac{\partial^2 \phi}{\partial y^2} + \sigma_z \frac{\partial^2 \phi}{\partial z^2} = 0
\]

(3)

Here, two dimensions, the fiber direction and the thickness direction, are considered. When a coordinate axis affine transformation is carried out as in Eq. (4), the Laplace equation shown in Eq. (5) is obtained.

\[
\begin{align*}
    \xi &= \frac{x}{\sqrt{\sigma_x}}, & \eta &= \frac{z}{\sqrt{\sigma_z}}
\end{align*}
\]

(4)

\[
\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} = 0
\]

(5)

Eq. (5) resembles the velocity potential of an irrotational ideal fluid. Fig. 2(a) shows a schematic drawing of a CFRP beam with thickness $t$, which is the object of the anisotropic electric potential function. If there are current and earth electrodes on the same side of the CFRP beam, the electric current density analysis can be carried out on the CFRP using the velocity potential of an ideal fluid by setting the current electrode as a source point and the earth electrodes as a sink point. Assuming that the source (electric current load point) coordinates are $(-a, 0)$, and the sink (ground point) coordinates are $(a, 0)$ where $a > 0$, the electric current density is given by [28]

\[
\begin{align*}
    i_x &= \frac{i}{\pi \sqrt{x_a \sigma_x}} \left\{ \frac{x_a}{\sqrt{x_a^2 - y^2}} - \frac{x}{\sqrt{x^2 - y^2}} \right\} \\
    i_z &= \frac{i}{\pi \sqrt{x_a \sigma_z}} \left\{ \frac{z}{\sqrt{z^2 - y^2}} - \frac{x}{\sqrt{x^2 - y^2}} \right\}
\end{align*}
\]

(6)

Furthermore, when analyzing thin laminates, an affine-transformed $\xi$-$\eta$ coordinate system contains an isotropic space; therefore, it is
possible to simulate the current density in the thickness direction by installing a number of mirror images in the same way as the velocity potential of an ideal fluid [29]. Here, taking N as the set of positive and negative mirror images, if the mirror images are drawn as shown in Fig. 2(b), the current density for the N = n set is derived as shown below, by summing from N = −n to N = n.

\[ i_k = \sum_{k=-n}^{n} \frac{1}{\pi \sigma_x} \left( \frac{x_k z_0}{x_k z_0^2 + z_0^2} - \frac{x_k z_0}{x_k z_0^2 + z_0^2} \right) \]

\[ i_k = \sum_{k=-n}^{n} \frac{1}{\pi \sigma_x} \left( \frac{x_k z_0}{x_k z_0^2 + z_0^2} - \frac{x_k z_0}{x_k z_0^2 + z_0^2} \right) \]

(7)

2.3. Delamination analysis using anisotropic doublets

The problem of delamination in a laminated plate can be modeled as a plate existing inside a fluid, because delamination cuts off the electric current density in the thickness direction at the desired position. Consider a situation in which a crack is positioned from \((x_1, z_0)\) on the left to \((x_2, z_0)\) on the right; in this area the doublet’s strength is set such that the electric current density at the median position.

\[ i_{x, d}(x, z') = -\frac{\sqrt{\sigma_z}}{2\pi} \int_{x_1}^{x_2} \mu(s) \left( \frac{x - s}{\sqrt{s^2 + z_0^2}} \right)^2 - \left( \frac{z'}{\sqrt{s^2 + z_0^2}} \right)^2 \right) \right) ds \]

(8)

If the electric current density in the thickness direction as created by the anisotropic electric potential function is set as \(i_k(x, z')\), then this leads to delamination by satisfying the integral equation below, which determines the doublet’s strength.

\[ i_1(x, 0) + i_{x, d}(x, z) = 0 \]

(9)

Because \(i_{x, d}(x, 0)\) is an improper integral equation, this peculiarity is eliminated by approximating \(z' = c\) (an extremely small, positive number). From the strength of the doublets referred to in Eq. (9), a voltage distribution with a crack is given by

\[ V(x, z) = \phi'(x, z_0) - \phi_0'(x, z_0) \]

\[ = -\frac{1}{2\pi \sqrt{\sigma_x}} \int_{x_1}^{x_2} \mu(s) \left( \frac{x - s}{\sqrt{s^2 + z_0^2}} \right)^2 + \left( \frac{z}{\sqrt{s^2 + z_0^2}} \right)^2 \right) ds \]

