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Modal flexibility-based damage detection of cantilever beam-type structures using baseline modification



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ABSTRACT

This paper presents a new damage detection approach for cantilever beam-type structures using the damage-induced inter-storey deflection (DIID) estimated by modal flexibility matrix. This approach can be utilized for damage detection of cantilever beam-type structures such as super high-rise buildings, high-rise apartment buildings, etc. Analytical studies on the DIID of cantilever beam-type structures have shown that the DIID abruptly occurs from damage location. Baseline modification concept was newly introduced to detect multiple damages in cantilever beam-type structures by changing the baseline to the prior damage location. This approach has a clear theoretical base and directly identifies damage location(s) without the use of a finite element (FE) model. For validating the applicability of the proposed approach to cantilever beam-type structures, a series of numerical and experimental studies on a 10-storey building model were carried out. From the tests, it was found that the damage locations can be successfully identified by the proposed approach for multiple damages as well as a single damage. In order to confirm the superiority of the proposed approach, a comparative study was carried out on two well-known damage metrics such as modal strain-based damage index approach and uniform load surface curvature approach.

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1. Introduction

Civil infrastructure such as bridges and buildings is exposed to severe environmental and service loadings over time due to fatigue, corrosion, natural hazards, etc. It is necessary to acquire accurate and real-time information on the damage state of these structures to prevent catastrophic failure, increase cost-effectiveness of maintenance, and prolong service life. For this purpose, structural health monitoring (SHM) for civil engineering has received considerable attention. Especially, research on global approach, called vibration-based approach has been rapidly expanding using various vibration characteristics, such as natural frequencies, mode shapes, mode shape curvatures, modal damping ratio, and modal flexibility, obtained from measured vibration data. The crucial advantage of the vibration-based approach is that structural condition of a massive civil structure can be assessed using a relatively small number of sensors. Extensive literature reviews on the vibration-based damage detection method can be found in [1–3].

Modal flexibility has shown itself to be a promising damage descriptor due to its high sensitivity to damage [4]. Pandey and Biswas [5] proposed a damage detection approach that uses changes in modal flexibility for the first time. In practice,

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the structural modes are rarely all identified from the measured vibration data. But, the flexibility matrix can be accurately estimated from only a few lower modes because it is inversely proportional to the squares of the natural frequencies. This approach was experimentally demonstrated on a three-span reinforced-concrete highway bridge [6]. Zhang and Aktan [7] and Park et al. [8] studied the uniform load surface (ULS) and its curvature. The ULS was found to have much less truncation effect and was least sensitive to experimental error. Displacement coefficients and profiles are presented as promising kernel condition and damage indices along with real-life examples by Catbas et al. [9]. Wu and Law [10] applied the ULS curvature to plate-type structures for damage detection and quantification. It was found that the ULS curvature is sensitive to damages, even with truncated, incomplete, and noisy measurements. In combination with the ULS curvature, two new damage detection algorithms (i.e., the generalized fractal dimension and simplified gapped-smoothing methods) are proposed by Wang and Qiao [11]. Catbas et al. [12] compared the performance of both the ULS and its curvature, and confirmed that the curvature is more advantageous. Another well-known flexibility-based approach is the damage locating vector (DLV) approach, which is based on changes in modal flexibility and uses an intact finite element model. The DLV have the property of inducing stress fields whose magnitude is zero in the damage elements [13]. Many researchers [14–16] carried out a series of studies on using the DLV approach to identify various damages in a truss structure. Subsequently, Bernal [17] developed the stochastic dynamic DLV (SDDLV) method to achieve damage localization using output-only information and to provide richer information on structural damage employing dynamic flexibility matrices. Koo et al. [18] developed the damage-induced deflection approach based on modal flexibility to localize multiple damages without the finite element model. Yu and Chen [19] are studied by combining the structural modal flexibility and the particle swarm optimization (PSO) technology. Kazemi et al. [20] developed a new two-phase procedure in order to localize the faults and corresponding severity in thin plate structures. Two effective damage detection methods for localizing and quantifying structural damage in shear frames are presented by Amiri et al. [21].

