Seismic damage identification in buildings using neural networks and modal data

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Abstract

A seismic damage identification method intended for buildings with steel moment-frame structure is presented in this paper. The method has a statistical approach and is based on artificial neural networks and modal variables. It consists of two main stages. The initial one is devoted to the calibration of the undamaged structure and the final one to the identification of the damaged structure after an earthquake. The inputs of the nets are the first flexural modes (frequencies and mode shapes) at each principal direction of the structure and the outputs are the spatial variables (mass and stiffness). A damage index at each storey is determined by comparing the initial and final stiffness. A simplified finite element model was used to generate the data needed to train the nets. This model is consistent with available modal data and damage definition. The method was simulated on a 5-storey office building under conditions as close as possible to reality. The robustness of the method was verified with simulated data. Latter on, a sensitivity analysis of the mass variability was also carried out. Finally, the influence of modal error in the accuracy of damage predictions was statistically studied. Results are successful as concern as the robustness of the method. However, it is found that this approach is quite sensitive to modal errors.

Keywords: Seismic damage identification; Neural networks; Steel frames; Modal data; Mass sensitivity; Transmission of errors

1. Introduction

Recent large earthquakes have shown that the structures in general are not completely protected against these events. When damage is not evident, it is useful to have information about the state of the structure, which is crucial in the case of important and public buildings as hospitals, communication centres, etc. If a severe damage is found, an immediate evacuation of people is pertinent, so as to prevent risks derived of aftershocks or posterior earthquakes. If low damage is predicted, the building can be returned to use reducing the economic effect of the earthquake.

Seismic design codes have significantly improved during last century [1]. The fundamental principle of modern codes is that the structures should remain elastic under small to medium earthquakes. In the case of large earthquakes, a controlled damage is permitted without collapse. This behaviour is achieved by means of plastic hinges, which are usually located in the beams.

Welded Steel Moment Frame (WSMF) buildings are very common in seismic areas due to its easy-to-build characteristics and ductile response. However, the post-seismic inspections carried out after the Northridge and Hyogoken-Nambu earthquakes revealed brittle fractures in some joints [2], which caused important reductions to the stiffness and ductility of the structures. This leads to a new design of the joints called “performance-based design”, which guarantees a good ductile performance of the joint. Even so, a lot of structures with old joints are likely to experience brittle fracture during a strong earthquake. On the other hand, formation of hinges implies a reduction of stiffness, which can weaken the structure and becoming vulnerable to aftershocks or new earthquakes.
the development of post-seismic methods for damage identification is justified in order to control the global state of the structure.

The aforementioned assessment of a structure can be carried out by local methods, as visual inspection, acoustic emission, radiography, etc. [3]. However, these methods are generally slow, expensive and their application requires the covering materials to be removed at the zones inspected. Global methods as vibration-based methods are non-destructive and give a global damage assessment of a structure using its dynamic response. A further local inspection focused only in the zones where the damage is predicted can be done using local methods. The advances in sensors and computing have motivated an extensive research in vibration-based methods. In the literature review presented by Doebling et al. [4] is concluded that robust methodologies focused on specific applications are needed.

A method based on neural networks (NNs) is proposed in this paper. The objectives of the method are to detect and quantify the global damage at each storey of a WSMF building using its low natural frequencies and mode shapes. The method is an attempt of generalisation of previous authors’ works [5–7] and constitutes an extension of an earlier method that is only based on natural frequencies [8]. Neural networks have particular characteristics that make them appropriate for this case. They can give the diagnostic of a structure almost instantaneously and their generalisation capability can be known beforehand during the training and testing processes. The aim of this work is to analyse the robustness of the proposed method under conditions as close as possible to those of the actual structures.

2. Damage identification method

2.1. General approach

The method proposed here is intended for buildings designed under modern codes with the following conditions:

- Steel moment-frame structure with regular geometry both in plan and elevation.
- Under seismic action, floors move as a rigid body within their plane.
- Lateral loads are absorbed by beam to column joints.
- Lateral displacements due to axial column deformation are negligible when comparing with those due to lateral deformation.

