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Uncertainty analysis for bulk power systems reliability evaluation using Taylor series and nonparametric probability density estimation

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ABSTRACT

The uncertainties in reliability evaluation model are fundamentally classified into aleatory and epistemic types. Aleatory uncertainty arises from the intrinsic randomness associated with a physical system, such as components stochastic failure and repair process. Epistemic uncertainty, on the other hand, results from an incomplete or inaccurate scientific understanding of the underlying process, such as component reliability parameters uncertainty. It's significant for risk based decision to distinguish the two kinds of uncertainties and quantify their impacts on reliability analysis. In literatures, most of papers focused on aleatory uncertainty, and only a few of them discussed the epistemic uncertainty. This paper is aimed to address uncertainty analysis of reliability indices considering the randomness of reliability parameters. Two goals are achieved in this paper. Firstly, the reliability indices are approximated through Taylor series with high efficiency after component parameters change, and its accuracy is compared with rerunning reliability evaluation. Secondly, to uncover the uncertainty propagation from input reliability parameters level to reliability evaluation output level, two methods, i.e. Taylor series Approximation and Monte Carlo simulation combined with nonparametric probability density estimation are proposed. Results obtained for the RBTS and IEEE-RTS79 power systems are presented and the validity of the proposed methods is verified.

Introduction

Reliability evaluation of bulk power systems is a powerful tool to quantify the risk of electric service interruption incurred by uncontrollable and unpredictable failure events using uncertainty analysis theory [1–4]. Through the comprehensive quantitative analysis of the possibility and impact of the random failures, the probabilistic measures of interruption risk for system or delivery points can be identified by condensing contingency likelihood and severity into probabilistic risk indices. The purpose of the probabilistic risk evaluation for bulk power systems is to quantitatively assess the impacts of various kinds of random factors on power systems performance, and provide valuable reference information for risk management, risk control and risk based decision. Using the probabilistic risk evaluation technology, not only the coupling relation between the uncertain factors and system performance can be uncovered, but also the system bottlenecks and dominant random factors can be effectively revealed.

The reliability performance of a power system is often affected by unavoidable uncertainties, and probabilistic uncertainty analysis can quantify the effect of input random variables on the output results of reliability evaluation model. The uncertainty in reliability evaluation model is fundamentally classified into aleatory and epistemic types. Quantitative uncertainty analysis has become an integral and essential part of risk based design and decision making, and the clear distinction of these two kinds of uncertainties is useful for taking the reliability/risk informed decisions with confidence [5–9]. The problem of acknowledging and treating uncertainty is vital for practical usability of reliability analysis results because the reliability indices mixing both the uncertainties means that one cannot see how much of the total uncertainty comes from epistemic and aleatory uncertainties.

Aleatory uncertainty is also termed in the literature as objective, irreducible, inherent, and stochastic uncertainty. It describes the inherent randomness (variation) associated with a physical system or environment, such as failure and repair time of equipments in a power system. This type of uncertainty cannot be reduced or eliminated because it is an intrinsic nature of the system itself. Aleatory uncertainty is dealt with by probability theory. Given the failure logic of system and probability density functions of failure and repair time, sequential Monte Carlo simulation can be used to obtain the probability density of reliability indices [10,11], while analytical approach and nonsequential Monte Carlo

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simulation can be used to get the point estimation(usually the expected value) of reliability indices.

However, both Monte Carlo simulation and analytical approach are built on a number of model parameters that are based on what is currently known about the physics of the relevant processes and the behavior of systems. The model parameters, such as failure rates and repair rates, are not exactly known because of deterioration or lack of data. This kind of uncertainty associated with state of knowledge, is referred as epistemic uncertainty. Epistemic uncertainty derives from some level of ignorance or incomplete information about a system, and it's reducible if more information is collected. Because of this reason, it is also termed as reducible, subjective, and model form uncertainty.

In the published literatures, aleatory uncertainty have been the main focus of bulk power systems reliability evaluation, and only few of them are involved in parameter uncertainty. The equipment reliability parameters are the input data in system risk evaluation and can be estimated or measured from historical failure statistics using parameter estimation technique. The correctness and accuracy of the historical data is critical in risk evaluation, as we know uncertainties or even errors in historical statistics are unavoidable. An essential task in the parameter estimation is to reduce the impacts of the uncertainties or errors and enhance the accuracy. The parameter estimation methods can be divided into point, interval and distribution estimations among which the distribution estimation is the most sophisticated and can offer a probability distribution function (PDF) of a parameter. Using the probability distribution of input data can greatly enhance the accuracy in system risk evaluation. It needs more calculation efforts and higher requirements for assessment techniques when probability distribution of input data are used, however, utilizing the approach presented in this paper this problem can be effectually solved with nearly no extra computational effort.

The impact of parameter uncertainty must be addressed if the analysis is to serve as a tool in the decision making process. To uncover the impact of parameter uncertainty on reliability evaluation results, two critical questions must be deeply explored.

- How do we characterize the uncertainty of equipment reliability parameters?
- How does the uncertainty propagate from parameter level to model output level?

There are several methods available in the literature to research parameter uncertainty propagation such as evidence theory [7], interval arithmetic [8], fuzzy arithmetic [9], classical probability theory, and so on. They are different from each other, in terms of characterizing the input parameter uncertainty and also in kind of propagation from parameter level to model output level. Because the most widely known and developed methods are available within the mathematics of probability theory, in this paper before the uncertainty analysis can be performed a description of model parameters uncertainty must be available, i.e., failure rate/repair rate are characterized by a probability distribution and then how they are propagated to the system level is investigated. So the above questions are turned into how to get the analytical expression of the reliability indices with respect to reliability parameters and how to obtain the PDF of reliability indices when PDFs of reliability parameters are given.

Through sensitivity analysis of reliability indices with respect to equipment failure and repair rates, the first order Taylor series of reliability indices at the mean values of component reliability parameters can be achieved [12,13]. So the reliability indices can be analytically calculated with high accuracy after reliability parameters are slightly changed. But the results using first order Taylor series have larger error if reliability parameters have significant changes. To improve the accuracy, the second-order partial differentials of reliability indices with respect to reliability parameters and then the second order Taylor series are deduced in this paper.

After the approximate analytical expressions of reliability indices are developed, how to obtain the PDFs of reliability indices assuming the PDFs of reliability parameters are known become a vital step for parameter uncertainty analysis. Using the second order Taylor series, the reliability indices can be regarded as random sums consisting of both linear and quadratic items of uncertain reliability parameters. To avoid the direct convolution of the PDFs of the reliability parameters, several methods are presented in literatures, such as saddlepoint approximation [6], Gram–Charlier's expansion [14], and characteristic function method [15]. However there is a common limitation among these methods that the constituent items in the random sum must be mutually independent random variables, which is true for first order Taylor series but invalid for second order Taylor expansion because the linear item and quadratic item are correlated with each other. Because of the above reason a new approach termed as nonparametric probability density estimation has been applied to solve this problem in this paper.

The aim of this work is to develop an efficient and accurate method, which is expected to have the approximate accuracy as Monte Carlo simulation depicted in section IV, but with much higher efficiency. The proposed method is detailed in the following section.

This paper is organized as follows. Section II gives the mathematical description of parameter uncertainties for bulk power systems reliability evaluation model. Section III describes the first and second order derivatives of reliability indices with respect to component reliability parameters. Section IV depicts the fundamental principle to evaluate probability distribution of reliability indices through nonparametric probability density estimation technique, and two methods are presented there, i.e. Taylor series approximation and Monte Carlo simulation. The study results and the effectiveness of the proposed approach are illustrated in Section V. The conclusion drawn from the analysis is provided in Section VI. Details are provided in Appendix.

Mathematical description of epistemic uncertainty

The reliability index Y can be expressed with the joint function of aleatory and epistemic uncertainties.

$$Y = R(\mathbf{U}, \mathbf{V}), \tag{1}$$

where

- **U** set of all epistemic uncertainties (uncertain reliability parameters known as failure rate $\lambda$and repair rate μ),
- **V** set of all aleatory uncertainties (stochastic variables associated with component failure or repair, i.e. the time to failure and the time to repair),
- R computational model for reliability evaluation considered as a deterministic function of both uncertainties mentioned above,
- Y reliability index.

R represents the computational model which describes the functional relationship between reliability index Y and both uncertainties for bulk power reliability evaluation, and it's a black-box and computationally expensive because it's too complicated for us to derive its explicit functional form. From (1), we can see that reliability index Y is actually a random variable affected by both aleatory and epistemic uncertainties. When holding the epistemic variables $\mathbf{U}$ fixed at a value $\mathbf{u}$, i.e. $\mathbf{U} = \mathbf{u}$, the resulting output Y is a function of the aleatory uncertainties $\mathbf{V}$ solely.