However, conventional flexibility-based approaches have several drawbacks such as no obvious relationship between damage and damage features, noise vulnerable characteristic, and the requirement of an intact finite element model. Those drawbacks have to be overcome to increase the robustness, reliability, and applicability in damage detection for civil infrastructure under various environmental effects, and inevitable measurement noise.

This paper proposes a new damage detection approach for cantilever beam-type structures that uses the damageinduced inter-storey deflection (DIID) obtained from modal flexibility matrices. This approach can be utilized for damage detection of cantilever beam-type structures such as super high-rise buildings, high-rise apartment buildings, etc. A bending moment is dominant in a total behavior of the structures [22]. The proposed approach has a clear theoretical base and directly identifies multiple damage locations as well as a single damage location without the use of a finite element model. Baseline modification concept was newly proposed to detect multiple damages in cantilever beam-type structures by changing the baseline to the prior damage location. In order to verify the feasibility of the proposed method, the theoretical background is introduced first. Next, numerical and experimental investigations are presented for a 10-storey building model with two damage scenarios. Finally, the modal strain-based damage index approach [23,24] and ULS curvature approach [7,10,11] are compared to show the effectiveness of the proposed approach.

2. Theory

2.1. Estimation of inter-storey deflection using modal flexibility

The modal flexibility matrix \mathbf{G}_m using *m* lower mode can be expressed as

$$\mathbf{G}_m = \mathbf{\Phi}_m \boldsymbol{\Lambda}_m^{-1} \mathbf{\Phi}_m^T = \sum_{i=1}^m \frac{\mathbf{\Phi}_i \mathbf{\Phi}_i^T}{\omega_i^2} \tag{1}$$

where $\Lambda_m = [\omega_i^2 \setminus]$ for which ω_i is the *i*-th structural natural frequency, i = 1, 2, ..., m; $\Phi_m = \{\phi_1, \phi_2, ..., \phi_m\}$; and ϕ_i is the *i*-th mass normalized mode shape. The mass-normalization on un-scaled mode shapes can be carried out by (1) system mass matrix, or (2) the known mass perturbation approaches [25,26].

The deflections under an arbitrary load **f** using modal flexibility matrix can be estimated as

$$\mathbf{u} = \mathbf{G}_m \mathbf{f} \tag{2}$$

where **u** is the deflection vector corresponding to the force vector **f**. The positive bending inspection load (PBIL), which is the vector composed of unit value at all the sensor locations, is used as the force vector **f** to obtain modal flexibility-based deflections, which produces only positive bending moments at all the floors [18]. This load vector is beneficial to average all the sensor noises through the equivalent summations of all the sensor contributions.

Finally, by using modal flexibility-based deflections obtained from Eq. (2), the inter-storey deflection can be estimated as

$$\mathbf{u}_{i}^{\mathrm{S}} = \mathbf{u}_{i+1} - \mathbf{u}_{i} \tag{3}$$

where \mathbf{u}_i^{is} is the inter-storey deflection and \mathbf{u}_i is the modal flexibility-based deflection at the *i*-th storey.

ι

2.2. General equation of damage-induced inter-storey deflection for damage detection of cantilever beam-type structures

By Hooke's law, the relationship between the deflection \mathbf{u}_0 and the applied force F can be expressed as

$$\mathbf{K}_{0}\mathbf{u}_{0} = \mathbf{F} \tag{4}$$

where \mathbf{K}_0 is the stiffness matrix of the intact structure.

A similar relationship under the same external force **F** with a reduction in the stiffness matrix $\Delta \mathbf{K}$ due to damage can be expressed as

$$(\mathbf{K}_0 - \Delta \mathbf{K})(\mathbf{u}_0 + \Delta \mathbf{u}) = \mathbf{F}$$
⁽⁵⁾

where $\Delta \mathbf{u} = \mathbf{u}_D - \mathbf{u}_I$ is the damage-induced deflection, \mathbf{u}_I and \mathbf{u}_D are deflections for intact/damage, respectively. By subtracting Eq. (4) from Eq. (5), the general equation of the DID can be obtained as

$$\Delta \mathbf{u} = \mathbf{G}_D(\Delta \mathbf{K} \mathbf{u}_0) = \mathbf{G}_D \Delta \mathbf{F} \tag{6}$$