The dynamic features selected for the identification where the natural frequencies and the mode shapes corresponding to the flexural vibration modes of the building in each transversal principal direction. In practice only a reduced number of low natural frequencies and mode shapes can be precisely identified with the common available techniques of modal testing and identification in buildings [9]. Taking into account this limitation, it is assumed that only the first three natural frequencies and mode shapes are known in each principal direction.

As the dynamic information is limited and both mass and stiffness affects the dynamic response of the structure, the identification process is carried out in each principal direction in two main stages. The first one corresponds to the initial state of the building. In this stage both the stiffness $K_0$ and the mass $M_0$ of the undamaged building is calibrated. The second one corresponds to a possible damage state of the building after a significant earthquake. The stiffness $K_f$ and the mass $M_f$ are recalculated. The presence and extension of the damage is finally estimated by comparing the initial and final values of stiffness.

At each stage, a NN is used to obtain the corresponding unknown spatial variables from the experimental modal properties of the building. Both nets are referred to as initial neural network (INN) and final neural network (FNN), respectively, in the remainder of the paper. Considering that abrupt changes of stiffness may occur in this case, inverse methods are not appropriate because they are relatively slow, and the convergence to the solution is not always guaranteed. NNs, however, can supply the spatial variables of the structure from the modal data in a short period of time. On the other hand, their convergence is guaranteed, as often as the modal data are within the training domain. The characteristics of these NNs are explained in the next section.

2.2. Neural networks

NNs are computational models inspired in the architecture and operation of the human brain. They are an assembling of connected processing units called neurons. The strength of the connection is represented by the “weights” or parameters, which are determined in the training process.

Radial basis function and multi-layer perceptron (MLP) are the typical configurations used for damage detection. In this work, a two-layer feed-forward MLP was used.

The inputs ($x_i$) of the MLP were the natural frequencies and the mode shapes, while the spatial variables constituted the output of the net ($y_i$). Output variables are obtained from the input in a concatenated way through linear combinations of previous layer values and weights ($w$) transformed by activation functions. A tangent hyperbolic activation function was used for the hidden layer ($g$) and a linear one for the output layer $g$ (Fig. 1).

$$y_k = \hat{g} \left[ \sum_{j=0}^{c} w_{kj}^{(2)} \cdot g \left( \sum_{i=0}^{d} w_{nj}^{(1)} \cdot x_i \right) \right]$$

This constitutes a black-box model based on mother basis functions of a single variable, which can approximate any continuous non-linear multivariate function within a given finite domain just by adjusting its weights. The accuracy of the approximation increases with the number of hidden units [10].
In the literature, the process of fitting a MLP to a given function is known as **learning** or **training**. The learning is based on a set of patterns of mapped input and output vectors, which is called learning data, and is obtained either analytically or experimentally. The process consists on minimising a quadratic error function, which represents the discrepancies between the predictions of the MLP corresponding of the input data and the target ones, with respect to the MLP weights. The optimisation process is divided in two steps. Firstly, the derivatives of the error function respect to the parameters of the net are computed. Error Back Propagation algorithm with batch strategy was used for this step. In the second step the optimisation of the parameters was done using Scaled Conjugate Gradients (SCG). SCG is an efficient algorithm of optimisation that takes the minimum number of cycles to minimise the error function. It belongs to the “line search” family where the minimum is obtained in a determined direction and the next search direction is the conjugated of the previous one. The method garantuises that the minimum is reached after \( W \) cycles for a quadratic error function, \( W \) being the number of weights. As the error function is not usually quadratic, SCG tends to degenerate during the running of the algorithm. Therefore, it should be restarted if convergence is not reached after several cycles. The learning was managed so as to restart the SCG by resetting the search vector to the negative gradient directions when the number of cycles equals the number of adaptive weights (\( W \)) of the MLP.