In the available literature, most of them carry out researches under the assumption that $\mathbf{U}$ are constants (usually the mean values $\boldsymbol{\mu}_\mathbf{U}$ of the component reliability parameters), and then $Y|\mathbf{U}=\boldsymbol{\mu}_\mathbf{U}$ is evaluated and analyzed. $Y|\mathbf{U}=\boldsymbol{\mu}_\mathbf{U}$ is still a random variable, and its conditional probability distribution,which quantifies the corresponding aleatory uncertainty in Y, can be attained through sequential Monte Carlo simulation and its conditional expected value $E(Y|\mathbf{U}=\boldsymbol{\mu}_\mathbf{U})$ can be estimated by either Monte-Carlo simulation or analytical approach. However, up to now few of papers have been presented to investigate the impact of parameter uncertainties $\mathbf{U}$ on reliability index Y. As we know, when the epistemic variables $\mathbf{U}$ are fixed at a specified value $\mathbf{u}$, the expected value $E(Y|\mathbf{U}=\mathbf{u})$ is a constant value uniquely determined by $\mathbf{u}$. When each uncertain reliability parameter U_i is viewed as a random variable with known probability density distribution that can be estimated or measured from the historical record, statistical analysis, or engineering judgment, obviously uncertainty of reliability parameters $\mathbf{U}$ cause stochastic variation in the $E(Y|\mathbf{U}=\mathbf{u})$. Because $\mathbf{U}$ is a stochastic vector representing parameter uncertainties, the expression $E(Y|\mathbf{U})$ which denotes the conditional expectation as function of the epistemic uncertainties $\mathbf{U}$ should also be seen as a random quantity.

$$E(Y|\mathbf{U}) = g(\mathbf{U}). \tag{2}$$

Therefore, the principal aim of parameter uncertainty analysis is to determine the subjective probability distribution of the conditional expectation $E(Y|\mathbf{U})$, which provides visual specification of quantitative uncertainty analysis. In traditional bulk power system reliability evaluation, $E(Y|\mathbf{U})$ denotes the conventional reliability indices as follows, which are expected indices essentially.

$$\mathrm{LOLP} = \sum_{\boldsymbol{x}\in\boldsymbol{X}} I_f(\boldsymbol{x})P(\boldsymbol{x}), \tag{3}$$

$$\mathrm{LOLF} = \sum_{\boldsymbol{x}\in\boldsymbol{X}} (I_f(\boldsymbol{x})\sum_{k=1}^{m}\lambda_{\boldsymbol{x}}^{in}(k))P(\boldsymbol{x}), \tag{4}$$

$$\mathrm{EENS} = 8760\sum_{\boldsymbol{x}\in\boldsymbol{X}} I_f(\boldsymbol{x})L_C(\boldsymbol{x})P(\boldsymbol{x}). \tag{5}$$

LOLP, LOLF and EENS are loss of load probability, loss of load frequency and expected energy not supplied respectively. $\boldsymbol{X}$ is system state space. A system state can be represented by a vector $\boldsymbol{x}=\{S_1, S_2, \ldots, S_m\}$, where S_k is the state of the kth component and m is the number of components. For notational simplicity we assume that the system components only have two states and $S_k = 0$ indicates the kth component is in down state, otherwise $S_k = 1$ shows the kth component is in up state. The indicator function $I_f(\boldsymbol{x})$ is used to identify the failed system state. If system is in failure state $I_f(\boldsymbol{x}) = 1$, if not, $I_f(\boldsymbol{x}) = 0$. $Lc(\boldsymbol{x})$ is load shedding amount under failure state $\boldsymbol{x}$. $\lambda_x^{in}(k)$ is the *incremental transition rate* of component k [12].

$$\lambda_{\boldsymbol{x}}^{in}(k) = (1-S_k)\mu_k - S_k\lambda_k, \tag{6}$$

where μ_k and λ_k are repair rate and failure rate of component k.

$P(\boldsymbol{x})$ is the probability of system state $\boldsymbol{x}$. If component failures are independent random variables, $P(\boldsymbol{x})$ is given by the product of the probabilities of each component as follows.

$$P(\boldsymbol{x}) = \prod_{k=1}^{m}P(S_k) = \prod_{k=1}^{m}\left(\frac{S_k\mu_k}{\lambda_k+\mu_k} + \frac{(1-S_k)\lambda_k}{\lambda_k+\mu_k}\right). \tag{7}$$

From (3)–(5) we can see that LOLP, LOLF and EENS are nonlinear functions of component reliability parameters. As this is a typically combinatorial problem, reliability evaluation of bulk power system is computationally extensive and time consuming. Because of the computational burden it's very difficult to research the impact of parameter variations on reliability indices.

The simplest evaluation of the epistemic distribution of $E(Y|\mathbf{U}=\mathbf{u})$ is through Monte Carlo simulation (MCS). This method requires that the each of the reliability parameters involved to be assigned a probability distribution that characterizes the possible variation. The random values from these distributions are sampled and arrive at an estimate of the uncertainty of reliability indices. A large computation effort is required for the MCS method. One realistic solution of this problem is to perform sensitivity analysis of reliability indices with respect to reliability parameters [12,13]. Through sensitivity analysis, (3)–(5) can be expressed as first order Taylor series expansions, and reliability indices can be directly calculated with high accuracy after reliability parameters have minor changes. However, for a system with poor reliability (3)–(5) are sometimes highly nonlinear, the approximate treatment using first order Taylor series may induce unacceptable error. For this reason, in order to efficiently and accurately estimate the uncertainty involved in the reliability indices computation, this paper explored the second order Taylor series at the mean values of random reliability parameters, which is more preferable option than the above one, and the general expression of the reliability indices $E(Y|\mathbf{U})$ in (2) is as follows.

$$\begin{aligned} E(Y|\mathbf{U}) &= g(\mathbf{U}) \\ &\approx g(\boldsymbol{\mu}_\mathbf{U}) + \sum_{i=1}^{N}\frac{\partial g}{\partial U_i}\bigg|_{\boldsymbol{\mu}_\mathbf{U}}\left(U_i-\mu_{U_i}\right) \\ &\quad + \frac{1}{2!}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial^2 g}{\partial U_i \partial U_j}\bigg|_{\boldsymbol{\mu}_\mathbf{U}}\left(U_i-\mu_{U_i}\right)\left(U_j-\mu_{U_j}\right), \end{aligned} \tag{8}$$

where $\boldsymbol{\mu} = [\mu_{U_1}, \mu_{U_2}, \ldots, \mu_{U_N}]^T$ is the vector of mean values of $\mathbf{U} = [U_1, U_2, \ldots, U_N]^T$, $g(\boldsymbol{\mu}_\mathbf{U})$ is conditional mean of reliability indices, which can be calculated from (3)–(5) either by MCS or enumeration method with reliability parameters $\mathbf{U}$ fixed at its mean values $\boldsymbol{\mu}_U$. Based on (8), the mean value of reliability indices can be calculated by

$$E(E(Y|\mathbf{U})) = E(g(\mathbf{U})) \approx g(\boldsymbol{\mu}_\mathbf{U}) + \frac{1}{2}\sum_{i=1}^{N}\sigma_{U_i}^2 \cdot \frac{\partial^2 g}{\partial U_i^2}\bigg|_{\mu_{U_i}}. \tag{9}$$

From (9), it can be seen clearly that the mean value $E(g(\mathbf{U}))$ for reliability indices $g(\mathbf{U})$ is larger than the conditional expectation $g(\boldsymbol{\mu}_\mathbf{U})$ when uncertainty of reliability parameter is involved.

It should be noted that the second-order partial derivatives of reliability indices with respect to reliability parameters must be calculated before we can use (8). The next section gives the derivation of the first and second-order partial derivatives.

First and second order partial derivatives of reliability indices

Partial derivatives of LOLP

Based on the idea of conditional probability, LOLP can be represented by the following formula and the details are listed in the Appendix.

$$\mathrm{LOLP} = \frac{\mu_i}{\lambda_i+\mu_i}(K_i^{(1)} - K_i^{(2)}) + K_i^{(2)}, \tag{10}$$

$$K_i^{(1)} = \sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=1} I_f(\boldsymbol{x}) \prod_{k=1,k\neq i}^{m} P(S_k), \tag{11}$$

$$K_i^{(2)} = \sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=0} I_f(\boldsymbol{x}) \prod_{k=1,k\neq i}^{m} P(S_k), \tag{12}$$

where $K_i^{(1)}$ is the LOLP under the condition that component i is always in up state. Similarly, $K_i^{(2)}$ is conditional LOLP when it is given that component i is in down state forever. For a specified power system, $K_i^{(1)}$ and $K_i^{(2)}$ are constants between 0 and 1 and can be evaluated simultaneously with the reliability indices in the reliability assessment process, so (10) can be referred to as the analytical expression of LOLP, which has only two variables μ_i and λ_i. Because component i is always in failure state for $K_i^{(2)}$, it can be concluded that $K_i^{(2)} - K_i^{(1)} \geqslant 0$. Utilizing (10), the following first-order and second-order partial differentials can be easily deduced.

$$\frac{\partial \mathrm{LOLP}}{\partial \lambda_i} = \frac{a_i\left(K_i^{(2)} - K_i^{(1)}\right)}{\lambda_i + \mu_i}, \tag{13}$$

$$\frac{\partial \mathrm{LOLP}}{\partial \mu_i} = \frac{u_i\left(K_i^{(1)} - K_i^{(2)}\right)}{\lambda_i + \mu_i}, \tag{14}$$

$$\frac{\partial^2 \mathrm{LOLP}}{\partial \lambda_i^2} = \frac{2a_i}{(\lambda_i + \mu_i)^2}\left(K_i^{(1)} - K_i^{(2)}\right), \tag{15}$$

$$\frac{\partial^2 \mathrm{LOLP}}{\partial \mu_i^2} = \frac{2u_i}{(\lambda_i + \mu_i)^2}\left(K_i^{(2)} - K_i^{(1)}\right), \tag{16}$$

$$\frac{\partial^2 \mathrm{LOLP}}{\partial \lambda_i \partial \mu_i} = \frac{\lambda_i - \mu_i}{(\lambda_i + \mu_i)^3}\left(K_i^{(2)} - K_i^{(1)}\right), \tag{17}$$

