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Out-of-plane (SH) soil-structure interaction: Semi-circular rigid foundation revisited

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ABSTRACT

The model studied in this paper presents an extension of previous work for a shear wall on a semi-circular rigid foundation in an isotropic homogeneous and elastic half-space. The objective is to develop a soil-structure interaction model that can later be applied to the case of a flexible foundation. As shown in the Introduction below, Luco considered the case of a rigid foundation subjected to vertical incident plane SH waves, and Trifunac extended the solution for the same rigid foundation subjected to SH waves but for arbitrary angles of the incidence. In this paper, a new approach and model are presented for the same semi-circular rigid foundation with a tapered-shape (instead of rectangular) superstructure. The analytical expression for the deformation of the semi-circular rigid foundation below this tapered shear wall with soil-structure interaction in an isotropic homogeneous and elastic half-space is thus derived. Results are then compared with those of Trifunac discussed in the section below. This problem formulation can and will later be extended in the case of a flexible foundation that is semi-circular or arbitrarily shaped.

1. Introduction

1.1. Brief history

Soil-structure interaction (SSI) is a process in which the effects of wave propagation in the half space are modified by the response characteristic of a structure and vice versa. SSI continues to be an active area of research especially at the interface of soil and structural dynamics. During the first half of the 20th century, engineered structures were designed with the assumption that a foundation was fixed to a rigid underlying medium; soil inertia was not considered and at best the soil was modeled as a spring element in structural models.

Reissner [3] studied the soil inertia and discovered that the material damping in the soil could modify to the response of the structure. In 1969, Luco [1] took up the topic and solved a two-dimensional (2D) interaction of a shear wall for an incident plane SH. Trifunac [2] generalized Luco's solution for arbitrary incidence of SH waves. Trifunac and Wong [4] then presented solutions for shallow and deep elliptical-rigid foundations, as well as multiple buildings and foundations (Wong and Trifunac [5]). Wong and Trifunac [6] also studied the effects of nearby canyons on soil-structure interaction and Abdel-Ghaffar and Trifunac [7] investigated the interaction of a simple 2D bridge excited by SH waves. Lee [8] presented the first three-dimensional (3D) analytical solution of interaction for a single degree-of-freedom oscillator resting on a semi-spherical foundation for the incidence of harmonic P, SV, and SH waves. Todorovska [9,10] described in-plane foundation-soil interaction for an embedded circular foundation and the effect of wave passage and embedment depth for in-plane building-soil interaction.

The purpose of this investigation is to develop new tools to solve more realistic models. The model studied in this paper presents an extension of the elastic shear wall with a circular rigid foundation studied by Luco [1] and by Trifunac [2]. However, our work will continue to be limited to plane-wave representation of incident waves, which was recently shown to be a good approximation Kara and Trifunac [11] and to the homogeneous half space – hence, any effects of the local soil layers will be neglected Liang et al. [12]. Finally, the structure and soil will be assumed to be linear and the only energy loss in the system will be associated with radiation of scattered waves into the half space. It is known that the nonlinear soil response can be a powerful sink of incident seismic energy Gicev and Trifunac [13,14]; Gicev et al. [15]; Trifunac and Todorovska [16–19]. We will study the related energy loss when we generalize the analysis presented here to the case of flexible foundation.

In practice, buildings are supported by concrete footings, piles and grade beams or mat foundation, and do not behave as rigid bodies when excited by seismic waves Trifunac et al. [20]. These

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foundations are not rigid; thus the ideal objective would be to solve for the soil-structure interaction of a shear wall with a flexible foundation. The methodology of Trifunac [2] cannot be modified to the case of a flexible or semi-rigid foundation, thus a new approach and model are developed in this paper to solve the soil-structure interaction of a tapered shear wall (structure) for rigid, flexible, and semi-rigid foundations using a “big arc approximation.” Further, it is not uncommon for high-rise buildings to gradually taper from the bottom to the top so that the top of the building is slightly narrower than the bottom.

1.2. Review of Trifunac [2] paper

The model studied in the Trifunac [2] paper is a 2D, infinitely long elastic shear wall resting on a semi-circular rigid foundation of radius a embedded in a half-space. It is subjected to plane incident SH waves with harmonic frequency ω. All materials here are homogeneous, elastic, and isotropic. Contact between the soil, the foundation, and the shear wall is assumed to be welded and no slippage exists. The material constants, namely the shear modulus and wave speed of the half-space soils and shear wall, are denoted by μ, C_β and μ_b, C_{β_b}, respectively. Since Trifunac's model used $exp\,(+i\omega t)$ as the time-dependent term and a polar- coordinate system that measured the angle with respect to the vertical y-axis, and our model uses $exp\,(-i\omega t)$ as the time-dependent term and a polar-coordinate system that measures the angle with respect to the horizontal x-axis (Fig. 1), we will first re-derive the Trifunac [2] result with respect to the notation used in this paper. For the coordinate system shown in Fig. 1, the analytical expression for the out-of-plane (SH) deformation of the rigid foundation, Δ, will now take the form

$$\Delta=\frac{\left[J_1(ka)-\left(\frac{J_o(ka)}{H_o^{(1)}(ka)}\right)H_1^{(1)}(ka)\right]a_o}{\frac{ka}{2}\left[\frac{M_f}{M_s}+\frac{M_b}{M_s}\left(\frac{\tan(k_bH)}{k_bH}\right)\right]-\frac{H_1^{(1)}(ka)}{H_o^{(1)}(ka)}}, \tag{1}$$

which is independent of the angle of incidence *SH* waves. Here M_s is the mass per unit length of soil to be replaced by the rigid foundation and M_f is the mass per unit length of the rigid foundation.