(10)

where \(\phi'\) and \(\phi_0'\) are the electric potential on the surface of the CFRP laminates before and after crack occurrence, respectively. Because the conductivity of CFRP in the thickness direction is miniscule compared to that in the direction of the fibers, and the electrical current flowing in the thickness direction is generally small, the mirror image is only on the upper part of \(N = 0\), as shown in Fig. 2(b). The remaining mirror images are disregarded so as to simplify calculations. According to the crack in the mirror image at \(N = 0\), the surface voltage distribution created by the doublet is twice as large; thus, Eq. (10) can be transformed into the following equation.

\[ V(x, z) = 2 \left( \phi'(x, z_0) - \phi_0'(x, z_0) \right) \]

\[ = -\frac{1}{\pi \sqrt{\sigma_x}} \int_{x_1}^{x_2} \mu(s) \left( \frac{x - s}{\sqrt{s^2 + z_0^2}} \right)^2 + \left( \frac{z}{\sqrt{s^2 + z_0^2}} \right)^2 \right) ds \]

(11)

In CSI, the current electrode interval (integral interval) that constitutes the search area is divided into equal parts \(N_s\). Within each interval \(N_s\), the doublet strength is set at a fixed value. If a doublet is set such that the electric current density at the median \(x_i^C\) of each interval is \(i_c(x_i^C, 0)\), then Eq. (9) can be written as follows.

\[ i_c(x_i^C, 0) - \sum_{j=1}^{N_s} \mu_j B_{ij} = 0 \]

(12)

\[ B_{ij} = \int_{x_{j-1}}^{x_j} \frac{\sqrt{\sigma_z}}{2\pi} \left( \frac{z_0^2}{\sqrt{s^2 + z_0^2}} \right) - \left( \frac{z}{\sqrt{s^2 + z_0^2}} \right)^2 \right) ds \]

(13)

Eq. (12) can be written as the following matrix.

\[
\begin{pmatrix}
B_{1,1} & \cdots & B_{1,N_s} \\
\vdots & \ddots & \vdots \\
B_{N_s,1} & \cdots & B_{N_s,N_s}
\end{pmatrix}
\begin{pmatrix}
\mu_1 \\
\vdots \\
\mu_{N_s}
\end{pmatrix}
= 
\begin{pmatrix}
i_1(x_1^C, 0) \\
\vdots \\
i_{N_s}(x_{N_s}^C, 0)
\end{pmatrix}
\]

(14)

The doublet strength of each interval \(\mu_j (j = 1, 2, \ldots, N_s)\) can be found by calculating the inverse of matrix B. The surface voltage is derived from Eq. (11) as follows.
\[
V(x,0) = -\frac{1}{2\pi\sqrt{\sigma_x}} \sum_{i=1}^{N_i} \frac{\sum_{j=1}^{N_j} H_j \int_{\gamma_i} \frac{\partial \phi_j}{\partial n} ds}{\left(\frac{\partial \phi_j}{\partial n}\right)^2 + \left(\frac{\partial \phi_j}{\partial \gamma_i}\right)^2}
\]

(15)

2.4. Estimating crack location with optimization

The optimization method in CSI uses GA [32] for estimation of crack position and size. The optimization process is described below.

Step 1: Initial swarm creation

In CSI, a large number of unit cracks are created in the space between current electrodes. Here, to translate the condition of the unit cracks between electrodes into individuals in GA, the crack existence constant \( \zeta \) is introduced. If a crack is present, then the crack existence constant \( \zeta \) is set at \( \zeta = 1 \), and if there is no crack present, then it is set at \( \zeta = 0 \). For example, the relationships between phenotype (PTYPE) and genotype (GTYPE) are shown in Fig. 3 for scenarios where a 2-mm crack is present at position \( x = -2 \) in an electrode space (\( x = -3 \) to 3 mm), and where two 1-mm cracks are present at positions \( x = -2 \) and \( x = 1 \). The interval is split into 1-mm increments. In this way, the unit crack position is expressed as its genotype. When the current electrode space is split into \( N_s \) intervals, the crack existence constant is expressed as a vector, as in the equation below.

\[
\zeta_i \in \{0, 1\} \quad (i = 1, 2, \ldots, N_s)
\]

(16)

Step 2: Assessing fitness

The fitness of each individual is defined by the difference between the true voltage on the surface of the laminates, \( V_{true}(x,0) \), and the voltage of each individual, \( V_{ind}(x,0) \).

\[
\text{fitness} = \sum_{i=1}^{n_m} \left( V_{true}(x,i) - V_{ind}(x,i) \right)^2
\]

(17)

where \( n_m \) is the number of voltage measurement points. The position and size of the crack are estimated by minimizing the fitness in Eq. (17).