After training, the capacity of generalisation of the MLP is tested through a data set, which is similar to that used for training but containing different patterns. The **testing** is developed by comparing the predictions of the trained MLP from the input data with the output ones. This is especially important when the data contain noise. If the MLP has too many hidden units, it reproduces not only the information underlying in the data, but also the noise. This undesirable result is called **over-fitting**. More details about MLPs can be found in Bishop [11].

The Netlab package was used to compute the MLP. This is a library freely offered by its authors from the Aston University [12]. Netlab is implemented as a set of functions written in the Matlab language using only core functions.

### 2.3. Analytical model

#### 2.3.1. Description

A Finite Element (FE) model is used to generate the data for training and testing the NNs. As the information contained in the features is limited, the definition of both the damage and the FE model should be consistent with it. Thus, the damage should be defined by a number of independent parameters less than or equal to the number of available modal parameters. Otherwise, the NNs would be unable to generalise. This is because different states of damage lead to very close sets of modal parameters [13].

For this type of structures, the damage scenario is defined on the basis of their past performance in earthquakes and the previsions of modern codes. Thus, the selected potential damage zones are the beams and column–foundation connections. These conditions are accomplished with a simplified semi-frame model containing a number of spatial variables equal to the number of available modal data (Fig. 2).

In this model, the Degrees of Freedom (DoFs) represent the horizontal translations of each storey of the original structure. As the modal information corresponds only to the translations, the rotational DoFs were dynamically condensed. For each storey, \( k_c, k_b, m \) are representative of the overall column stiffness, beam stiffness and mass of the corresponding storey. The global mass and stiffness matrices of the simplified model can be assembled from the individual masses and stiffnesses. \( k_{fj} \) represents the column–foundation joint stiffness. A completely rigid connection corresponds to \( k_{fj} = \infty \), while a perfect hinge corresponds to \( k_{fj} = 0 \). This variable also includes the foundation flexibility itself and, to some extend, the soil–structure interaction. In order to avoid large values of \( k_{fj} \), a new variable \( \gamma \) was defined.

\[
\gamma = \frac{k_{fj}}{k_{fj} + k_{c1}}
\]
where $k_c$ is the stiffness of the pier of the first floor in the simplified model. This variable is referred to as foundation joint fixity factor and takes values in the [0–1] interval, $\gamma = 1$ for a rigid joint and $\gamma = 0$ for a pinned one.

The damage to the structure is considered to be localized in beams and column–foundation joints, the column stiffness being considered constant during the life of the structure.

### 2.3.2. Starting values

In order to obtain an approximate starting value of the spatial variables of the simplified model, each storey is split into a series of identical 1-bay frames (see Fig. 3). Thus, all of them have the same modal properties. Hence, the 1-bay frame represents dynamically the complete storey.

Another advantage is that they have anti-symmetric mode shapes. This allows the 1-bay frame to be reduced to an equivalent semi-frame having half beam length (see Fig. 4).

Finally, the physical properties of the equivalent semi-frame are approximated in each storey by equating the sum of all the values of each original variable (see Fig. 3a) to those of the 1-bay frame series (see Fig. 3b). This yields the following values of the variables of the equivalent semi-frame (Fig. 4):

$$
\begin{align*}
 k_{eq} &= k_c = \frac{\sum k_c^i}{(2N_c - 2) \cdot M_c}, \\
 k_{bo} &= k_b = \frac{\sum k_b^i}{(N_c - 1) \cdot M_c}, \\
 L_{beq} &= L_b = \frac{\sum L_b^i}{2(N_c - 1) \cdot M_c}, \\
 m_{eq} &= m = \frac{\sum m^i}{(2N_c - 2) \cdot M_c}
\end{align*}
$$

where $N_c$ and $M_c$ are the number of columns parallel and perpendicular to the considered direction, respectively. This procedure allows the starting values of the variables of the simplified model to be expeditiously calculated from those of the complete one.

### 2.3.3. Damage definition

When damage is present in a structure and there is not external evidence, which are the conditions the actual method is intended for, it is generally due to brittle fractures. Other damage mechanisms, such as low-cycle fatigue and local buckling, might also contribute to the structure deterioration. The damage is usually located at the connections of the beams to other elements and splices, and it is randomly distributed through the structure. As a consequence of the damage there is a reduction in the flexural stiffness of the beams, which affects the lateral stiffness of the structure.