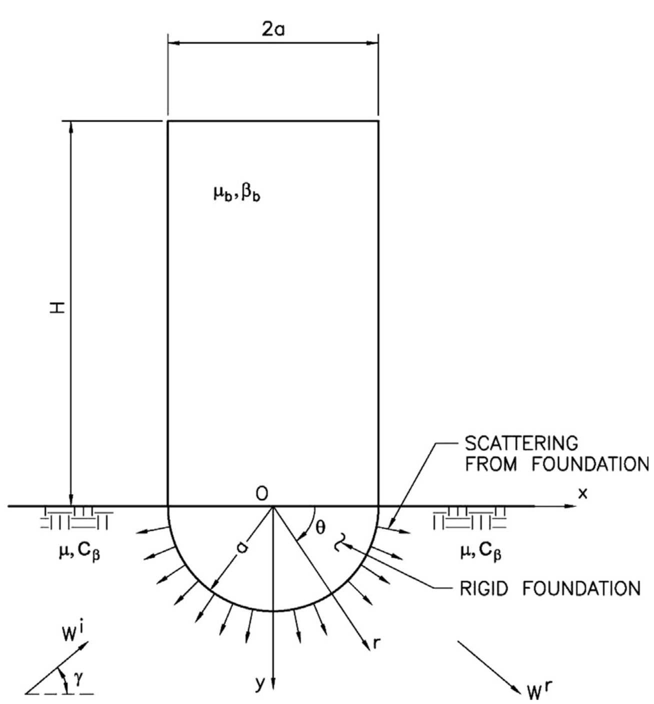

Fig. 1. Shear wall, foundation, and soil Trifunac [2].

A train of plane harmonic incident SH waves impinges on the model from deep earth with an incidence angle γ with respect to the horizontal axis. A Cartesian coordinate system (x, y) and a corresponding polar coordinate system (r, θ) have been defined with the origin at the center of the semi-circular foundation.

The incident wave field consists of a train of plane *SH* waves of unit amplitude with harmonic frequency ω, wave speed C_β, shear wave number $k=k_\beta=\omega/C_\beta$, and incidence angle γ. The incident $w^{(i)}$ and reflected $w^{(r)}$ waves can be expressed together as follows:

$$\begin{aligned} w^{i+r}(x,y)&=w^{(i)}+w^{(r)}=e^{ik(x\cos\gamma-y\sin\gamma)-i\omega t}+e^{ik(x\cos\gamma+y\sin\gamma)-i\omega t}\\ w^{i+r}(r,\theta)&=w^{(i)}+w^{(r)}=(e^{ikr\cos(\gamma+\theta)}+e^{ikr\cos(\gamma-\theta)})e^{-i\omega t}, \end{aligned} \tag{2}$$

where the out-of-plane motions are all in the z direction, perpendicular to the x, y plane (Fig. 1). Eq. (2) represents waves, which propagate in the positive x direction with phase velocity $c=C_\beta/\cos\gamma$. From here on, the harmonic term $exp\,(-i\omega t)$ will be understood and omitted in all subsequent equations.

The free-field incident and reflected waves w^{i+r} given by Eq. (2) can be expanded into a Fourier–Bessel series as

$$w^{i+r}(r,\theta)=\sum_{n=0}^{\infty}2\varepsilon_n i^n J_n(kr)\cos n\gamma\cos n\theta=\sum_{n=0}^{\infty}a_nJ_n(kr)\cos n\theta, \tag{3}$$

where for $n=0,1,2\ldots J_n(kr)$ is the Bessel function of the first kind with argument kr and order n , and $a_n=2\varepsilon_n i^n\cos n\gamma$. For n=0, 1, 2, 3….a_n are the coefficients of the free-field waves: $\varepsilon_o=1$ and $\varepsilon_n=2$ for $n>0$, so that $a_o=2$. The scattered and diffracted wave from the foundation w^R must satisfy the Helmholz wave equation for harmonic waves with frequency ω:

$$\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}+k^2w=0, \tag{4}$$

for $r\geq a$ and $|\theta|\geq\pi/2$, it must satisfy the boundary conditions given by

$$\sigma_{\theta z}=\frac{1}{r}\frac{\partial w_z}{\partial\theta}=0 \quad \text{at} \quad \theta=0,\pi \text{ and } r\geq a \tag{5}$$

and

$$w^{i+r}+w^R=\Delta \quad \text{at} \quad 0\leq\theta\leq\pi \text{ and } r=a \tag{6}$$

Here Δ is the unknown movement of the rigid foundation. The motion w^R represents an outgoing wave from the cylindrical foundation. It must also satisfy Eq. (4) and boundary condition (6). This wave can be represented as follows:

$$w^R(r,\theta)=\sum_{n=0}^{\infty}A_nH_n^{(1)}(kr)\cos n\theta, \tag{7}$$

Where A_n are the unknown complex numbers to be determined by boundary conditions and the wave functions and $H_p^{(1)}(kr)$ are the Hankel functions of the first kind with argument kr and order$p$.

The displacement of the shear wall, also in the z-direction (out-of-plane), has the same harmonic frequency ω, and w^b and must satisfy the Helmholtz wave equation with y the axis pointing vertically down (Fig. 1):

$$\frac{\partial^2 w^b}{\partial y^2}+k_b^2w^b=0 \quad \text{for} \quad -H\leq y\leq 0, \tag{8}$$

with $k_b=\omega/C_{\beta_b}$ being the building shear wave number, and C_{β_b} the wave speed in the shear wall. The shear wall must satisfy the boundary conditions of

$$\begin{aligned} &\sigma_{yz}=\mu_b\frac{\partial w^b}{\partial y}=0 \quad \text{at} \quad y=-H, \quad \text{top of shear wall}\\ &w^b=\Delta e^{-i\omega t} \qquad \text{at} \quad y=0. \end{aligned} \tag{9}$$