Step 3: Selection

The selection method makes use of roulette selection [32]. Roulette is a selection method that allocates a higher probability of selection to individuals with a higher degree of fitness. However, with roulette selection, it remains possible that individuals with a low degree of fitness will be selected, and a possibility that individuals with the highest degree of fitness will not be selected. To prevent individuals with a high degree of fitness from being eliminated, an elite saving strategy [32] is also used, whereby individuals with the highest degree of fitness are saved and retrieved if they are not chosen as part of the selection process.

Step 4: Crossing and mutation

Two selected individuals are crossed, and two new individuals’ genes are generated. Mutations are reflected in these two individuals’ genes, and new individuals are defined. If the generation created in steps 3 and 4 satisfies the end conditions, then the process ends. If it does not satisfy the end conditions, step 2 is repeated. The end condition is a valid when the positions of unit cracks have converged.

3. CSI verification

3.1. Analytical object and conditions

A 60-mm long, 2-mm wide, 16-layer \((0.125\text{mm thick layers})\) laminated plate was treated as the analytical object. As shown in Fig. 4, the origin of the \( x \)-\( z \) axis was established in the center of the test specimen, 1 A/m of current per unit width was applied at the position \((x, z) = (-15, 0)\), and a 0-V ground voltage was established at the position \((x, z) = (15, 0)\). The materials are modeled as a woven CFRP fabric; the conductivity is in-plane isotropic, whereas it is anisotropic in the thickness direction. The electric conductivity in the fiber direction is set at \( \sigma_x = 1.0 \text{ S/m} \). The electric conductivity in the thickness direction, the number of voltage measurement points (voltage electrode numbers), the crack depth and position are varied as shown in Table 1, which confirms that there is an effect on crack estimation accuracy. It should be noted that the electrodes placed on the specimen surface are assumed to be points having no width. Because practical applications of CFRP surfaces usually involve a coating with varnish or paint on the CFRP surface, it is recommended that the electrode be co-cured during CFRP processing [27] or that the surface undergo laser abrasion [33] to obtain good electric contacts between the CFRP and the electrodes. To produce multiple electrodes for practical applications, electrode pattern sheets fabricated by photolithography [34,35] and electroplating methods [36] were proposed.

A number (a maximum of 28) of 1-mm unit cracks were generated in a search region in the same layer as an estimated crack, at intervals of \( x = -14 \) to 14 mm. With CSI, a fixed doublet strength was calculated in the unit cracks, the true-value voltage distribution was calculated in a search region split into 0.2-mm intervals. A fixed doublet strength was calculated with high accuracy in the doublet intervals. An estimated crack is set at 7 mm in length. After the CSI search, if the cracks in the search area were below 2 mm,
they were ignored because they were very small, and the estimation accuracy and error rate were calculated as described in the following section. The GA parameters used in this analysis are shown in Table 2.

3.2. Results and discussion

3.2.1. Effect of electric conductivity in the thickness direction on the accuracy of crack estimation

To improve the fracture toughness, in recent aircraft structures, interlaminar thermoplastic resins are sometimes added to composite laminates [37]. It is necessary to verify cases of low electric conductivity in the thickness direction. Here, estimation accuracy, $Acc$ (overall accuracy), is defined as follows:

$$ Acc = \frac{1}{N} \sum_{i=1}^{N} b_i $$

(18)

$$ b_i = \begin{cases} 1 & \text{(if the estimation at interval } i \text{ matches)} \\ 0 & \text{(otherwise)} \end{cases} \quad (19) $$

As shown in Eq. (19), $b_i$ is 1 if there is agreement between the presence or absence of a crack in the interval $i$ of the search area and the true presence or absence of a crack; otherwise, its value is 0 (i.e., if there is no agreement). Therefore, $Acc$ in Eq. (18) is the accuracy index, taking into account the location and size of the crack. In addition, commission errors are introduced, whereby unit cracks do not exist where true cracks do not exist, and omission errors are introduced, whereby unit cracks do not exist where true-value cracks do exist. Both of these errors are calculated.