where $\mathbf{G}_D = (\mathbf{K}_0 - \Delta \mathbf{K})^{-1}$ is the structural flexibility matrix in the damage states, and $\Delta \mathbf{F} = \Delta \mathbf{K} \mathbf{u}_0$ is the force induced by the stiffness loss of the intact system which produces the DID to the damage system [27–29]. $\Delta \mathbf{F}$ may be considered as 'the lost resisting force by the damage', since $\Delta \mathbf{F}$ is the force carried by the damaged portion of the structure in the intact states. Then, the damage-induced inter-storey deflection (DIID) can be evaluated from Eqs. (3)–(6) as

$$\Delta \mathbf{u}^{\rm IS} = \mathbf{u}^{\rm IS}_{\rm D} - \mathbf{u}^{\rm IS}_{\rm I} \tag{7}$$

where $\Delta \mathbf{u}^{IS}$ is the DIID, \mathbf{u}_{I}^{IS} and \mathbf{u}_{D}^{IS} are inter-storey deflection for intact/damage cases, respectively.

For the cantilever beam-type structure with a column damage ΔK , the forces ΔF due to damage proportional to the stiffness reduction can be obtained as

$$\Delta \mathbf{F} = \Delta \mathbf{K} \mathbf{u} = \left\{ \begin{array}{c} \mathbf{0} \\ \alpha_e \mathbf{f}_e \\ \mathbf{0} \end{array} \right\}$$
(8)



K_R

Fig. 1. Structure with torsional spring representation of damage.



Fig. 2. (a) Deflection due to damage and applied force of a damaged cantilever beam, (b) damage-induced deflection and (c) damage-induced inter-storey deflection.

where $\Delta \mathbf{K} = \text{diag}(0, \alpha_e \mathbf{k}_e, 0)$, α_e is the damage ratio, $0 < \alpha_e < 1$, \mathbf{k}_e is the elementary stiffness matrix representing the intact columns at the damage system, and $\mathbf{f}_e = \mathbf{k}_e \mathbf{u}_e = \{V, M, -V, M\}$ is the stress resultant of the element in the intact state.

2.3. Damage detection of cantilever beam-type structures by DIID

Consider a cantilever beam in pure bending as an ideal case of cantilever beam-type structures. One convenient modeling technique for a beam with a crack is the concept of the weightless torsional spring [30], as shown in Fig. 1. The



Fig. 3. Multiple damage detection through baseline modification (a) initial baseline and (b) modified baseline.



Fig. 5. Outlier analysis based on standard normal distribution.

damaged cantilever beam applied by the external force **F** can be modeled as Fig. 2(a) according to Eqs. (5) and (6). The DID of the damaged cantilever beam can be estimated based on the mechanics of materials theory, as shown in Fig. 2(b). Thus, the DID occurs from damaged location (*l*) with the same value $\Delta \mathbf{u}^{IS}(l)$, as depicted in Fig. 2(c). In this study, baseline modification concept was newly introduced to detect multiple damages in cantilever beam-type structures, as shown in Fig. 3. If the damage was localized at the *i*-th location, a baseline is newly defined as the value of DIID at the *i*-th location (i. e., the value of DIID at the prior damage location). After that, damage detection can be carried out through the newly defined baseline. Thus, the *l*-th location is the damage location estimated from initial baseline as shown in Fig. 3(a). In Fig. 3(b), the *l*-th location is the newly defined baseline and the (*l*+*m*)-th location is newly estimated damage location from modified baseline, respectively.

Table 1Structural model parameters.

Column (beam element)		Floor (shell element)		
Parameters	Value	Parameters	Value	
Mass density (ρ) Poisson's ratio (v) Elasticity modulus (E) Length (L)	7850 kg m ⁻³ 0.28 200 GPa 0.2 m	Mass density (ρ) Poisson's ratio (v) Elasticity modulus (E) Area (m^2) Thickness (mm)	7850 kg m ⁻³ 0.28 200 GPa 0.2 by 0.2	



Fig. 6. Numerical model (a) damage locations and assumed sensors and (b) ANSYS model.