Dependence on x in the shear wall is eliminated in Eq. (10) by the assumption that the foundation is rigid. The solution of

Eqs. (8) and (9) is then given by

$$w^b = \Delta e^{-i\omega t}\left[\cos k_b y - \tan k_b H \sin k_b y\right]. \tag{10}$$

The base shear force per unit length of the shear wall f_z^b can be expressed as

$$f_z^b = -\omega^2 M_b \left(\frac{\tan k_b H}{k_b H}\right)\Delta e^{-i\omega t}, \tag{11}$$

where $M_b = \rho_b 2aH$ is the mass of the shear wall per unit length and the natural frequencies of the shear wall on the fixed foundation are

$$k_b H = (2n+1)\frac{\pi}{2} \tag{12}$$

To find the displacement Δ in terms of u_z, it is necessary to write the dynamic equilibrium equation for the rigid foundation. This equation, assuming there is no slippage between the soil and the foundation, is

$$-\omega M_f \Delta e^{-i\omega t} = (f_z^s + f_z^b), \tag{13}$$

Where M_f is the mass per unit length of the foundation. The f_z^b, the action force per unit length of the shear wall on the foundation, is given by Eq. (11) as shown above. The f_z^s, the action force of the soil on the foundation, is obtained by integrating the forces due to stresses of the waves in the half-space at the surface of the rigid foundation. From Eqs. (2) and (5), we obtain the following:

$$f_z^s = -a\int_{-\pi/2}^{\pi/2} \sigma_{rz}\Big|_{r=a} d\theta\left[\mu a\pi\left\{ka_n J_1(ka) + kA_n H_1^{(1)}(ka)\right\}\right]e^{-i\omega t} \tag{14}$$

The movement of the rigid foundation can then be expressed using Eqs. (11), (13), and (14) Trifunac [2]:

$$\Delta = \frac{\left[J_1(ka) - \left(\frac{J_o(ka)}{H_o(ka)}\right)H_1^{(1)}(ka)\right]a_o}{\frac{ka}{2}\left[\frac{M_f}{M_s} + \frac{M_b}{M_s}\left(\frac{\tan(k_b H)}{k_b H}\right)\right] - \frac{H_1^{(1)}(ka)}{H_o^{(1)}(ka)}}, \tag{15}$$

where M_s is the mass per unit length of soil to be replaced by the rigid foundation, and M_f is the mass per unit length of the rigid foundation.

2. The new model: tapered shear wall

2.1. The model

The model studied in this paper is a 2D, elastic tapered shear wall supported by a semi-circular rigid foundation of radius a attached to the elastic half-space, as illustrated in Fig. 2. All materials are homogeneous, elastic, and isotropic. The material constants—i.e., shear modulus and wave speed of the elastic half-space soil and shear wall—are denoted by μ, C_β and μ_b, C_{β_b}, respectively. Contact between the soil-to-foundation and foundation-to-shear wall is assumed to be bonded so that no slippage can occur between the contact surfaces. The structure on top of the foundation is an elastic shear wall of which a section is a circular sector $0 \le \theta \le \nu\pi$ of large radius R. The center of the circular sector is at O', a point high above the structure, so that the base of the shear wall in contact with the foundation is at a radius R and of width $2a$. The shear wall has height H above the foundation, so that the top of the shear wall is a circular arc with radius $R = R_1 - H$. Here the radius R is assumed to be very large compared with its half width, $R \gg a$, so that the full width of the shear wall, which is also the diameter of the semi-circular foundation, $2a \sim \nu\pi R$ or $R \sim 2a/\nu\pi$.

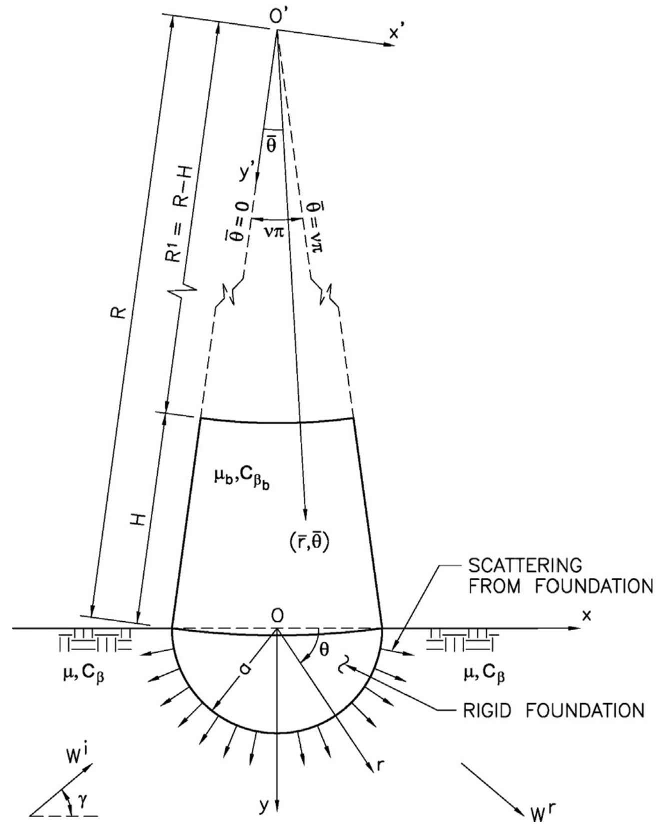

Fig. 2. The mathematical model of the tapered shear wall with a semi-circular rigid foundation.