Fig. 5(a) shows the resulting estimation accuracy if a crack of estimated length 7 mm occurs between $x = -12-5$ mm. $\sigma_z$ gradually decreases from an isotropic value of 0.01 S/m to the high anisotropic values of 0.008, 0.006, 0.004, 0.002, and 0.001 S/m. CSI is carried out ten times under the same conditions. As can be seen in the figure, until $\sigma_z = 0.004$ S/m, the estimation accuracy is shown to be high; however, after $\sigma_z = 0.002$ S/m, the omission error rate increases, and accurate crack estimations are no longer achieved. Fig. 5(b) shows the true voltage distribution if $\sigma_z = 0.01$ or 0.001 S/m. The reason for this phenomenon is thought to be that as the electric conductivity is lower, the electric current density in the thickness direction is lower in the crack location, and as it does not reach a local distribution, as in the case of $\sigma_z = 0.01$ S/m true value voltage distribution of Fig. 5(b), where there tend to be more errors. The computational time using the GA parameter in Table 1 is approximately 4.5 min using a PC (CPU: Core i7-2600, memory: 4.00 GB) for one calculation; it takes approximately 45 min for ten calculations.

3.2.2. Effect of the number of voltage measurement points on estimation accuracy

Based on real-world measurements, in practice, it is preferable to have fewer voltage electrodes. As shown in Table 3, estimation is carried out with five patterns of voltage measurement points. As shown in Fig. 6, the range of $x$ and the interval in Table 3 indicate the distance between the outermost electrodes and the distance between adjacent electrodes, respectively. Fig. 7 shows the estimation accuracy that resulted from ten estimations of 7-mm cracks between $x = -12-5$ mm. As can be seen in the figure, the crack

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Table 2

<table>
<thead>
<tr>
<th>Parameters of GA for CSI.</th>
</tr>
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<tbody>
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<td>Population</td>
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<td>Max generation</td>
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<td>Cross-over rate</td>
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</tr>
<tr>
<td>Gene length</td>
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<td>Selection style</td>
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Fig. 5. Result of crack estimation in unidirectional laminate using CSI. Relationship of (a) electric conductivity in thickness direction to estimation accuracy and error and (b) location $x$ and true electric voltage change on the specimen’s surface. (Delamination crack $x = -12-5$).
estimation accuracies are similar; the true values of cracks were estimated, except in the case of three voltage measurement points (Case 5). Based on this result, it is possible to detect cracks using electrode intervals smaller than the widths of the cracks to be detected.

3.2.3. Effect of crack depth on estimation accuracy

Fig. 8(a) shows an estimation accuracy distribution map that calculates the estimation accuracy while changing the location of the 7-mm wide cracks at 2-mm intervals for each layer. Representative center points of the 7-mm cracks are plotted. Ten crack estimations were carried out under the same conditions. Additionally, Fig. 8(b) shows the absolute value of the electric current density in the thickness direction at each crack location.

As can be seen in Fig. 8(a), 7-mm-wide cracks were generally detected with over 80% accuracy across the laminated plate. Furthermore, Fig. 8(a) and (b) show that the estimation accuracy is higher near a current electrode where the electric current density in the thickness direction is high. The closer the estimated crack is to the center of a current electrode or the deeper the layer, the lower the estimation accuracy. Fig. 9 shows the relationship between the omission error rate and the electric current density in the thickness direction at the corresponding position. We observe that the lower the electric current density in the thickness direction, the greater the omission error rate. To ensure a high estimation accuracy, the electric current in the thickness direction must have a high value, which is obtained by adjusting the electrode interval considering anisotropic electric conductivity.

4. Conclusions

This paper proposed crack swarm inspection, with the goal of estimating crack locations and sizes from the surface voltage.
distribution of a cracked composite material by introducing a genetic algorithm for crack analysis with doublets. By applying the technique to cracked CFRP laminated plates, the estimation accuracy was validated for the electric conductivity, voltage measurement points, and crack depth. The lower the electric conductivity in the thickness direction, the more difficult crack detection becomes, because the voltage change distribution caused by crack development does not have sharp peaks in regions close to the crack. It is possible to detect cracks with voltage measurement points by installing electrodes at smaller intervals than the width of the cracks to be detected. Furthermore, it was confirmed that 7-mm cracks were detected with a degree of accuracy of over 80%. It was also confirmed that the electric current density in the thickness direction around the crack location affects the estimation accuracy. By calculating the equivalent electric conductivity, the proposed method can be extended to multidirectional laminates. Because the accuracy of the estimation depends on the electric conductivity ratio ($\sigma_t/\sigma_r$), this method would be more accurate in multidirectional laminates, which have smaller $\sigma_r$ compared with unidirectional laminates.

References