$$\frac{\partial^2 \mathrm{LOLP}}{\partial \lambda_i \partial \lambda_j} = \frac{a_i}{\lambda_i + \mu_i}\left(\frac{\partial K_i^{(2)}}{\partial \lambda_j} - \frac{\partial K_i^{(1)}}{\partial \lambda_j}\right) = \frac{a_j}{\lambda_j + \mu_j}\left(\frac{\partial K_j^{(2)}}{\partial \lambda_i} - \frac{\partial K_j^{(1)}}{\partial \lambda_i}\right), \tag{18}$$

$$\frac{\partial^2 \mathrm{LOLP}}{\partial \lambda_i \partial \mu_j} = \frac{a_i}{\lambda_i + \mu_i}\left(\frac{\partial K_i^{(2)}}{\partial \mu_j} - \frac{\partial K_i^{(1)}}{\partial \mu_j}\right) = \frac{u_j}{\lambda_j + \mu_j}\left(\frac{\partial K_j^{(1)}}{\partial \lambda_i} - \frac{\partial K_j^{(2)}}{\partial \lambda_i}\right), \tag{19}$$

$$\frac{\partial^2 \mathrm{LOLP}}{\partial \mu_i \partial \mu_j} = \frac{u_i}{\lambda_i + \mu_i}\left(\frac{\partial K_i^{(1)}}{\partial \mu_j} - \frac{\partial K_i^{(2)}}{\partial \mu_j}\right) = \frac{u_j}{\lambda_j + \mu_j}\left(\frac{\partial K_j^{(1)}}{\partial \mu_i} - \frac{\partial K_j^{(2)}}{\partial \mu_i}\right), \tag{20}$$

where a_i, u_i are availability and unavailability of component i respectively.

Partial derivatives of LOLF

Based on the same principle, the analytical expressions and partial differentials of LOLF with respect to reliability parameters can be deduced like LOLP.

$$\mathrm{LOLF} = \frac{\lambda_i \mu_i (K_i^{(2)} - K_i^{(1)})}{\lambda_i + \mu_i} + \frac{\mu_i (K_i^{(3)} - K_i^{(4)})}{\lambda_i + \mu_i} + K_i^{(4)}, \tag{21}$$

$$K_i^{(3)} = \sum_{\boldsymbol{x} \in \boldsymbol{X}, S_i = 1} I_f(\mathbf{x}) \left(\sum_{j=1, j \neq i}^{m} \lambda_{\boldsymbol{x}}^{in}(j) \right) \prod_{k=1, k \neq i}^{m} P(S_k), \tag{22}$$

$$K_i^{(4)} = \sum_{\boldsymbol{x} \in \boldsymbol{X}, S_i = 0} I_f(\mathbf{x}) \left(\sum_{j=1, j \neq i}^{m} \lambda_{\boldsymbol{x}}^{in}(j) \right) \prod_{k=1, k \neq i}^{m} P(S_k), \tag{23}$$

where $K_i^{(3)}$ and $K_i^{(4)}$ are conditional LOLF under the assumption that component i is always in success state and failure state respectively. The following partial differentials can be derived from (21).

$$\frac{\partial \mathrm{LOLF}}{\partial \lambda_i} = a_i^2 (K_i^{(2)} - K_i^{(1)}) - \frac{a_i (K_i^{(3)} - K_i^{(4)})}{\lambda_i + \mu_i}, \tag{24}$$

$$\frac{\partial \mathrm{LOLF}}{\partial \mu_i} = u_i^2 (K_i^{(2)} - K_i^{(1)}) + \frac{u_i (K_i^{(3)} - K_i^{(4)})}{\lambda_i + \mu_i}, \tag{25}$$

$$\frac{\partial^2 \mathrm{LOLF}}{\partial \lambda_i^2} = \frac{-2a_i^2 (K_i^{(2)} - K_i^{(1)})}{\lambda_i + \mu_i} - \frac{2a_i (K_i^{(4)} - K_i^{(3)})}{(\lambda_i + \mu_i)^2}, \tag{26}$$

$$\frac{\partial^2 \mathrm{LOLF}}{\partial \mu_i^2} = -\frac{2u_i^2 (K_i^{(2)} - K_i^{(1)})}{\lambda_i + \mu_i} + \frac{2u_i (K_i^{(4)} - K_i^{(3)})}{(\lambda_i + \mu_i)^2}, \tag{27}$$

$$\frac{\partial^2 \mathrm{LOLF}}{\partial \lambda_i \partial \mu_i} = \frac{2a_i u_i (K_i^{(2)} - K_i^{(1)})}{\lambda_i + \mu_i} - \frac{(\lambda_i - \mu_i)(K_i^{(3)} - K_i^{(4)})}{(\lambda_i + \mu_i)^3}, \tag{28}$$

$$\begin{aligned} \frac{\partial^2 \mathrm{LOLF}}{\partial \lambda_i \partial \lambda_j} &= a_i^2 \left(\frac{\partial K_i^{(2)}}{\partial \lambda_j} - \frac{\partial K_i^{(1)}}{\partial \lambda_j}\right) - \frac{a_i}{\lambda_i + \mu_i}\left(\frac{\partial K_i^{(3)}}{\partial \lambda_j} - \frac{\partial K_i^{(4)}}{\partial \lambda_j}\right) \\ &= a_j^2 \left(\frac{\partial K_j^{(2)}}{\partial \lambda_i} - \frac{\partial K_j^{(1)}}{\partial \lambda_i}\right) - \frac{a_j}{\lambda_j + \mu_j}\left(\frac{\partial K_j^{(3)}}{\partial \lambda_i} - \frac{\partial K_j^{(4)}}{\partial \lambda_i}\right), \end{aligned} \tag{29}$$

$$\begin{aligned} \frac{\partial^2 \mathrm{LOLF}}{\partial \lambda_i \partial \mu_j} &= a_i^2 \left(\frac{\partial K_i^{(2)}}{\partial \mu_j} - \frac{\partial K_i^{(1)}}{\partial \mu_j}\right) - \frac{a_i}{\lambda_i + \mu_i}\left(\frac{\partial K_i^{(3)}}{\partial \mu_j} - \frac{\partial K_i^{(4)}}{\partial \mu_j}\right) \\ &= u_j^2 \left(\frac{\partial K_j^{(2)}}{\partial \lambda_i} - \frac{\partial K_j^{(1)}}{\partial \lambda_i}\right) + \frac{u_j}{\lambda_j + \mu_j}\left(\frac{\partial K_j^{(3)}}{\partial \lambda_i} - \frac{\partial K_j^{(4)}}{\partial \lambda_i}\right), \end{aligned} \tag{30}$$

$$\begin{aligned} \frac{\partial^2 \mathrm{LOLF}}{\partial \mu_i \partial \mu_j} &= u_i^2 \left(\frac{\partial K_i^{(2)}}{\partial \mu_j} - \frac{\partial K_i^{(1)}}{\partial \mu_j}\right) + \frac{u_i}{\lambda_i + \mu_i}\left(\frac{\partial K_i^{(3)}}{\partial \mu_j} - \frac{\partial K_i^{(4)}}{\partial \mu_j}\right) \\ &= u_j^2 \left(\frac{\partial K_j^{(2)}}{\partial \mu_i} - \frac{\partial K_j^{(1)}}{\partial \mu_i}\right) + \frac{u_j}{\lambda_j + \mu_j}\left(\frac{\partial K_j^{(3)}}{\partial \mu_i} - \frac{\partial K_j^{(4)}}{\partial \mu_i}\right). \end{aligned} \tag{31}$$

Partial derivatives of EENS

$$\mathrm{EENS} = \frac{\mu_i}{\lambda_i + \mu_i}(E_i^{(1)} - E_i^{(2)}) + E_i^{(2)}, \tag{32}$$

$$E_i^{(1)} = 8760 \sum_{\boldsymbol{x} \in \boldsymbol{X}, S_i = 1} I_f(\boldsymbol{x}) L_C(\boldsymbol{x}) \prod_{k=1, k \neq i}^{m} P(S_k), \tag{33}$$

$$E_i^{(2)} = 8760 \sum_{\boldsymbol{x} \in \boldsymbol{X}, S_i = 0} I_f(\boldsymbol{x}) L_C(\boldsymbol{x}) \prod_{k=1, k \neq i}^{m} P(S_k), \tag{34}$$

where $E_i^{(1)}$ and $E_i^{(2)}$ are conditional EENS under the assumption that component i is always in success state and failure state respectively. Through (32) the following partial differentials can be obtained.

$$\frac{\partial \mathrm{EENS}}{\partial \lambda_i} = \frac{a_i (E_i^{(2)} - E_i^{(1)})}{\lambda_i + \mu_i}, \tag{35}$$

$$\frac{\partial \mathrm{EENS}}{\partial \mu_i} = \frac{u_i (E_i^{(1)} - E_i^{(2)})}{\lambda_i + \mu_i}, \tag{36}$$

$$\frac{\partial^2 \mathrm{EENS}}{\partial \lambda_i^2} = \frac{2a_i}{(\lambda_i + \mu_i)^2}(E_i^{(1)} - E_i^{(2)}), \tag{37}$$

$$\frac{\partial^2 \mathrm{EENS}}{\partial \mu_i^2} = \frac{2u_i}{(\lambda_i + \mu_i)^2}(E_i^{(2)} - E_i^{(1)}), \tag{38}$$

$$\frac{\partial^2 \mathrm{EENS}}{\partial \lambda_i \partial \mu_i} = \frac{\lambda_i - \mu_i}{(\lambda_i + \mu_i)^3}(E_i^{(2)} - E_i^{(1)}), \tag{39}$$

$$\frac{\partial^2 \mathrm{EENS}}{\partial \lambda_i \partial \lambda_j} = \frac{a_i}{\lambda_i + \mu_i}\left(\frac{\partial E_i^{(2)}}{\partial \lambda_j} - \frac{\partial E_i^{(1)}}{\partial \lambda_j}\right) = \frac{a_j}{\lambda_j + \mu_j}\left(\frac{\partial E_j^{(2)}}{\partial \lambda_i} - \frac{\partial E_j^{(1)}}{\partial \lambda_i}\right), \tag{40}$$

$$\frac{\partial^2 \mathrm{EENS}}{\partial \lambda_i \partial \mu_j} = \frac{a_i}{\lambda_i + \mu_i}\left(\frac{\partial E_i^{(2)}}{\partial \mu_j} - \frac{\partial E_i^{(1)}}{\partial \mu_j}\right) = \frac{u_j}{\lambda_j + \mu_j}\left(\frac{\partial E_j^{(1)}}{\partial \lambda_i} - \frac{\partial E_j^{(2)}}{\partial \lambda_i}\right), \tag{41}$$

$$\frac{\partial^2 \mathrm{EENS}}{\partial \mu_i \partial \mu_j} = \frac{u_i}{\lambda_i + \mu_i}\left(\frac{\partial E_i^{(1)}}{\partial \mu_j} - \frac{\partial E_i^{(2)}}{\partial \mu_j}\right) = \frac{u_j}{\lambda_j + \mu_j}\left(\frac{\partial E_j^{(1)}}{\partial \mu_i} - \frac{\partial E_j^{(2)}}{\partial \mu_i}\right). \tag{42}$$