As we mentioned, we considered a cantilever beam in pure bending as an ideal case of cantilever beam-type structures to distinctly explain the theory of the proposed approach. However, small errors of DIID may occur for general cantilever-type structures, since the structural behavior may be slightly affected by small shearing forces although the structure can be categorized into a bending moment dominant structure. Another reason that small errors of DIID can occur is measurement noises, since the noises may contaminate measurement data.

From the relationship between damage and DIID in a cantilever beam-type structure, it was found that the $\Delta \mathbf{u}^{IS}(i)$ induced by damage accumulates to the upper floors with the same value, as shown in Fig. 3. In order to simplify the damage detection procedure and simply estimate damage locations, a modified DIID is proposed by setting the values below the baseline, except for the initial value exceeding the baseline, to zero as shown in Fig. 4.

Finally, the damage detection of a cantilever beam-type structure can be performed by the following equation:

Damage occurs at the *i*-th floor if a modified DIID $\Delta \mathbf{u}_{m}^{IS}(i) > baseline at the$ *i*-th floor (9)

2.4. Damage detection by DIID under noisy measurements

From the relationship between damage and DIID in the previous section, damage detection can be performed using the DIID. However, unexpected DIID at the intact location may occur due to inevitable measurement noise. Therefore, statistical approaches are preferred for reducing false damage detection. Many of the outlier analysis techniques described in [31,32] are single dimensional or at best univariate. One such single dimensional approach is Grubbs' approach (extreme studentized deviate) which calculates a Zi value as the difference between the mean value for the attribute and the query value divided by the standard deviation for the attribute where the mean and standard deviation are calculated from all attribute values. In this study, an outlier index Zi based on Grubbs' approach is utilized to carry out damage localization of a cantilever beam-type structure as

$$Zi = \frac{\mathbf{u}^{IS}(i) - \overline{\mathbf{u}}_{I}^{IS}(i)}{\sigma(\mathbf{u}_{I}^{IS}(i))}$$
(10)



Fig. 7. Damage scenarios (a) intact column, (b) damage column, (c) damage column on the first floor (top view) and (d) damage column on the third floor (top view).

where $\mathbf{u}^{\text{IS}}(i)$ is the current inter-storey deflection, $\overline{\mathbf{u}}_{l}^{\text{IS}}(i)$ is the mean value of the inter-storey deflection for the intact structure and $\sigma(\mathbf{u}_{l}^{\text{IS}}(i))$ is the standard deviation of inter-storey deflection for the intact structures, respectively. In order to simplify the damage detection procedure and recognize damaged locations at a glance, Zi is also modified to Zi_m in accordance with modified DIID concept, introduced in Section 2.3.

Damage is located at the
$$i - \text{th floor if } Zi_m > Z^{\text{Threshold}}$$
 (11)

The calculation of $\mu \pm 2\sigma$ to estimate threshold values dividing background data from anomalies has been used after its introduction [33]. In this study, 2σ is also considered as threshold level. (Fig. 5)

3. Numerical simulation

In order to verify the feasibility of the proposed damage detection approach for cantilever beam-type structures, numerical simulations were carried out on a 10-storey building model with the material properties shown in Table 1. Columns and floors of the structure were modeled by beam and shell elements, respectively. The numerical model was totally composed of 4800 elements and 5974 nodes. Limited sensors were assumed to be installed at each floor and modal information of intact/damage structures was obtained by ANSYS. To consider sensor weight which was used in experiments (i.e., each 0.4 kg), point mass was introduced on each floor. Damages were simulated by reducing the bending stiffness (EI) of the affected column(s) with same severity for all of the damage cases, as shown in Figs. 6 and 7.

The first three natural frequencies and corresponding mass-normalized mode shapes were used to construct modal flexibility for intact/damage case, because the number of measurable modes in real structure is limited due to the rigidity of the structures and the excitation level. Changes in modal parameters due to progressive damages are depicted in Table 2 and Fig. 8. The natural frequencies were found to be reduced by about 0.53–1.73 percent, while modal assurance criterion (MAC) values did not change much.

Modal flexibility-based deflection and its inter-storey deflection under the PBIL are shown in Figs. 9 and 10. Figs. 11 and 12 show the DIID for each damage case. For DC1, it was clearly found that the DIID abruptly occurred at the first floor.