In order to be consistent with the practical available modal data, the damage is defined globally through the simplified FE model. Thus, in each storey, the damage is formulated as the variation of the bending stiffness relative to the undamaged one,

$$
d_b = \frac{k_{bi} - k_{bo}}{k_{bi}}
$$

where $k_{bo}$ and $k_{bi}$ are the bending stiffness in the plane to be analysed of the beam corresponding to the storey $i$ in the simplified FE model, for the undamaged and damaged states, respectively. This constitutes a bounded global damage index for each storey, in which a value 0 represents an undamaged storey, and a value 1 represents a complete damaged storey.

In the same way, the column–foundation joint damage severity $x$ is defined as,

$$
x = \frac{\gamma_f - \gamma_i}{\gamma_i}
$$

where $\gamma_f$ and $\gamma_i$ are the foundation joint fixity factor in the simplified FE model corresponding to the undamaged and damaged state, respectively.

This simplified model was coded in Matlab [14], and the related natural frequencies and mode shapes were computed through $fe_eig$ function of the Structural Dynamic Toolbox [15].

### 2.4. Modelling of spatial variables

The spatial properties (mass and stiffness) are considered as random functions of both time and position in order to obtain a representation as close as possible to reality.

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Fig. 3. (a) Original storey, (b) 1-bay frame series.

Fig. 4. Left: 1-bay frame. Right: equivalent semi-frame.
2.4.1. Stiffness
A different modelling is adopted in each stage of the structure. Three different stiffnesses are considered in the modelling: column, beam and foundation joint fixity factor.

- **Column stiffness.** It is assumed to remain constant during the life of the structure. Thus, the value obtained in the initial stage is used for the final one. The variations of stiffness of each structural element respect to its nominal value are assumed to be normally distributed.
- **Beam stiffness.** It is assumed to be constant in the period of life previous to a significant earthquake and normally distributed. In the final stage, a random and independent reduction of stiffness is assumed in the different storeys of the structure.
- **Foundation joint fixity factor.** This variable can take values in [0–1] interval in the initial stage. In this study, a nominally fixed joint, which can have values of $\gamma$ within the [0.891–1] interval according to the Eurocode [16], is considered. A Beta distribution, which accomplish for the aforementioned conditions, was adopted for this variable (Fig. 5). In the final stage, a uniform random reduction was selected for $\gamma$.

2.4.2. Mass modelling
A hierarchical model has been used for modelling the mass. The mass variability has been modelled dividing the total mass in two specific and independent different parts: the dead mass and the live mass.

The dead mass includes the mass of the structure and the mass of permanent non-structural elements and installations. This term does not vary significantly through the life of the structure. The discrepancies of the actual values respect to the nominal ones are usually considered to be normally distributed with some additive constant discrepancy [17].

In general, the live mass includes the furniture, equipment, stored objects and people. In this case, however, it is assumed that the building is not occupied during the experimental modal testing. Consequently, the part of the mass corresponding to people is not taken into account. Live mass varies at random in time and space. The spatial variations are assumed to be homogeneous, and the variation in time is divided in the sustained and intermittent components. The sustained mass includes the time average of the actual fluctuating mass and the uncertainties due to the short-term fluctuations around the average. The intermittent masses have small relative duration, and they are due to sources like gathering of people or furniture staking during remodelling. The latter component was not considered in this work.

The sustained mass corresponding to a floor was computed through an equivalent uniform distributed mass $q$, with the following statistical properties.

$$E[q] = m$$

$$\text{Var}[q] = \sigma_v^2 + \sigma_u^2 A_0 / A \kappa$$

where $m$ is the overall mean mass per unit of area. The variable $V$ represents the variability of the sustained mass between different floors and it is considered zero mean and normally distributed. The variable $U(x, y)$ describes the mass variability into the same floor and it is a zero mean random field with a characteristic skew to the right. The variables $U$ and $V$ are assumed to be stochastically independents. $A$ denotes the area of a floor and $A_0$ is the reference area. Finally, the parameter $\kappa$ depends on the shape of the influence surface. In this case, it takes the value 1, because the influence surface is constant as all the elemental masses have the same contribution to the total mass in each floor.