It should be emphasized here that in the above equations the partial differentials of $K_{\boldsymbol{i}}^{(1)} \sim K_{\boldsymbol{i}}^{(4)}$ and $E_{\boldsymbol{i}}^{(1)} \sim E_{\boldsymbol{i}}^{(2)}$ with respect to λ_j and μ_j are similar to the traditional reliability indices and all of them can be calculated in the reliability evaluation process. In other words, $K_{\boldsymbol{i}}^{(1)} \sim K_{\boldsymbol{i}}^{(4)}$, $E_{\boldsymbol{i}}^{(1)} \sim E_{\boldsymbol{i}}^{(2)}$ and their partial differentials can be treated as a new kinds of reliability indices. The analytical expressions of the partial differentials for $K_{\boldsymbol{i}}^{(1)} \sim K_{\boldsymbol{i}}^{(4)}$ and $E_{\boldsymbol{i}}^{(1)} \sim E_{\boldsymbol{i}}^{(2)}$ are listed in the Appendix.

Evaluating uncertainty of reliability indices

Taylor series approximation

Using the partial differentials deduced in the previous section and (8), the first-order and second-order Taylor expansions of reliability indices (3)–(5) can be got, and then the approximate results of reliability indices can be also directly obtained when multi-component parameters have changed. Now another question arises, given the PDFs of reliability parameters, how can we evaluate the PDFs of the reliability indices? When first-order Taylor expansion is employed, the reliability indices can be treated as a linear combination of uncertain parameters $\mathbf{U}$. With the assumption that the random deviations of reliability parameters are small enough so that reliability indices obtained by first-order Taylor approximation are accurate adequately, based on the additivity of normal distribution we can conclude that the reliability indices also follow normal distribution with mean value $E(g(\mathbf{U}))=E(E(Y|\mathbf{U}))=g(\boldsymbol{\mu}_{\mathbf{U}})$ and variance $V(g(\mathbf{U}))$ when reliability parameters follow normal distribution and are mutually independent. This idea has been adopted in the mean value first order Second Moment (MVFOSM) approach [6] for probabilistic uncertainty analysis.

$$V(g(\mathbf{U})) = V(E(Y|\mathbf{U})) \approx \sum_{i=1}^{N} \left(\frac{\partial g}{\partial U_i}\bigg|_{U_i} \right)^2 \sigma_{U_i}^2. \tag{43}$$

However, reliability parameters may follow different probability distributions, and above all reliability parameters may vary largely so that the linear approximation of reliability indices may be inaccurate, thus MVFOSM may lead to poor accuracy in spite of its high efficiency. Except MVFOSM, first order reliability method (FORM) and first order Saddlepoint Approximation (FOSA) [6] also use the first order Taylor series approximation to be the performance function $g(\mathbf{U})$. Such approximation may not accurately capture the nonlinearity of $g(\mathbf{U})$, and thus may not be suitable for the situations where highly nonlinear performance functions are involved.

In the available literatures, in order to avoid the direct convolution some useful approaches have been presented to compute approximate PDF for random function $g(\mathbf{U})$, such as saddlepoint approximation [6], Gram–Charlier series [14], and characteristic functions-based approach [15]. However, all these methods are only suitable for first-order Taylor series of $g(\mathbf{U})$, in which each item in the random sum must be independent of each other. As for the second-order Taylor series, the linear items and quadratic items are correlated with each other, so the above methods are invalid for it. Although Karl Pearson's system of frequency curves can be used to compute approximate percentage points of intractable or empirical distributions based on the knowledge of its first four moments [16,17], the determination of the most suitable Pearson distribution is troublesome. Moreover, the accuracy is limited because moments higher than four are ignored in Pearson curves.

The most attractive method used in statistical literature for modeling probability density distribution from sample points is the nonparametric probability density estimation [18–20].

Let $X_1, X_2, \ldots, X_n$ denote a sample of size n of a reliability index which is a random variable with density $f(x)$. The probability density estimation of $f(x)$ at the point x is given by

$$\hat{f}_h(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right), \tag{44}$$

where the smoothing parameter h is known as the bandwidth and in practice the kernel function $K()$ is generally chosen to be a unimodal probability density function symmetric about zero. The kernel K satisfies the conditions.

$$\int_{-\infty}^{\infty} K(x)dx = 1, \tag{45}$$

$$\int_{-\infty}^{\infty} xK(x)dx = 0, \tag{46}$$

$$\int_{-\infty}^{\infty} x^2K(x)dx = \mu_2(K) > 0. \tag{47}$$

It's well known that the performance of kernel density estimation depends crucially on the value of bandwidth h instead of the kernel function K, so Gaussian kernel (standard normal distribution) is usually a popular option in practice. $\hat{f}_h$ will inherit all the continuity and differentiability properties of the kernel K, so that if K is the normal density function, then $\hat{f}_h$ will be a smooth curve having derivatives of all orders.

The statistical properties of kernel density estimator are shown as follows.

$$\mathrm{Bias}(\hat{f}_h(x)) = E(\hat{f}_h(x)) - f(x) = \left(\frac{h^2}{2}\right)\mu_2(K)f''(x) + o(h^2), \tag{48}$$

$$\mathrm{Var}(\hat{f}_h(x)) = E(\hat{f}_h(x)^2) - (E(\hat{f}_h(x)))^2 = \frac{f(x)R(K)}{nh} + o\left(\frac{1}{nh}\right), \tag{49}$$

$$\begin{aligned}\mathrm{MISE}(\hat{f}_h) &= E\left(\int_{-\infty}^{\infty} (\hat{f}_h(x) - f(x))^2 dx\right) \\ &= \int_{-\infty}^{\infty} \mathrm{Bias}(\hat{f}_h(x))^2 dx + \int_{-\infty}^{\infty} \mathrm{Var}(\hat{f}_h(x))dx \\ &= \frac{h^4}{4}\mu_2(K)^2R(f'') + \frac{1}{nh}R(K) + o\left(\frac{1}{nh}\right) + o(h^4),\end{aligned} \tag{50}$$

$$\mathrm{AMISE}(\hat{f}_h) = \frac{h^4}{4}\mu_2(K)^2R(f'') + \frac{1}{nh}R(K), \tag{51}$$

where

$$R(K) = \int_{-\infty}^{\infty} K^2(x)dx, \tag{52}$$

$$R(f'') = \int_{-\infty}^{\infty} (f''(x))^2 dx. \tag{53}$$

From (48)–(50), it can be seen that we face the familiar trade-off between variance and bias because the bias and variance of the kernel density estimator cannot be reduced simultaneously. We would surely like to keep both variance and bias small but increasing h will lower the variance while it will raise the bias (decreasing h will do the opposite). Minimizing the MISE (mean Integrated Squared Error) which is the integration of the sum of variance and squared bias, represents a compromise between variance and bias. Ignoring higher order terms of MISE, an approximate formula for the MISE, called AMISE (asymptotic mean Integrated Squared Error), can be given as (51). The bandwidth that minimizes the AMISE is given by.

$$h_{\mathrm{AMISE}} = \left[\frac{R(K)}{\mu_2(K)^2R(f'')}\right]^{1/5} n^{-1/5}. \tag{54}$$

Eq. (54) gives the optimal bandwidth minimizing AMISE, but it's still difficult to directly use it because $R(f'')$ is an unknown part as the unknown $f(x)$.

In order to get the optimal bandwidth h, some methods are presented such as Cross-Validation, Silverman's Rules of Thumb (ROT), and Plug-in methods (PM). Because the ROT method is relatively easy and widely used, it's adopted in this paper.

Let IQR and σ denote the interquartile range and standard deviation of reliability indices, respectively. Take the kernel function $K()$ to be the usual Gaussian kernel and assume that the underlying distribution f is normal, Silverman [20] showed that (54) can be reduced to

$$h_{\text{AMISE}} = 1.06\sigma n^{-1/5} \tag{55}$$

or

$$h_{\text{AMISE}} = 1.06(\text{IQR}/1.34)n^{-1/5} = 0.79\text{IQR}n^{-1/5}. \tag{56}$$

Furthermore, Silverman recommended reducing the factor 1.06 to 0.9 so as not to miss bimodality and using the smaller h estimated by (55) and (55). This rule is commonly used in practice and is often referred to as Silverman's reference bandwidth or Silverman's rule of thumb. It's given by

$$h_{\text{ROT}} = 0.9\text{A}n^{-1/5}, \tag{57}$$

where A = min{sample standard deviation, (sample interquar- tile range)/1.34}.