2.2. Free-field waves in the half-space

The excitation consists of a series of plane SH waves incident onto the rigid foundation from a half-space at an incidence angle γ with respect to the horizontal axis. A Cartesian coordinate system (x, y) and a corresponding polar-coordinate system (r, θ) have been defined with the origin at the center of the semi-circular foundation. These waves are identical to the free-field waves expressed in the last section, and are now considered in more detail. The incident free-field wave consists of plane waves with unit ampl, wave speed C_β, and wave number $k = k_\beta = \omega/C_\beta$. The incident waves can be expressed in both the rectangular and polar coordinates as follows:

$$\begin{aligned} w^{(i)}(x,y) &= e^{\,ik(x\cos\gamma - y\sin\gamma)} \\ &= w^{(i)}(r,\theta) = e^{\,ikr(\cos\gamma\cos\theta - \sin\gamma\sin\theta)} = e^{\,ikr\cos(\gamma+\theta)}, \end{aligned} \tag{16}$$

and the reflected plane waves can be written as

$$\begin{aligned} w^{(r)}(x,y) &= e^{ik(x\cos\gamma + y\sin\gamma)} \\ &= w^{(r)}(r,\theta) = e^{ikr(\cos\gamma\cos\theta + \sin\gamma\sin\theta)} = e^{ikr\cos(\gamma-\theta)}, \end{aligned} \tag{17}$$

where γ is the angle of incidence or reflection with respect to the horizontal axis; $k_x = k\cos\gamma$ and $k_y = k\sin\gamma$ represent the components of the SH wave number along the x- and y-axes, respectively. The $e^{-i\omega t}$ harmonic time factor is understood and omitted from all wave equations. Applying the Jacobi-Anger Expansion (Pao and Mow 1973) [21], we have

$$\begin{aligned} e^{\pm ikr\cos\theta} &= \sum_{n=0}^{\infty}\varepsilon_n(\pm i)^n J_n(kr)\cos n\theta \\ e^{ikr\cos(\gamma\pm\theta)} &= \sum_{n=0}^{\infty}\varepsilon_n i^n J_n(kr)\cos[n(\gamma\pm\theta)] \\ &= \sum_{n=0}^{\infty}\varepsilon_n i^n J_n(kr)(\cos n\gamma\cos n\theta \pm \sin n\gamma\sin n\theta), \end{aligned} \tag{18}$$

Where $i=\sqrt{-1}$ is the imaginary complex unit, $J_n(.)$ is the Bessel function of the first kind with order n, and the two expressions in polar coordinate(r,θ) of Eqs. (16) and (17) can be expanded into an infinite series. The free-field wave field is then given by their sum as follows:

$$w^{(ff)}(r,\theta)=w^{(i)}+w^{(r)}=e^{ikr\cos(\gamma+\theta)}+e^{ikr\cos(\gamma-\theta)}$$
$$=\sum_{n=0}^{\infty}2\varepsilon_n i^n J_n(kr)\cos n\gamma\cos n\theta=\sum_{n=0}^{\infty}a_n J_n(kr)\cos n\theta, \tag{19}$$

where $a_n=2\varepsilon_n i^n\cos n\gamma$, exactly as in Eq. (3) above, which represents the free-field waves that are finite everywhere in the half-space. For $n=0,1,2,3\ldots a_n$ are the coefficients of the free-field waves;$\varepsilon_o=1$ and $\varepsilon_n=2$ for $n>0$, so that$a_o=2$.

The free-field waves will arrive towards the foundation, resulting in scattered and diffracted waves in the half-space. The wave field in the half-space scattered from the rigid foundation for $r\geq a$ and $0\leq\theta\leq\pi$ is given (as in the last section) by:

$$w^{(S)}(r,\theta)=\sum_{n=0}^{\infty}A_n H_n^{(1)}(kr)\cos n\theta, \tag{20}$$

Where A_n are the unknown complex numbers to be determined by boundary conditions and the wave functions and $H_n^{(1)}(.)e^{-i\omega t}$ represent outgoing waves satisfying Summerfield's radiation condition. The foundation is assumed to be rigid and thus the particles anywhere on and inside the foundation must have the same out-of-plane motion.

2.3. The wave field within the structure

Since the building structure on top is a shear wall that is defined as a circular sector with center at O', a point above the structure (Fig. 1), the building waves will be defined using the polar-coordinate system $(\overline{r},\overline{\theta})$ with an origin at O'. The out-of-plane motion is independent of coordinate x and can be represented as, for $R-H\leq\overline{r}\leq R$ and $0\leq\overline{\theta}\leq\nu\pi$:

$$w^{(B)}=w^{(B)}(\overline{r},\overline{\theta})=\sum_{n=0}^{\infty}\left[B_n^{(1)}H_{n/\nu}^{(1)}(k_b\overline{r})+B_n^{(2)}H_{n/\nu}^{(2)}(k_b\overline{r})\right]\cos\left(\frac{n\overline{\theta}}{\nu}\right), \tag{21}$$

where $H_{n/\nu}^{(1)}(k_b\overline{r})$ and $H_{n/\nu}^{(2)}(k_b\overline{r})$ are the Hankel functions of the first or second kind with argument $k_b\overline{r}$ and order n/ν;$B_n^{(1)}$ and $B_n^{(2)}$ are the unknown complex numbers to be determined by boundary conditions and the wave functions.

3. The boundary conditions

The boundary conditions on the flat ground surface are automatically satisfied by the free-field waves $w^{(ff)}$ and the scattered waves $w^{(S)}$. The stress and displacement continuity equations along the semi-circular interface of the rim and $0\leq\theta\leq\pi$ will be expressed as in the following.

3.1. Displacement continuity

$$\left(w^{(ff)}+w^{(S)}\right)\Big|_{r=a}=\Delta \qquad for\ 0\leq\theta\leq\pi, \tag{22}$$

is the displacement continuity requirement, where Δ is the amplitude of the rigid foundation displacement.