The values of the parameters depend on the use of the building, and a Gamma distribution is found to fit adequately the actual observations. More details of the modelling can be found in Probabilistic Model Code [18].

3. Simulation on a 5-storey building

3.1. Description of the building

The reference structure used for the simulation is intended to be representative of a typical office building. It was a 5-storey building with regular distribution both in plant and elevation. The dimensions of the building were 16 m width and 18 m length rectangular plant, the height being 4 m for the first storey and 3 m for the rest. The structure is designed with a moment-resistant orthogonal steel frame 4-bay by 3-bay, the columns being HEB-450 sections and the beams, IPE-450 sections. It had concrete floors and glass façade.

The nominal dead mass of the building was 480 kg/m² for the fifth storey and 380 for the remainders. A Gamma distribution was adopted for the live mass, the parameters of the model in Eq. (6) being [18] $m = 37.5$ kg/m², $\sigma_v = 15$ kg/m², $\sigma_u = 30$ kg/m² and $A_0 = 20$ m².
3.2. Application of the method

The method was simulated considering the major principal direction of the building $z$ (see Fig. 6). The inputs to the nets were the first modes of the structure. Namely, three natural frequencies and mode shapes for the INN, and three natural frequencies and two mode shapes for the FNN. The outputs were masses, beam stiffnesses, column stiffnesses of each storey and column–foundation joint fixity factor for the INN, and masses, beam stiffnesses of each storey and column–foundation joint fixity factor for the FNN.

If all the physical variables were considered in the initial stage, they could not be determined from the modal data, because proportional values of the variables yield the same modal properties. In order to break this indeterminacy, the stiffness of the first floor pier in the simplified model was fixed in the initial stage. Its value was set equal to that obtained by Eq. (3).

The number of input variables was selected to be at least the same as output variables, in order to have enough information to train the networks. The number of intermediate units was selected by trial and error adopting a trade-off between training time and precision of the testing results. Eventually, a $n:4n:n$ architecture was used for all the NNs. The configuration of both neural networks is shown in Fig. 7.

After selecting the architecture of the nets, the database for training and testing was generated. At each stage, the process of building the database started with the generation of the random output values. For this end, the Latin Hypercube (LH) method was used. LH method is an efficient algorithm that provides a uniform multivariate sampling and guarantees a good representation of the entire probability interval. In this process, each component was considered as an independent random variable with a given probability density function (pdf) in each stage (see Table 1). The pdfs of the variables should be selected as close as possible to reality so as to reduce the training process and improve the predictions of the nets. The database was then completed with the corresponding modal data, which was computed by the simplified model on the basis of the selected variables. Finally, the NNs were trained and tested.

In the initial stage, a Gaussian distribution was adopted for both the column stiffness and the beam stiffness, with coefficients of variation 5% and 11.5%, respectively. This distribution is intended to cover both the modelling errors and the deviations of each variable with respect to its nominal value. A Beta distribution was adopted for the variable $\gamma$. The parameters of this distribution were tuned to achieve a 95% probability in the [0.891–1] interval of the variable, which is representative of a nominally fixed joint (see Fig. 5). Only constants masses are considered in this stage. They are assumed to have Gaussian distribution with 7.5% coefficient of variation. A database containing 3000 data sets was generated with the aforementioned procedure. 2000 sets were used for training the INN, and the remaining 1000 sets for testing. The training lasted 300 h. The results of testing are shown in Table 2. The correlation coefficients between the target values and the INN predictions are good, with values greater than 0.82 for all the variables.

In practice, the INN would be used to calibrate the initial variables from the starting ones just by feeding the INN with the experimental modal data. As experimental data is not available in this study, they were replaced with analytical ones corresponding to the mean values of the variables. The calibrated values of the column stiffnesses obtained in this stage are used in the next one.