In order to evaluate the PDFs of the reliability indices under the assumption the PDFs of reliability parameters are given, we propose the following solution steps.

Step (1) Use the mean failure rate λ and mean time to repair r (reciprocal of the mean repair rate, that is $1/\mu$) of system components as input parameters, and then perform bulk power systems reliability evaluation. The expected value of reliability index and its first-order and second-order partial differentials with respect to failure rates and repair rates can be got using Eq. (10)–(42).

Step (2) Treat the failure rate and time to repair of system components as random variables, and draw random numbers from the given PDFs for these reliability parameters. In these random numbers convert the times to repair into repair rates through their reciprocal relation, and then the random sample $\mathbf{U}_i = [U_1, U_2, \ldots, U_N]$ for reliability parameters can be determined.

Step (3) Use Eq. (8) and random sample $\mathbf{U}_i$, the reliability index $X_i = E(Y|\mathbf{U}_i) = g(\mathbf{U}_i)$ can be approximately computed.

Step (4) Repeat Step (2) and Step (3) for n times, and the random sample of size n of reliability index, i.e. $X_1, X_2, \ldots, X_n$, are obtained. Then calculate the IQR and σ through random sample $X_1, X_2, \ldots, X_n$.

Step (5) Use Eq. (57) to obtain the optimal bandwidth h_{ROT} for kernel density estimation of reliability index

Step (6) Conduct the kernel density estimation using equation (44), and then the PDF for reliability index $E(Y|\mathbf{U})$ can be achieved.

Step (7) As is well known, the expected value $E(E(Y|\mathbf{U}))$ and variance $V(E(Y|\mathbf{U}))$ or standard variance $Std(E(Y|\mathbf{U}))$ can be directly available from the above PDF.

Monte Carlo simulation

The natural straightforward approach is to repeat reliability evaluation many times, each time with randomly selected values from the subjective probability distributions of the reliability parameters. The results are then summarized in form of probability distribution functions representing the effect of parameter uncertainties on outcomes of reliability evaluation model.

Given the PDFs for various uncertain reliability parameters, to discover the law of propagation of parameter uncertainty through the reliability evaluation model, a two-phase simulation approach is proposed here. Although this method will suffer from huge computational burden unavoidably and will not be a practicable option in practical engineering application, it can be served as a benchmark case to verify the validity and accuracy of Taylor series approximation method. The algorithm procedure is depicted as follows, which are somewhat different from the above described solution steps for Taylor series approximation approach.

Step (1) Characterize the uncertainty of reliability parameters by an appropriate probability distribution, which can be lognormal distribution, normal distribution, triangular distribution or empirical distribution from engineering experts.

Step (2) Draw samples from the PDFs for reliability parameters of system components by any sampling approach, like crude or Latin-hypercude sampling approach, and the random sample $\mathbf{U}_i = [U_1, U_2, \ldots, U_N]$ of reliability parameters can be obtained. This action takes place in the first stage of the two-phase algorithm, which provides input data for the later reliability evaluation.

Step (3) Treat the epistemic parameters as constants inside the second stage, i.e., the sampled values from step 2 are passed on to the second stage. In this stage the bulk power systems reliability evaluation will be rerun to quantify the effect of aleatory uncertainties using the sampled reliability parameters as their distributional parameters, then the reliability index X_i can be obtained accurately.

Step (4) Repeat Step (2) and Step (3) for n times until the coefficient of variation (CV) from sample data of reliability index $X_1, X_2, \ldots, X_n$ is smaller than the predefined threshold value or until n is larger than a reasonably large value, thus the random sample of size n for reliability index are obtained.

Step (5) Use Eq. (57) to obtain the optimal bandwidth h_{ROT} for kernel density estimation of reliability index.

Step (6) Conduct the kernel density estimation using equation (44), and then the PDF for reliability index $E(Y|\mathbf{U})$ can be achieved.

Step (7) Get the expected value $E(E(Y|\mathbf{U}))$,variance $V(E(Y|\mathbf{U}))$ and standard variance $Std(E(Y|\mathbf{U}))$ using the PDF obtained in Step 6.

However, this two-phase simulation approach needs to run reliability evaluation for a number of times, so it's complex and expensive to run even for a small power system, and the computational effort makes it impractical, but it can provide reference results (benchmark) for comparison with Taylor series approximation proposed in this paper.

Study results

In order to verify the performance of the proposed method, reliability evaluation is conducted using the RBTS test system [21] under the peak load condition. State enumeration method is used, and line outages, generating unit outages, and combined generating unit and line outages have been considered up to 4th, 6th and 5th levels respectively. Firstly the variation tendency of reliability indices is observed below when failure rate λ and mean time to repair MTTR (inverse of repair rate μ) of all the system components simultaneously change into k times of their original values, where k is between 0.75 and 1.25. The annualized reliability indices after parameters changed are obtained by three ways: first-order Taylor series approximation(Linear Approximation), second-order Taylor series approximation (Quadratic Approximation) and rerunning reliability evaluation with the modified parameters(Actual Value). Results are shown Fig. 1–3.

Using the actual value as a basis of comparison, it can be seen from the results that the second-order Taylor series approximation is more accurate than the first-order Taylor series approximation, especially for LOLF index. As Fig. 3 shows, the result of second-order Taylor series approximation is almost identical to the actual value while the result of first-order Taylor series approximation has relatively large errors. The reason lies in that the second-order Taylor series approximation is practically a quadratic approximation of the nonlinear function $E(Y|\mathbf{U})$, while the first-order Taylor series approximation is a linear approximation which results in a line tangential at the point $\boldsymbol{\mu}_{\mathbf{U}}$ to render the actual relationship between reliability indices $E(Y|\mathbf{U})$ and reliability parameters $\mathbf{U}$. In other words, the linear approximation is local. The error which comes from linearization of the nonlinear function $E(Y|\mathbf{U})$ becomes particularly noticeable when reliability parameters have large variations. For example, let λ and MTTR of all components be 0.75 times of their original values, the LOLP and EENS indices calculated by first-order Taylor series approximation become negative unexpectedly. It can be seen from this unacceptable error that first-order Taylor series approximation is only suitable for the circumstance that reliability parameters have minor variations. In this special case example, results from second-order Taylor series approximation may also encounter somewhat errors because of the extreme assumption of synchronous changes for all components reliability parameters. In reality, the reliability parameters are mutually independent random variables and the perfect correlation for them does not exist, considering this point the second-order Taylor series approximation can provide closer outcome to rerunning reliability evaluation than what Fig. 1–3 shows.

It's somewhat complicated to deduce the second-order partial differentials of reliability indices with respect to reliability parameters, in addition the linear items and the quadratic items are correlated to each other in the second-order Taylor series, so the first-order Taylor series is almost the primary option to address the parameter uncertainty analysis. Because of the larger error associated with the linearization and the second-order partial differentials of reliability indices have already been successfully derived in Section III, it's more suitable and practicable to use the second-order Taylor series to identify the impact of uncertain reliability parameter $\mathbf{U}$ on reliability indices $E(Y|\mathbf{U})$.

To perform a quantitative uncertainty analysis, probability distributions must be assigned to each of the uncertain parameters. The distributions may result directly from data obtained from a proper experimental design, but usually subjective judgment must be used to reflect the degree of belief that the unknown value for a parameter lies within a specified range. Where data are limited but uncertainty is relatively low, a range may be used to specify a uniform distribution. If there is knowledge about a most likely value or midpoint, in addition to a range, a triangular distribution may be assigned. When the range of uncertainty is very large, a log-uniform or log-triangular distribution may be more appropriate than the uniform or triangular distribution.

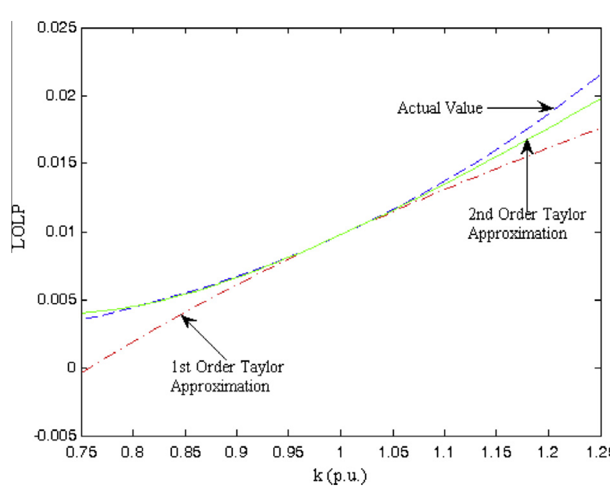

Fig. 1. LOLP index of RBTS after λ and MTTR of all components changed into k times of their original values.

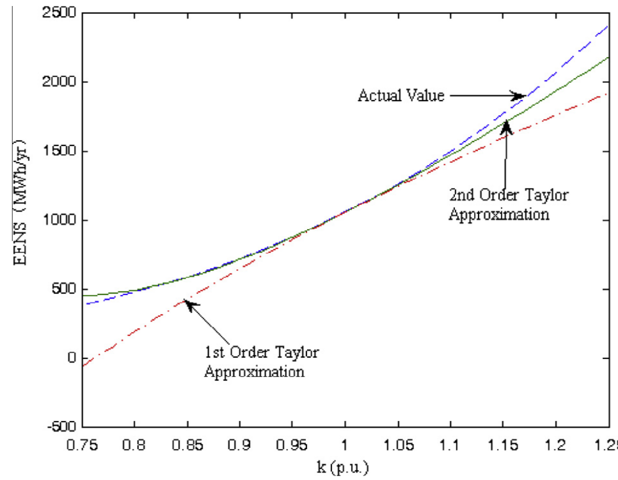

Fig. 2. EENS index of RBTS after λ and MTTR of all components changed into k times of their original values.