The substitution of Eqs. (19) and (20) with Eq. (22) leads to the following two boundary conditions:

$$a_n J_n(ka)+A_n H_n^{(1)}(ka)=\begin{cases}\Delta & for\ n=0\\ 0 & for\ n=1,\ 2,\ 3,\ \ldots\end{cases}$$

$$A_n=\infty\begin{cases}\frac{\Delta-a_o J_o(ka)}{H_o^{(1)}(ka)} & for\ n=0\\ \frac{-a_n J_n(ka)}{H_n^{(1)}(ka)} & for\ n=1,2,3,\ldots\end{cases} \tag{23}$$

3.2. The stress-free and stress-continuity equations in the building

The stress at the top of the building for $n=1,\ 2,\ 3,\ \ldots$ is

$$\tau_{z\overline{r}}\Big|_{\overline{r}=R_1}=\mu_b\frac{\partial w^{(B)}}{\partial\overline{r}}\Big|_{\overline{r}=R_1}=\mu_b k_b\left[B_n^{(1)}H_{n/v}^{(1)}{}'(k_bR_1)+B_n^{(2)}H_{n/v}^{(2)}{}'(k_bR_1)\right]=0$$

$$B_n^{(1)}H_{n/v}^{(1)}{}'(k_bR_1)+B_n^{(2)}H_{n/v}^{(2)}{}'(k_bR_1)=0$$

$$B_n^{(2)}=-\left[\frac{H_{n/v}^{(1)}{}'(k_bR_1)}{H_{n/v}^{(2)}{}'(k_bR_1)}\right]B_n^{(1)} \qquad for\ \ n=0,1,2,3\ldots \tag{24}$$

Substituting Eq. (24) with (21), the out-of-plane motion of the shear wall is simplified to the following:

$$w^{(B)}=\sum_{n=0}^{\infty}B_n^{(1)}\left[H_{n/v}^{(1)}(k_b\overline{r})-\frac{H_{n/v}^{(1)}{}'(k_bR_1)H_{n/v}^{(2)}(k_b\overline{r})}{H_{n/v}^{(2)}{}'(k_bR_1)}\right]cos\left(\frac{n}{v}\overline{\theta}\right)$$
$$w^{(B)}=\sum_{n=0}^{\infty}B_n^{(1)}\left[\frac{H_{n/v}^{(1)}(k_b\overline{r})H_{n/v}^{(2)}{}'(k_bR_1)-H_{n/v}^{(1)}{}'(k_bR_1)H_{n/v}^{(2)}(k_b\overline{r})}{H_{n/v}^{(2)}{}'(k_bR_1)}\right]cos\left(\frac{n}{v}\overline{\theta}\right)$$
$$w^{(B)}=\sum_{n=0}^{\infty}B_n^{(1)}H_{n/v}^{\wedge}(k_bR_1,k_b\overline{r})cos\left(\frac{n}{v}\overline{\theta}\right) \tag{25}$$

where $H_{n/v}^{\wedge}(k_b\overline{r})=H_{n/v}^{\wedge}(k_bR_1,k_b\overline{r})=H_{n/v}^{(1)}(k_b\overline{r})H_{n/v}^{(2)}{}'(k_bR_1)-H_{n/v}^{(1)}{}'(k_bR_1)H_{n/v}^{(2)}(k_b\overline{r})/H_{n/v}^{(2)}{}'(k_bR_1)$ is a "scaled" Hankel function defined as a linear combination of Hankel functions of the first and second kind. The boundary condition at the interface of shear wall and rigid foundation $w^{(B)}|_{\overline{r}=R}=\Delta$, gives

$$B_n\,H_{n/v}^{\wedge}(k_b\,R_1,\,k_b\,R)=\begin{cases}\Delta & for\ \ n=0\\ 0 & for\ \ n=1,2,3,\ldots\end{cases} \tag{26a}$$

resulting in $B_o=\Delta/H_o^{\wedge}(k_b\,R_1,\,k_b\,R)$ and $B_n=0$ for $n>0$, so that the building wave

$$w^{(B)}=w^{(B)}(k_b r)=\frac{H_o^{\wedge}(k_b\,R_1,k_b\,\overline{r})}{H_o^{\wedge}(k_b\,R_1,k_b\,R)}\Delta, \tag{26b}$$

becomes a one-term expression.

3.3. The dynamic equation for the rigid foundation, $w^f=\Delta e^{-i\omega t}$

As pointed out by Luco [1] and Trifunac [2], the displacement of the foundation Δ can be determined by applying the dynamic equilibrium equation for the rigid foundation as follows:

$$M_f\ddot{w}^f=-M_f\,\Delta\omega^2e^{-i\omega t}=(f_s+f_b)e^{-i\omega t}$$
$$-M_f\omega^2\Delta=f_s+f_b, \tag{27}$$

where $M_f=1/2\ \ \rho_f\,\pi a^2$ is the mass of the rigid foundation per unit length in the z-axis, ρ_f is the mass density of the foundation; f_b is the force of the shear wall acting on the foundation per unit length at $\overline{r}=R$ and $0\leq\overline{\theta}\leq\nu\pi$, and f_S denotes the force due to *total (free–field + scattered)* waves at $r=a$ and $0\leq\theta\leq\pi$.

$$f_b=\int_o^{\nu\pi}\mu_b\frac{\partial w^{(B)}}{\partial\overline{r}}\Big|_{\overline{r}=R}Rd\overline{\theta}=\mu_b k_b R\nu\pi\ \left[B_oH_o^{\wedge}{}'(k_bR_1,k_bR)\right]$$
$$f_b=-\mu_b k_b R\nu\pi\left[\frac{H_1^{\wedge}(k_bR_1,k_bR)}{H_o^{\wedge}(k_bR_1,k_bR)}\right]\Delta, \tag{28a}$$

and

$$f_s=\int_o^{\pi}\tau_{rz}\big|_{\overline{r}=a}ad\theta$$

$$f_s = \mu \mathrm{ka} \sum_{n=0}^{\infty} \left[a_n J_n{}'(\mathrm{ka}) + A_n H_n^{(1)}{}'(\mathrm{ka}) \right] \int_0^{\pi} \cos n\theta d\theta$$

$$f_s = \mu k \, \pi a \sum_{n=0}^{\infty} \left[a_o J_o{}'(\mathrm{ka}) + A_o H_o^{(1)}{}'(\mathrm{ka}) \right]$$

$$f_s = -\mu k \, \pi a \left[a_o J_1(\mathrm{ka}) + A_o H_1^{(1)}(\mathrm{ka}) \right], \tag{28b}$$

where the integral: $\int_0^{\pi} cosn\theta d\theta = \begin{cases} \pi & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}.$