In the final stage, a uniform distribution was adopted for the final beam stiffness and column–foundation stiffness. A variation relative to the initial values within the $[-30\%, +10\%]$ interval was assumed for these variables. Even though a negative variation of stiffness is expected in practice, the predictions of the FNN could be positive variations in some cases due to the interpolation errors of the FNN and the errors of the input data. That is why a positive bound of the interval was adopted. It tries to avoid practical extrapolation errors in the FNN. The masses include the dead ones previously calibrated plus the live ones. These are assumed to have a Gamma distribution with the parameters given by Eq. (6). The database had 1500 data sets in this case. 1000 sets were used for training...
the FNN, and the remaining 500 sets for testing. The training lasted 11 h. The results of testing are also shown in Table 2. The correlation coefficients between the target values and the FNN predictions are even better than the previous ones, with values greater than 0.9993 for all the variables. Consequently, the NNs are capable of giving accurate predictions from unseen data.

### 3.3. Mass sensitivity analysis

The influence of the mass variability in the accuracy of the damage predictions is studied in this subsection. For this purpose, an additional NN was developed for the final stage. It is referred to as Constant Mass Neural Network (CMNN) in the following. The inputs to this NN were the first two natural frequencies and the first mode shape, while the outputs were the beam stiffnesses of each storey and the column–foundation joint fixity factor. A 6:24:6 architecture was adopted for this NN (see Fig. 8).

A database similar to that used for the FNN and containing 750 data sets was generated through the LH method and the calibrated simplified model. The same value of the mass of each storey was considered in all the sets. They were computed on the basis of the mean values of the mass variables. 500 data sets were used to training, and the rest for testing the CMNN.

Once trained the CMNN, another database with 1000 data sets was generated in order to compare the accuracy of the damage predictions of both the FNN and the CMNN. This was intended to be representative of actual data. Hence, it was generated by taking random values of the live mass combined with independent random values of the damage. The variables were also selected through the LH method by using the adopted Gamma pdf for the mass and the uniform one for the damage. Then, the corresponding modal properties of the structure were computed by the calibrated simplified FE model. Finally, this database was used to feed both the CMNN and the FNN, and the accuracy of their predictions was studied.

Fig. 9 shows the results of both the CMNN and the FNN compared with the target values. The discrepancies in the results of the FNN are only due to its accuracy, and those of the CMNN include the effects of the mass variability, too. It is visually evident a high scatter in the results of the CMNN, while those of the FNN are within a narrow band around the exact solution.

These results are numerically shown in Table 3. The correlation coefficients between the target values and the predictions for both nets are included. The values for

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**Table 1**

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**Table 2**

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<td>$\gamma_f$</td>
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</table>

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Fig. 8. Neural networks with constant mass.
the CMNN are very poor, while those of the FNN are excellent.

3.4. Influence of modal data errors

In previous subsections, the natural frequencies and mode shapes have been considered as deterministic values. However, in practice, modal data includes uncertainties due to measurement and identification errors that affect the precision of the damage predictions. The transmission of these uncertainties and their influence in the damage predictions are statistically studied in this subsection. Even though both the INN and the FNN contribute to the damage prediction errors, only the effect of the FNN was considered herein. This is because in the final stage a quick response is necessary, while in the initial one there is generally enough time to reach an accurate calibration of the structure.

In order to distinguish the influence of every type of modal variable in the final damage error, two separate analyses were carried out. The first one considering only the frequency error and the second one considering only the mode shapes error.

The process consisted on the comparison of the reference damage values with those obtained through the FNN. For this end, a database containing 10,000 values was generated. This database is similar to that used for training the FNN, but with a $[-20\%, 0\%]$ interval for the variation of the stiffnesses. This was completed by computing the damage indexes using Eqs. (4) and (5), which constitutes the reference database.