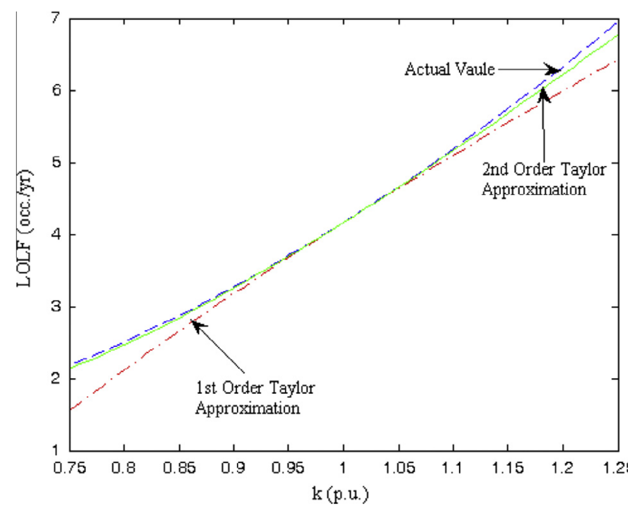

Fig. 3. LOLF index of RBTS after λ and MTTR of all components changed into k times of their original values.

To investigate the validity of the proposed method, parameter uncertainty analysis is implemented assuming uniform distribution (a, b), triangular distribution (a, c, b) and normal distribution (μ, σ) for components reliability parameters. The lower bound a and upper bound b for both of the uniform and triangular distributions are 0.5 and 1.5 times of the original reliability parameter values while the mode c for triangular distribution is 1.0 times of the original value. For the normal distribution its mean value μ and standard variance σ are 1.0 and 0.2 times of the original value. In fact the proposed approach in this paper can accommodate any desired PDFs, and the normal, uniform and triangular distribution used here are only for case study. In realistic engineering application, the type of PDF for reliability parameters can be determined by historical data and expert judgment. When there is doubt about the effect of different distributions, then different distributions should be assumed and the effect analyzed.

In order to demonstrate the accuracy and efficiency of the proposed method, the comparisons between the Monte Carlo simulation and the Taylor series approximation are carried out with sample size $n = 2000$. The expected value and standard variance

Table I
Expected value and standard deviation for LOLP indices of RBTS system under triangular distribution.

Method	Reliability index	$E(E(Y\|\mathbf{U}))$	$Std(E(Y\|\mathbf{U}))$
Linear approximation	LOLP	0.0089	0.0020
Quadratic approximation		0.0099	0.0017
Monte Carlo simulation (CV = 0.43%)		0.0098	0.0019
Linear approximation	EENS (MWh/yr)	960.7	242.6
Quadratic approximation		1070.2	201.6
Monte Carlo simulation (CV = 0.61%)		1051.2	228.7
Linear approximation	LOLF (occ./yr)	4.0381	0.5396
Quadratic approximation		4.1846	0.5125
Monte Carlo simulation (CV = 0.43%)		4.1537	0.5292

Table II
Expected value and standard deviation for reliability indices of RBTS system under uniform distribution.

Method	Reliability index	$E(E(Y\|\mathbf{U}))$	$Std(E(Y\|\mathbf{U}))$
Linear approximation	LOLP	0.0080	0.0029
Quadratic approximation		0.0101	0.0022
Monte Carlo simulation (CV = 0.61%)		0.0096	0.0026
Linear approximation	EENS (MWh/yr)	861.2	357.8
Quadratic approximation		1090.6	264.7
Monte Carlo simulation (CV = 0.69%)		1043.1	319.9
Linear approximation	LOLF (occ./yr)	3.8994	0.7713
Quadratic approximation		4.2033	0.7120
Monte Carlo simulation (CV = 0.41%)		4.1457	0.7546

Table III
Expected value and standard deviation for reliability indices of RBTS system under normal distribution.

Method	Reliability index	$E(E(Y\|\mathbf{U}))$	$Std(E(Y\|\mathbf{U}))$
Linear approximation	LOLP	0.0090	0.0021
Quadratic approximation		0.0098	0.0039
Monte Carlo simulation (CV = 0.43%)		0.0096	0.0018
Linear approximation	EENS (MWh/yr)	965.9	257.5
Quadratic approximation		1074.7	487.5
Monte Carlo simulation (CV = 0.48%)		1052.6	229.0
Linear approximation	LOLF (occ./yr)	4.0458	0.5494
Quadratic approximation		4.2070	0.7204
Monte Carlo simulation (CV = 0.28%)		4.1720	0.5175

of LOLP, EENS, and LOLF index for RBTS system under different distributions are listed in Table I, II and III. It can be seen from these tables the coefficient of variation (CV) are adequately smaller than 1% so that we can assume Monte Carlo simulation (MCS) with 2000 trials could catch the stochastic feature of the problem studied and provide "true" results.

Using the results obtained from MCS approach with 2000 trials as the basis, it is shown in Table I, II and III that second-order Taylor series approximation (Quadratic Approximation) has apparently smaller errors than first-order Taylor series approximation (Linear Approximation) in estimating expected value $E(E(Y|\mathbf{U}))$ and standard variance $std(E(Y|\mathbf{U}))$ of reliability indices when considering parameter uncertainty. Besides $E(E(Y|\mathbf{U}))$ and $std(E(Y|\mathbf{U}))$, the probability density distribution for $E(Y|\mathbf{U})$ can also be obtained which are shown in Fig. 4–6.

From Fig. 4–6, it can be seen whatever distribution the reliability parameters follow, the PDF curves obtained from Linear Approximation method always lean to the left side when comparing with Quadratic Approximation and MCS methods. This is especially obvious for uniform and normal distributions because the distribution range of EENS index has extended wrongly to the negative domain. However when comparing results obtained from Quadratic Approximation against those obtained from MCS, it shows that PDF curves resulted from these two methods almost have the identical distribution range and feature point such as the central point. As Fig. 1–3 show, when there are larger varia-

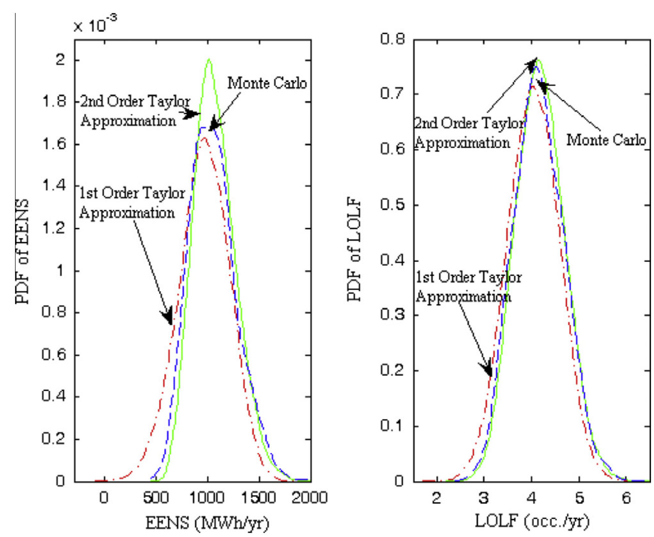

Fig. 4. Probability distributions of EENS and LOLF index in the case of triangular distribution of λ and MTTR for RBTS.

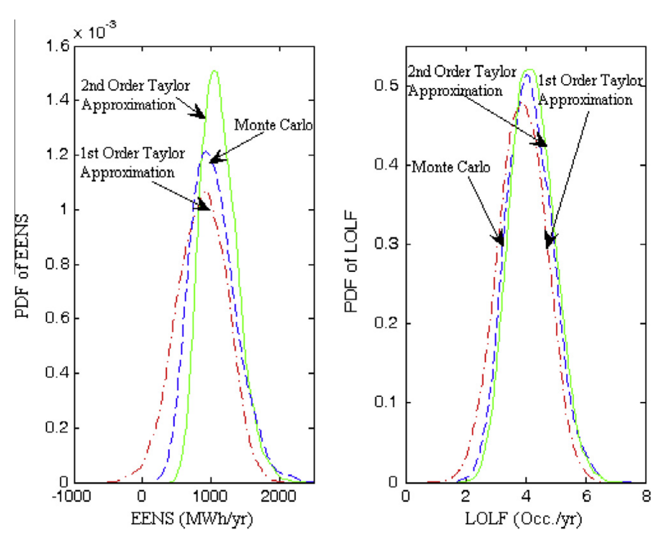

Fig. 5. Probability distributions of EENS and LOLF index in the case of uniform distribution of λ and MTTR for RBTS.

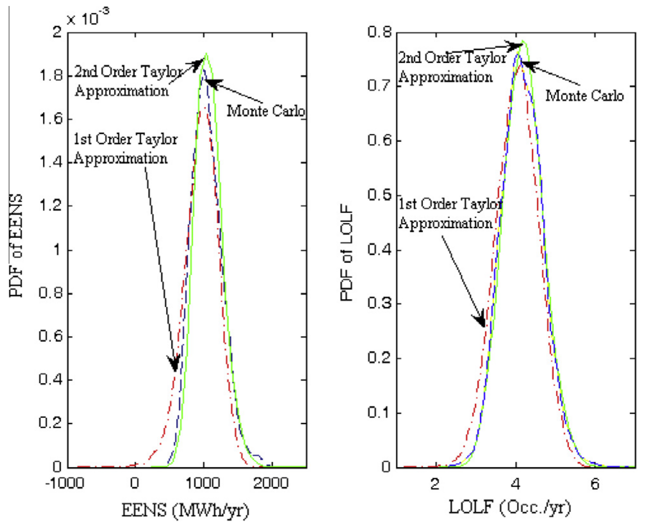

Fig. 6. Probability distributions of EENS and LOLF index in the case of normal distribution of λ and MTTR for RBTS.