The displacement of the rigid foundation can be determined by solving Eqs. (28a), (28b) and (27) for $n=0$.

$$\Delta = \frac{\left[J_1(\mathrm{ka}) - \left(\frac{J_o(\mathrm{ka})}{H_o^{(1)}(\mathrm{ka})} \right) H_1^{(1)}(\mathrm{ka}) \right] a_o}{\frac{M_f \ \omega^2}{\mu k \ \pi a} + \left(\frac{\mu_b \ k_b R v \pi}{\mu k \ \pi a} \right) \left(-\frac{H_1^{\wedge}(k_b \ R_1, \ k_b \ R)}{H_o^{\wedge}(k_b \ R_1, \ k_b \ R)} \right) - \frac{H_1^{(1)}(\mathrm{ka})}{H_o^{(1)}(\mathrm{ka})}} \quad . \tag{29a}$$

Let $M_s = 1/2 \quad \rho \pi a^2$ again (as in Section 1 above) be defined as the mass per unit length of the soil to be replaced by the rigid foundation, ρ, the mass density of the soil. For $R = R_1 + H > > a$, $R v \pi \sim 2\mathrm{a}$, so $M_B \sim \rho_B 2aH$ is again the mass of the building per unit length, where ρ_B here is the mass density of the building (with the tapered shear wall approaching the shape of a rectangular shear wall with width 2a and height H). Thus $\mu_b k_b R_2 v \pi / \mu k \pi a \sim (\mathrm{ka}/2)\,(M_b/M_s)\,(1/k_b H)$, Eq. (29a) can further be simplified to

$$\Delta = \frac{\left[J_1(\mathrm{ka}) - \left(\frac{J_o(\mathrm{ka})}{H_o^{(1)}(\mathrm{ka})} \right) H_1^{(1)}(\mathrm{ka}) \right] a_o}{\frac{\mathrm{ka}}{2} \left[\frac{M_f}{M_s} + \frac{M_b}{M_s} \left(\frac{1}{k_b H} \right) \left(-\frac{H_1^{\wedge} k_b R_1, k_b R}{H_o^{\wedge} k_b R_1, k_b R} \right) \right] - \frac{H_1^{(1)}(\mathrm{ka})}{H_o^{(1)}(\mathrm{ka})}}. \tag{29b}$$

Using the asymptotic approximation $H_1^{\wedge}(k_b R_1, k_b R)/H_o^{\wedge}(k_b R_1, k_b R) \sim -\tan(k_b H)$ as $R \gg a$, $\mathrm{R} \to \infty$ (see Eq. (A16), Appendix A), the displacement of the rigid foundation Eq. (29b) will be identical to Eq. (1) above in Trifunac [2]:

$$\Delta = \frac{\left[J_1(ka) - \left(\frac{J_o(ka)}{H_o^{(1)}(ka)} \right) H_1^{(1)}(ka) \right] a_o}{\frac{ka}{2} \left[\frac{M_f}{M_s} + \frac{M_b}{M_s} \left(\frac{\tan(k_b H)}{k_b H} \right) \right] - \frac{H_1^{(1)}(ka)}{H_o^{(1)}(ka)}}. \tag{30}$$

4. Numerical analysis of the displacements

As pointed out by Trifunac [2], the envelope of the rigid foundation displacement Δ is given by

$$\Delta_e = \left[J_1(\mathrm{ka}) - \frac{J_o(\mathrm{ka})}{H_o^{(1)}(\mathrm{ka})} H_1^{(1)}(\mathrm{ka}) \right] \left[\frac{J_o^2(\mathrm{ka}) + Y_o^2(\mathrm{ka})}{Y_o(\mathrm{ka}) J_1(\mathrm{ka}) - Y_1(\mathrm{ka}) J_o(\mathrm{ka})} \right] a_o \tag{31}$$

The backbone curve of Δ could be understood as the displacement of the rigid foundation whose density is identical to that of