Table 2	2				
Modal	parameter	changes	due	to	damage.

Case	The first mode		The second mode			The third mode			
	$f_1(Hz)$	$\Delta f_1/f_1~(\%)$	MAC	$f_2(Hz)$	$\Delta f_2/f_2~(\%)$	MAC	<i>f</i> ₃ (Hz)	$\Delta f_3/f_3~(\%)$	MAC
Intact case(IC) Damage case 1 (DC1) Damage case 2 (DC2)	3.405 3.366 3.346	- - 1.15 - 1.73	1.0000 0.9999 0.9999	20.87 20.71 20.70	- -0.77 -0.81	1.0000 0.9999 0.9999	58.09 57.78 57.52	- - 0.53 - 0.98	1.0000 0.9999 0.9999



Fig. 8. Mode shapes for (a) the first mode, (b) the second mode and (c) the third mode.





Fig. 10. Inter-storey deflections (a) IC, (b) DC1 and (c) DC2.

By introducing modified DIID, damage at the first floor was clearly identified without any noisy signal at the intact floors as shown in Fig. 11. For DC2, it was also clear that the modified DIID appeared at the first and the third floor only, as expected (see Fig. 12).

4. Experimental validation

Experimental validations were carried out on a 10-storey building model and the structure is supported by bolt connection, as shown in Fig. 13. Structural parameters of the experimental structure are essentially the same with those of the numerical simulation as shown in Table 1. Ambient vibration tests were performed under shaking table excitation with random loads and ten accelerometers (i.e., Model name: PCB 393B12) were installed on each floor. Two damage scenarios, which are also the same with numerical simulation, were considered, as shown in Fig. 7. Damages were described by replacement of the intact column to the damage column, as shown in Fig. 14.

The sampling frequency is 100 Hz and anti aliasing filter (i.e., 50 Hz) was applied to raw measurement signals. The typical time domain signals and their power spectra are depicted in Fig. 15. To see the uncertainty of the modal parameters and damage detection results, this measurement was repeatedly performed eight times for each intact/damage case. The lower three modes were utilized to calculate the modal flexibility matrices for each intact/damage case. The stochastic subspace identification method [34] was used to identify the modal parameters of the test structure. The changes in the modal parameters due to damages are shown in Table 3 and Figs. 16 and 17. The mass-normalized mode shapes were obtained from system mass matrix. The natural frequencies were found to be reduced by about 1.05–2.20 percent, while there are no observable changes in the MAC values.



Fig. 11. (a) DIID of DC1 from IC and (b) modified DIID of DC1 from IC.



Fig. 12. (a) DIID of DC2 from IC and (b) modified DIID of DC2 from IC.

The deflections under the PBIL are shown in Fig. 18 with 1σ deviation. The 1σ deviation is used to show the deviations of the deflections according to measurement noises for each case. From one differentiation of the deflections, inter-storey deflections of each case were calculated from the deflections, as shown in Fig. 19. The changes in inter-storey deflections



Fig. 13. (a) Experimental model, (b) structural boundary condition and (c) connection between columns and floors.



Fig. 14. (a) Intact column and (b) damaged column.

(i.e., DIID) between one damage measurement and one intact measurement with 2σ deviations are shown in Fig. 20. The 2σ deviations (threshold level 2) are used to show that the 2σ deviations of the intact inter-storey deflections due to measurement noises are below DIIDs by progressive damages.

The outlier index Zi and modified damage index Zi_m , considering measurement noise, are plotted in Figs. 21 and 22 for each damage case. Zi_m can more simply estimate damaged locations than Zi as shown in Figs. 21 and 22. For DC1, damage at the first floor was clearly identified by the modified damage index Zi_m and there is no value exceeding the second baseline (i.e., single damage occurred in the structure on the first floor only). For DC2, it was also clear that Zi_m identifies damage on the first floor. Moreover, Zi_m value at the third floor exceeded the second baseline, which means that other damages may have occurred in the structure. From additional process to localize multiple damage based on the second baseline, the damage on the third floor was exactly identified without any false negative damage detection, as shown in Fig. 22.