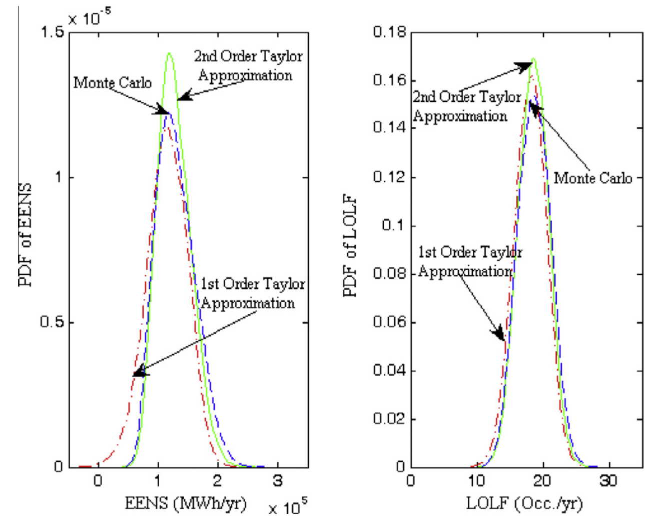

Fig. 8. Probability distributions of EENS and LOLF index in the case of triangular distribution of λ and MTTR for IEEE-RTS79.

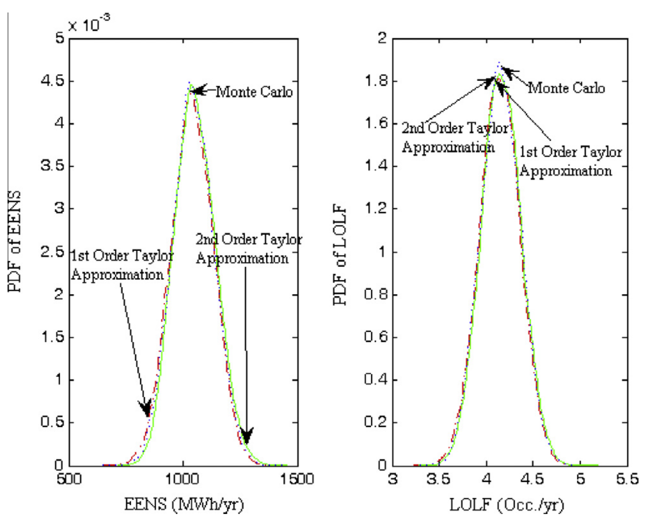

Fig. 7. Probability distributions of EENS and LOLF index in the case of modified triangular distribution of λ and MTTR for RBTS.

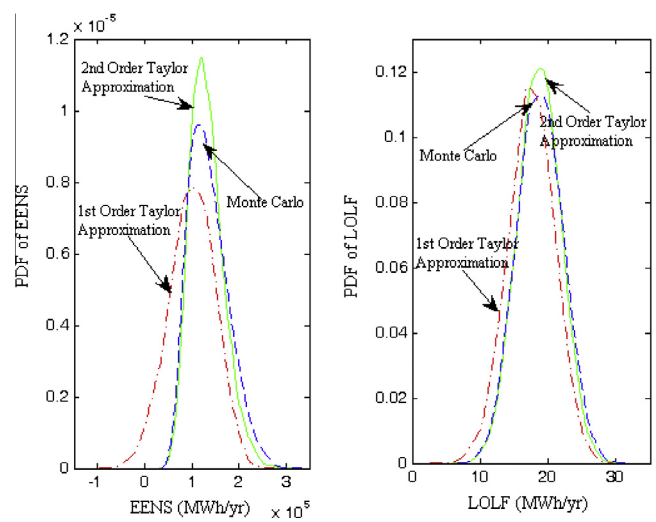

Fig. 9. Probability distributions of EENS and LOLF index in the case of uniform distribution of λ and MTTR for IEEE-RTS79.

tions in reliability parameters, reliability indices gained from Linear Approximation are smaller than actual values, especially when a larger decrease of reliability parameters is encountered the Linear Approximation results become negative unexpectedly. Because the distribution ranges for reliability parameters are actually somewhat wide in this case study, these reliability parameters are susceptible to large stochastic variation, and this is the true reason why PDF curves obtained from Linear Approximation lean to the left side or even lay on the negative side of the axis.

It can be extrapolated from the above that if these reliability parameters have only small random perturbations around their mean values, the PDF curves produced by Linear Approximation, Quadratic Approximation and Monte Carlo simulation will almost coincide with each other. Given the lower bound a and upper bound b for triangular distribution (a, c, b) are 0.8 and 1.2 times of its nominal values, the PDF curves are reported in Fig. 7, and it validates this conclusion well.

To further clarify the difference between Linear and Quadratic Approximation, reliability evaluation is also performed using the IEEE-RTS79 test system [22]. Assuming the lower bound a and upper bound b for triangular and uniform distributions are 0.5 and 1.5 times of the original reliability parameter values, the annualized reliability indices are visualized in Fig. 8 and 9. It can be seen again that the PDF curves produced by Linear Approximation tilt to the right side because Linear Approximation always underestimates reliability indices when reliability parameters vary.

Conclusion

To obtain adequate information about the adequacy and reliability of a bulk power system, reliability parameters uncertainties have to be considered in the reliability evaluation. To take uncertainty of components work duration and outage duration into account in the reliability evaluation procedure, this paper proposes the Taylor series approximation approach combined with nonparametric probability density estimation technique. Test results have indicated that if the uncertain parameters considered can be measured or estimated, the distributions of reliability indices can be accurately and efficiently evaluated with the proposed quadratic approximation method.

The proposed method is tested and verified by comparison with results from the Monte Carlo simulation on RBTS. Using the results obtained from Monte Carlo simulation as a reference basis, the proposed quadratic approximation method can achieve similar results with less effort in the numerical computations and has a better performance than the linear approximation method. The information about uncertainty of reliability indices obtained from the proposed method will provide system planning and operation engineers more confidence in system reliability and lead to less need for conservative operation of the power grid.

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Analytical formula of LOLP and EENS with respect to reliability parameters of component *i*

$$\begin{aligned}
\mathrm{LOLP} &= \sum_{\boldsymbol{x}\in\boldsymbol{X}} I_f(\boldsymbol{x})P(\boldsymbol{x}) = \sum_{\boldsymbol{x}\in\boldsymbol{X}} I_f(\boldsymbol{x})\prod_{k=1}^{m}P(S_k)\\
&= \sum_{\boldsymbol{x}\in\boldsymbol{X}} I_f(\boldsymbol{x})P(S_i)\prod_{k=1,k\notin i}^{m}P(S_k)\\
&= \sum_{\boldsymbol{x}\in\boldsymbol{X}} I_f(\boldsymbol{x})\left(\frac{S_i\mu_i+(1-S_i)\lambda_i}{\lambda_i+\mu_i}\right)\prod_{k=1,k\notin i}^{m}P(S_k)\\
&= \sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=1}\frac{\mu_i}{\lambda_i+\mu_i}I_f(\boldsymbol{x})\prod_{k=1,k\neq i}^{m}P(S_k)\\
&\quad+\sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=0}\frac{\lambda_i}{\lambda_i+\mu_i}I_f(\boldsymbol{x})\prod_{k=1,k\neq i}^{m}P(S_k).
\end{aligned}\tag{A1.1}$$

In the above formula (A1.1), when define $K_i^{(1)}$ and $K_i^{(2)}$ as

$$K_i^{(1)} = \sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=1} I_f(\boldsymbol{x})\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A1.2}$$

$$K_i^{(2)} = \sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=0} I_f(\boldsymbol{x})\prod_{k=1,k\neq i}^{m}P(S_k).\tag{A1.3}$$

LOLP index can be expressed as

$$\begin{aligned}
&\mathrm{LOLP} = \frac{\mu_i}{\lambda_i+\mu_i}K_i^{(1)}+\frac{\lambda_i}{\lambda_i+\mu_i}K_i^{(2)}\\
&= \frac{\mu_i}{\lambda_i+\mu_i}(K_i^{(1)}-K_i^{(2)})+K_i^{(2)}.
\end{aligned}\tag{A1.4}$$

By following the above method, the analytical formula of EENS can be easily derived in the same manner as (A1.5), (A1.6) and (A1.7).

$$\mathrm{EENS} = \frac{\mu_i}{\lambda_i+\mu_i}(E_i^{(1)}-E_i^{(2)})+E_i^{(2)},\tag{A1.5}$$

$$E_i^{(1)} = 8760\sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=1} I_f(\boldsymbol{x})L_C(\boldsymbol{x})\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A1.6}$$

$$E_i^{(2)} = 8760\sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=0} I_f(\boldsymbol{x})L_C(\boldsymbol{x})\prod_{k=1,k\neq i}^{m}P(S_k).\tag{A1.7}$$