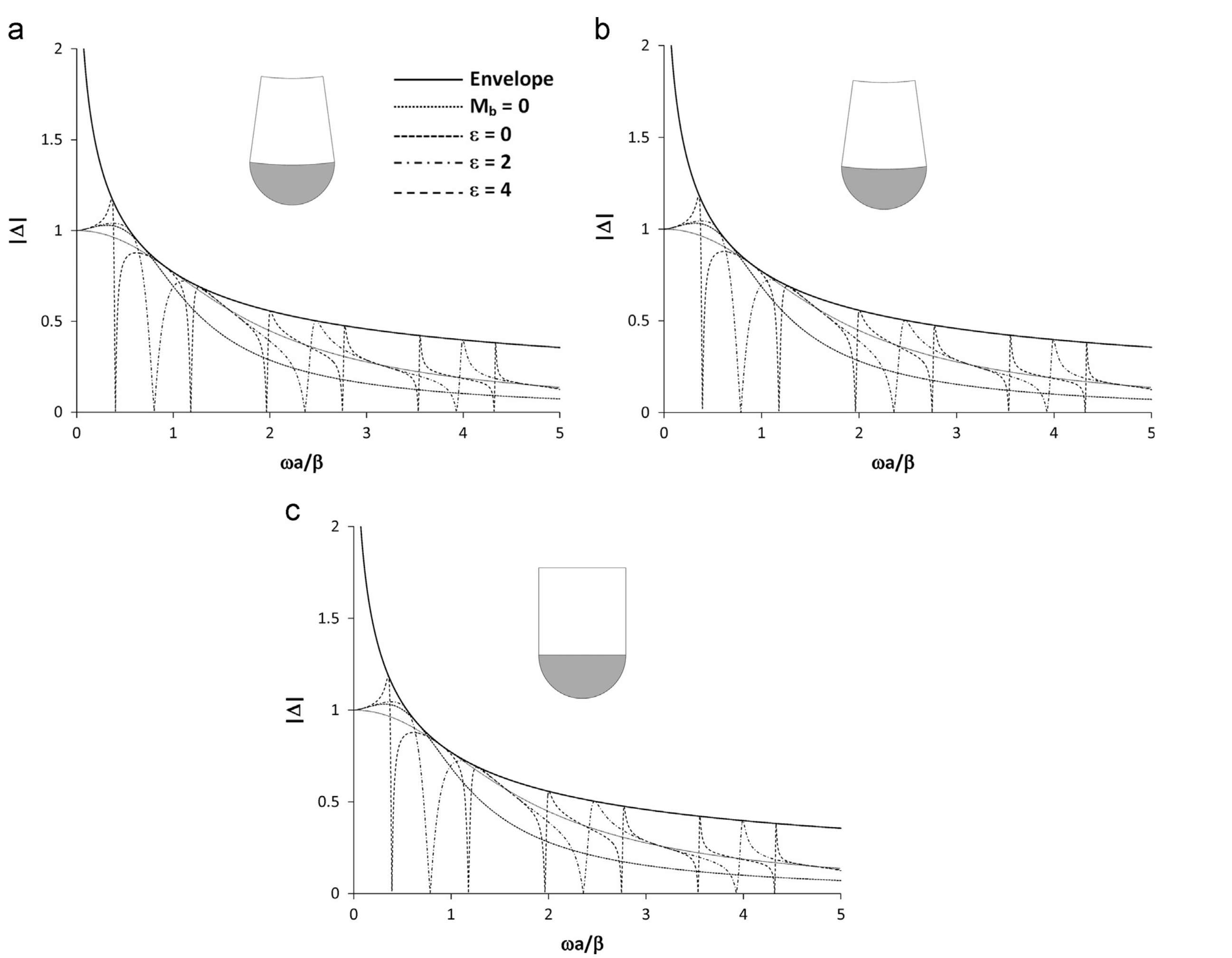

Fig. 3. The effect of interaction on Δ: the amplitude of foundation vibration. (a) R/H = 10, M_b/M_s = 1, M_f/M_s = 1, ε = 0, 2, 4, (b) R/H = 100, M_b/M_s = 1, M_f/M_s = 1, ε = 0, 2, 4 and (c) M_b/M_s = 1, M_f/M_s = 1, ε = 0, 2, 4 (Trifunac, 1972).