5. Comparative study

Comparative study was carried out on two well-known damage detection approaches in the literature: (1) modal strainbased damage index approach [23,24] and (2) ULS curvature approach [7,10,11]. Explicit forms of the damage metrics used



Fig. 15. (a) Typical time history and (b) corresponding power spectra.

Table 3Modal parameter changes due to damage.

Case	The first mode			The second mode			The third mode		
	$f_1(Hz)$	$\Delta f_1/f_1~(\%)$	MAC	$f_2(Hz)$	$\Delta f_2/f_2~(\%)$	MAC	$f_3(Hz)$	$\Delta f_3/f_3$ (%)	MAC
IC DC1 DC2	3.153 3.120 3.087	- - 1.05 - 2.09	1.0000 0.9999 0.9999	16.50 16.27 16.27	- - 1.39 - 1.39	1.0000 0.9999 0.9998	43.25 42.48 42.30	- - 1.78 - 2.20	1.0000 0.9996 0.9994



Fig. 16. Evolutions of the natural frequencies for (a) the first mode, (b) the second mode and (c) the third mode.

in this study are shown in the literature [27,35]. The lower three modes obtained from the experimental study were also used. Outlier index Zi was calculated based on the intact deviations and changes due to the damage, in the same sense in Eq. (10) as follows:

$$Zi = \frac{\overline{m}_d(i) - \overline{m}_0(i)}{\sigma(m_0(i))}$$
(12)

where $m_d(i)$ is the damage metric at the *i*-th location between one damage measurement and one intact measurement; $m_0(i)$ is the damage metric at the *i*-th location between two intact measurements; $\overline{m}_d(i)$, $\overline{m}_0(i)$ and $\sigma(m_0(i))$ denote the mean values of $m_d(i)$ and $m_0(i)$, and the standard deviation of $m_0(i)$, respectively.

For damage index approach, the damage at the first floor was successfully identified for all of the damage cases, but the approach missed damage on the third floor for DC2 as shown in Fig. 23.

For ULS curvature approach, damage on the first floor was also clearly identified for all of the damage cases. Although outlier index appeared on the third floor for DC2, it could not exceed threshold level, indicating no damage as shown in



Fig. 17. Mode shapes for (a) the first mode, (b) the second mode and (c) the third mode.



Fig. 18. Deflections with 1σ deviations (a) IC, (b) DC1, and (c) DC2.

Fig. 24. From the comparison, the proposed approach also performed better in a cantilever beam-type structure compared to other two approaches in terms of sensitivity to damage and robustness to measurement noises.

6. Conclusion

In this paper, a new damage detection approach for cantilever beam-type structures was developed using the DIID estimated by modal flexibility matrix. The proposed approach can be utilized for damage detection of cantilever beam-type structures such as super high-rise buildings, high-rise apartment buildings, etc. A bending moment is dominant in a total behavior of the structures. Analytical studies on the DIID of cantilever beam-type structures have shown that changes in inter-storey deflections abruptly occur from damage location. Baseline modification concept was newly introduced to detect



Fig. 19. Inter-storey deflections with 2σ deviations (a) IC, (b) DC1 and (c) DC2.



Fig. 20. DIID with 2σ deviations (a) intact deviation, (b) DC1 and (c) DC2.



Fig. 21. Damage detection results for DC1 (a) Zi and (b) Modified Zi (Zi_m).



Fig. 22. Damage detection results for DC2 (a) Zi, (b) Modified Zi (Zi_m) and (c) Modified Zi (Zi_m) estimated from the second baseline.



Fig. 23. Investigation of the damage location(s) by the damage index approach (a) DC1 and (b) DC2.

multiple damages in cantilever beam-type structures by changing the baseline to the prior damage location. For validating the applicability of the proposed approach to cantilever beam-type structures, a series of numerical and experimental studies on a 10-storey building model were carried out. From the tests, it was found that the damage locations can be



Fig. 24. Investigation of the damage location(s) by ULS curvatures approach (a) DC1 and (b) DC2.

successfully identified by the proposed approach for multiple damages as well as single damage. In order to confirm the superiority of the proposed approach, a comparative study was carried out on two well-known damage metrics such as modal strain-based damage index approach and ULS curvature approach.

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