Analytical formula of LOLF with respect to reliability parameters of component *i*

$$\begin{aligned}
\mathrm{LOLF} &= \sum_{\boldsymbol{x}\in\boldsymbol{X}}(I_f(\boldsymbol{x})\sum_{k=1}^{m}\lambda_{\boldsymbol{x}}^{in}(k))P(\boldsymbol{x})\\
&= \sum_{\boldsymbol{x}\in\boldsymbol{X}}(I_f(\boldsymbol{x})\sum_{k=1}^{m}\lambda_{\boldsymbol{x}}^{in}(k))\prod_{k=1}^{m}P(S_k)\\
&= \sum_{\boldsymbol{x}\in\boldsymbol{X}}(I_f(\boldsymbol{x})\sum_{k=1}^{m}\lambda_{\boldsymbol{x}}^{in}(k))\left(\tfrac{S_i\mu_i+(1-S_i)\lambda_i}{\lambda_i+\mu_i}\right)\prod_{k=1,k\notin i}^{m}P(S_k)\\
&= \sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=1}\tfrac{\mu_i}{\lambda_i+\mu_i}\left(I_f(\boldsymbol{x})\sum_{k=1}^{m}\lambda_{\boldsymbol{x}}^{in}(k)\right)\prod_{k=1,k\notin i}^{m}P(S_k)\\
&\quad+\sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=0}\tfrac{\lambda_i}{\lambda_i+\mu_i}\left(I_f(\boldsymbol{x})\sum_{k=1}^{m}\lambda_{\boldsymbol{x}}^{in}(k)\right)\prod_{k=1,k\notin i}^{m}P(S_k)\\
&= \sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=1}\tfrac{\mu_i I_f(\mathbf{x})}{\lambda_i+\mu_i}\left(\sum_{k=1,k\neq i}^{m}\lambda_{\boldsymbol{x}}^{in}(k)-\lambda_i\right)\prod_{k=1,k\notin i}^{m}P(S_k)\\
&\quad+\sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=0}\tfrac{\lambda_i I_f(\mathbf{x})}{\lambda_i+\mu_i}\left(\sum_{k=1}^{m}\lambda_{\boldsymbol{x}}^{in}(k)+\mu_i\right)\prod_{k=1,k\notin i}^{m}P(S_k).
\end{aligned}\tag{A2.1}$$

In the above formula (A2.1), when define $K_i^{(3)}$ and $K_i^{(4)}$ as

$$K_i^{(3)} = \sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=1} I_f(\mathbf{x})\left(\sum_{j=1,j\neq i}^{m}\lambda_{\boldsymbol{x}}^{in}(j)\right)\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A2.2}$$

$$K_i^{(4)} = \sum_{\boldsymbol{x}\in\boldsymbol{X},S_i=0} I_f(\mathbf{x})\left(\sum_{j=1,j\neq i}^{m}\lambda_{\boldsymbol{x}}^{in}(j)\right)\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A2.3}$$

$$\begin{aligned}
\mathrm{LOLF} &= \frac{\lambda_i\mu_i(K_i^{(2)}-K_i^{(1)})}{\lambda_i+\mu_i}+\frac{\mu_iK_i^{(3)}}{\lambda_i+\mu_i}+\frac{\lambda_iK_i^{(4)}}{\lambda_i+\mu_i}\\
&= \frac{\lambda_i\mu_i(K_i^{(2)}-K_i^{(1)})}{\lambda_i+\mu_i}+\frac{\mu_i(K_i^{(3)}-K_i^{(4)})}{\lambda_i+\mu_i}+K_i^{(4)}.
\end{aligned}\tag{A2.4}$$

Analytical expressions of the partial differentials for $K_i^{(1)} \sim K_i^{(4)}$ and $E_i^{(1)} \sim E_i^{(2)}$

From the equation (A1.2) and (A1.3), (A1.6) and (A1.7) and (A2.2) and (A2.3), the partial differentials of $K_i^{(1)} \sim K_i^{(4)}$ and $E_i^{(1)} \sim E_i^{(2)}$ with respect to λ_j and μ_j (failure rate and repair rate of component *j*) can be further derived respectively

$$\frac{\partial K_i^{(1)}}{\partial\lambda_j} = \sum_{\mathbf{x}\in\mathbf{X},S_i=1}\frac{I_f(\mathbf{x})(1-2S_j)\mu_j}{(\lambda_j+\mu_j)^2(S_ja_j+(1-S_j)u_j)}\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A3.1}$$

$$\frac{\partial K_i^{(1)}}{\partial\mu_j} = \sum_{\mathbf{x}\in\mathbf{X},S_i=1}\frac{I_f(\mathbf{x})(2S_j-1)\lambda_j}{(\lambda_j+\mu_j)^2(S_ja_j+(1-S_j)u_j)}\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A3.2}$$

$$\frac{\partial K_i^{(2)}}{\partial\lambda_j} = \sum_{\mathbf{x}\in\mathbf{X},S_i=0}\frac{I_f(\mathbf{x})(1-2S_j)\mu_j}{(\lambda_j+\mu_j)^2(S_ja_j+(1-S_j)u_j)}\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A3.3}$$

$$\frac{\partial K_i^{(2)}}{\partial\mu_j} = \sum_{\mathbf{x}\in\mathbf{X},S_i=0}\frac{I_f(\mathbf{x})(2S_j-1)\lambda_j}{(\lambda_j+\mu_j)^2(S_ja_j+(1-S_j)u_j)}\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A3.4}$$

$$\frac{\partial E_i^{(1)}}{\partial\lambda_j} = \sum_{\mathbf{x}\in\mathbf{X},S_i=1}\frac{8760I_f(\mathbf{x})Lc(\mathbf{x})(1-2S_j)\mu_j}{(\lambda_j+\mu_j)^2(S_ja_j+(1-S_j)u_j)}\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A3.5}$$

$$\frac{\partial E_i^{(1)}}{\partial\mu_j} = \sum_{\mathbf{x}\in\mathbf{X},S_i=1}\frac{8760I_f(\mathbf{x})Lc(\mathbf{x})(2S_j-1)\lambda_j}{(\lambda_j+\mu_j)^2(S_ja_j+(1-S_j)u_j)}\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A3.6}$$

$$\frac{\partial E_i^{(2)}}{\partial\lambda_j} = \sum_{\mathbf{x}\in\mathbf{X},S_i=0}\frac{8760I_f(\mathbf{x})Lc(\mathbf{x})(1-2S_j)\mu_j}{(\lambda_j+\mu_j)^2(S_ja_j+(1-S_j)u_j)}\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A3.7}$$

$$\frac{\partial E_i^{(2)}}{\partial\mu_j} = \sum_{\mathbf{x}\in\mathbf{X},S_i=0}\frac{8760I_f(\mathbf{x})Lc(\mathbf{x})(2S_j-1)\lambda_j}{(\lambda_j+\mu_j)^2(S_ja_j+(1-S_j)u_j)}\prod_{k=1,k\neq i}^{m}P(S_k),\tag{A3.8}$$

$$\frac{\partial K_i^{(3)}}{\partial \lambda_j} = -\sum_{\mathbf{x}\in\mathbf{X},S_i=1} \frac{I_f(\boldsymbol{x})S_j a_j}{(S_j a_j + (1-S_j)u_j)} \prod_{k=1,k\neq i}^{m} P(S_k) + \sum_{\mathbf{x}\in\mathbf{X},S_i=1} \frac{I_f(\boldsymbol{x})(1-2S_j)\mu_j \sum_{l=1,l\neq i}^{m} \lambda_{\mathbf{x}}^{in}(l)}{(\lambda_j+\mu_j)^2 (S_j a_j + (1-S_j)u_j)} \prod_{k=1,k\neq i}^{m} P(S_k), \tag{A3.9}$$

$$\frac{\partial K_i^{(3)}}{\partial \mu_j} = \sum_{\mathbf{x}\in\mathbf{X},S_i=1} \frac{I_f(\boldsymbol{x})(1-S_j)u_j}{(S_j a_j + (1-S_j)u_j)} \prod_{k=1,k\neq i}^{m} P(S_k) + \sum_{\mathbf{x}\in\mathbf{X},S_i=1} \frac{I_f(\boldsymbol{x})(2S_j-1)\lambda_j \sum_{l=1,l\neq i}^{m} \lambda_{\mathbf{x}}^{in}(l)}{(\lambda_j+\mu_j)^2 (S_j a_j + (1-S_j)u_j)} \prod_{k=1,k\neq i}^{m} P(S_k), \tag{A3.10}$$

$$\frac{\partial K_i^{(4)}}{\partial \lambda_j} = -\sum_{\mathbf{x}\in\mathbf{X},S_i=0} \frac{I_f(\boldsymbol{x})S_j a_j}{(S_j a_j + (1-S_j)u_j)} \prod_{k=1,k\neq i}^{m} P(S_k) + \sum_{\mathbf{x}\in\mathbf{X},S_i=0} \frac{I_f(\mathbf{x})(1-2S_j)\mu_j \sum_{l=1,l\neq i}^{m} \lambda_{\mathbf{x}}^{in}(l)}{(\lambda_j+\mu_j)^2 (S_j a_j + (1-S_j)u_j)} \prod_{k=1,k\neq i}^{m} P(S_k), \tag{A3.11}$$

$$\frac{\partial K_i^{(4)}}{\partial \mu_j} = \sum_{\mathbf{x}\in\mathbf{X},S_i=0} \frac{I_f(\boldsymbol{x})(1-S_j)u_j}{(S_j a_j + (1-S_j)u_j)} \prod_{k=1,k\neq i}^{m} P(S_k) + \sum_{\mathbf{x}\in\mathbf{X},S_i=0} \frac{I_f(\boldsymbol{x})(2S_j-1)\lambda_j \sum_{l=1,l\neq i}^{m} \lambda_{\mathbf{x}}^{in}(l)}{(\lambda_j+\mu_j)^2 (S_j a_j + (1-S_j)u_j)} \prod_{k=1,k\neq i}^{m} P(S_k). \tag{A3.12}$$

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