In order to obtain the modal data close to that obtained in practice, the reference natural frequencies and mode shapes were contaminated. For this purpose, Gaussian noise with a given coefficient of variation $\delta$ was independently added to every modal variable.

These contaminated modal variables were used to feed the FNN providing 10,000 values of damage index. The difference between each damage index and the corresponding one in the reference is referred to as the Damage Prediction Error (DPE). This process was repeated for different values of $\delta$, namely, from $0\%$ to $0.5\%$ with an increment of $0.1\%$ for the natural frequencies and from $0\%$ to $0.1\%$ with an increment of $0.02\%$ for the mode shapes.

Even though the input error is normally distributed, the output errors are not Gaussian due to the non-linearity of the process. Moreover, the FNN interpolation errors are added to the output. Under these conditions, it is very difficult to fit all the results to a given probability distribution. Eventually, it was decided to interpolate the obtained cumulative probability functions (cpf) of the damage error by means of a new neural network, which will be referred as statistical neural network (SNN). The advantage of this approach is that the cpf is not defined a priori, but it is implicit in the net. A 2:2:1 architecture was eventually adopted for this net. The inputs are the cpf of the damage error and the coefficient of variation of the modal error $\delta$, and the output is the corresponding DPE for each variable (Fig. 10).

From the data obtained in the numerical simulation, a discrete cumulative frequency function containing 150 values was obtained for each value of $\delta$. These data were used

![Fig. 9. Correlation between target and predicted damage. Left: CMNN. Right: FNN.](image)

![Table 3](image)

<table>
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<td>$\alpha$</td>
<td>0.9994</td>
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![Fig. 10. Statistical neural network.](image)
to train the SNN for each stiffness variable. Once trained, the SNN can supply the equiprobability curves of the DPE as a function of $\delta$. These results are plotted in Figs. 11 and 12.

As it can be seen, all the cases show the same trend. There is an initial error for $\delta = 0$ due to the approximation of the FNN. Then, the absolute value of DPE increases exponentially as a function of $\delta$. Additionally, mode shapes have more influence in the value of the output error than natural frequencies. Setting $\delta = 0.1$, for example, the DPE corresponding to the mode shapes is around six times greater than that of the natural frequencies.

These curves can be used to obtain the maximum allowable level of $\delta$ for a given maximum absolute value of the output error with a given confidence (Fig. 13). For example, it is found that $\delta$ should be less than 0.1% in natural frequencies to obtain absolute values of the damage prediction errors up to 0.05 with a 95% confidence (values between 2.5% and 97.5%). In the case of mode shapes, $\delta$ should be less than 0.02%, for the same conditions.

3.5. Conclusions

In this paper, a method for seismic damage identification using modal parameters and neural networks has been developed. The method is intended for buildings with steel moment-frame structure. The process includes two successive stages, starting with a calibration of the initial stiffness and mass of the structure. The second stage deals with the identification of the final stiffness and mass after a severe...
In each stage, a MLP was used to obtain the spatial variables of the structure using its flexural natural frequencies and mode shapes. The database used for training the NNs was generated through a simplified finite element model.

The method was simulated on a 5-storey office building. The testing process of the NNs shows that the damage predictions for unseen random data are very similar to target values, which demonstrates the robustness of the method. The sensitivity analysis of the live mass variability illustrates the influence of this variable in the results of the method. The analysis shows that the prediction errors can reach an order of magnitude even higher than the target values if mass is considered constant. This means that is

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**Fig. 12.** Equiprobability curves as a function of the coefficient of variation of the input error for mode shapes.

**Fig. 13.** Maximum allowable modal error for a given absolute damage error.
necessary to include the mass as an output variable in order to obtain accurate results. The influence of modal data errors was also studied. The statistical analysis shows that the method is quite sensitive to them, especially to mode shapes ones. For simulating the error, a Gaussian noise was added independently to the modal parameters and the damage obtained was compared to the reference one. It is shown that the coefficient of variation of the modal errors should be less than 0.1% for natural frequencies and 0.02% for mode shapes to obtain absolute values of the damage prediction errors up to 0.05 with a 95% confidence.

References