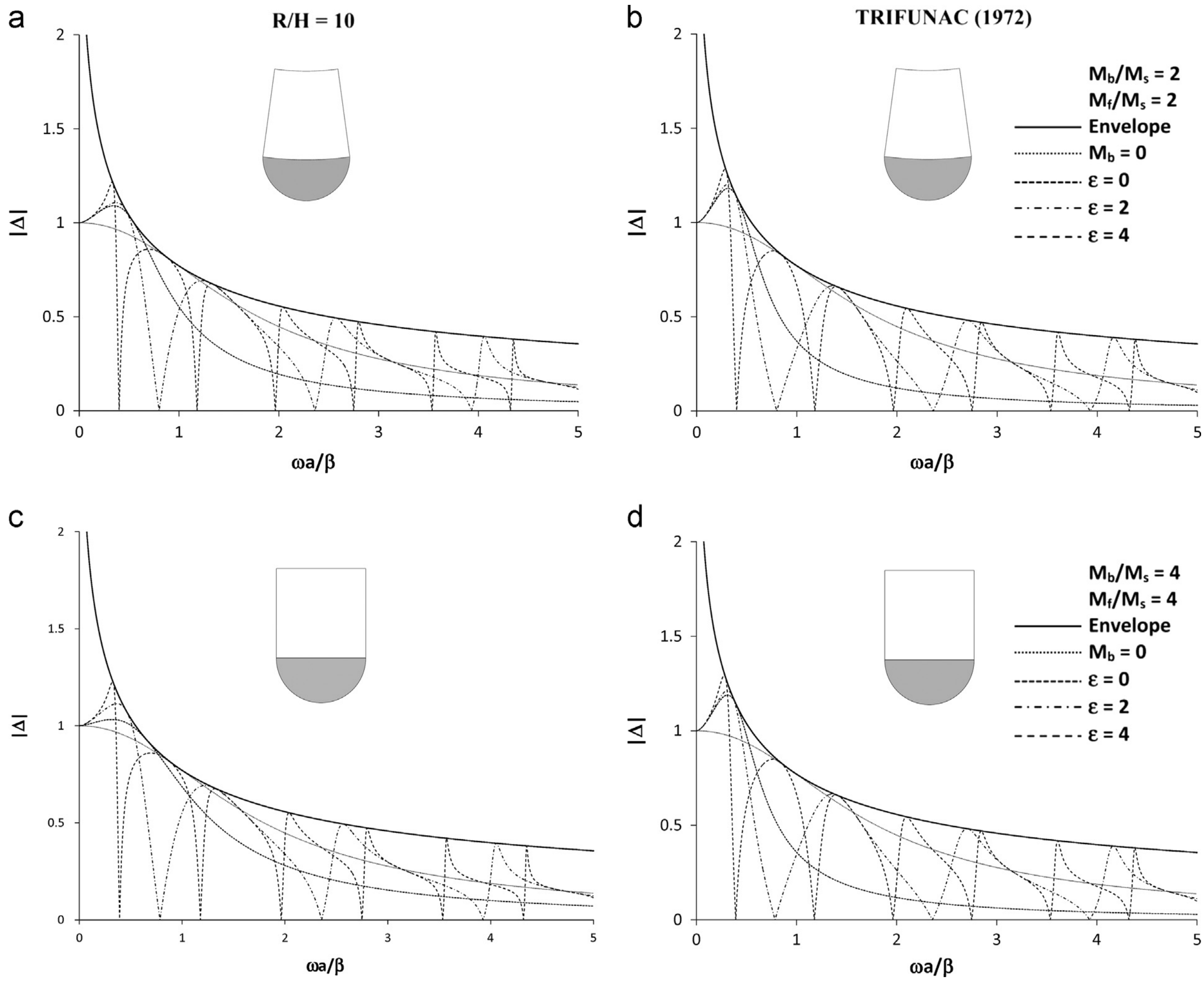

Fig. 4. Comparison of the displacement amplitudes of the Tapered-shape structures (a,b) and the rectangular structures (c,d) of Trifunac [2].

the surrounding soil by setting $M_b/M_s = 0$.

$$\Delta_o = \frac{\left[J_1(\mathrm{ka}) - \{J_o(\mathrm{ka})/H_o^{(1)}(\mathrm{ka})\}H_1^{(1)}(\mathrm{ka})\right] a_o}{\frac{\mathrm{ka}}{2}(M_f/M_s) - (H_1^{(1)}(\mathrm{ka})/H_o^{(1)}(\mathrm{ka}))} \tag{32}$$

To characterize the problem in terms of dimensionless parameters, we define $\varepsilon = k_b H/\mathrm{ka} = \beta H/\beta_b$ awhere $H = R - R_1$. The shape of the tapered shear wall is characterized by the ratio R/H, the ratio of the circular radius of the sector to its height. We plot the amplitude of the foundation vs. the dimensionless $\omega a/\beta$ for $R/H = 10$ *and* 100, $H/a = 10$, $M_f/M_s = 1$ and $M_b/M_s = 1, 2, 4$ with $\varepsilon = 0,2,4$ (Figs. 3(a), (b), (c), and 4(a, b, c and d)) and compare them with Trifunac [2]. The results agree well when $R/H = 10$, and are found to be almost identical when $R/H = 100$.

The two figures show that the new model of the tapered-shape shear wall is a legitimate and realistic model and the results show good agreement with the model studied by Luco [1] and Trifunac [2]. The existing model has one advantage over the models studied by Luco [1] and Trifunac [2]. While the previous models allow explicit analytic expression to be derived for the displacement of the semi-circular rigid foundation below a rectangular shear wall, it has the limitation that the derivation works only for a rigid foundation and cannot be extended to the case of a flexible foundation. The present model also allows explicit analytic expressions for the displacements of the same semi-circular foundation below a tapered-shape shear wall and when the shape of the shear wall is close enough to the rectangular one, the expressions for the displacement amplitudes agree at all angles of incidence and at all frequencies (Figs. 3 and 4). Further, results for the present model can and will be extended to cases in which the foundation is flexible and elastic with different elastic properties. This new model is thus formulated for future work on soil-structure interaction by flexible, elastic foundations.

5. Conclusion and further studies

In the majority of papers on soil-structure interaction – similar to the papers by Luco [1] and Trifunac [2], the foundation is simplified as a rigid body. Almost every building has a foundation that transfers upper loads to the soil evenly. With the exception of some towers and mast structures, most engineered structures cannot to be mathematically simplified to a single degree-of-freedom dynamic system. Therefore, a mathematical model with a flexible foundation is of considerable significance in soil-structure interaction research.

From the results of the numerical analyses of the proposed model in Fig. 2, it is concluded that the tapered-shape structure methodology works well with large-enough radius R. This methodology can be extended to solve for the soil-structure interaction of a shear wall supported by a flexible foundation as part of our future studies.

As seen in Trifunac [2], when a foundation is assumed to be rigid, every particle at any horizontal cross-section of the structure parallel to the half-space surface must have the same out-of-plane motion of the shear wall and is independent of coordinate y. Thus, dependence on y in the shear wall is eliminated in Eq. (10). This simplicity in the dependence of the displacement solution does not permit an extension of the solution for a flexible foundation. The wave field within the structure of the tapered-shape methodology only depends on the polar coordinate system, thus the methodology can be expanded to solve for both rigid and flexible elastic foundations.

Appendix A

Asymptotic Approximation for $R = R_1 + H \gg a$

The mass of the soils foundation, M_s per unit length is

$$M_s \sim \frac{\rho\pi a^2}{2} = \left(\frac{\mu\pi a^2}{2C^2}\right)\left(\frac{k}{k}\right)^2 = \frac{\mu\pi a^2 k^2}{2\omega^2} \tag{A1}$$

The mass of the building, M_s per unit length is

$$M_b \sim 2\rho_b aH = \left(\frac{2\mu_b aH}{C_b^2}\right)\left(\frac{k_b}{k_b}\right)^2 = \frac{2\mu_b k_b^2 aH}{\omega^2} \tag{A2}$$

$$\frac{\mu_b k_b R v\pi}{\mu k\pi a} \sim \left(\frac{\mu_b k_b R v\pi}{\mu k\pi a}\right)\left(\frac{k_b H}{k_b H}\right) = \frac{\mu_b k_b^2 (R v\pi H)}{\mu k\pi a(k_b H)} = \frac{\mu_b k_b^2 (2aH)}{\mu k\pi a(k_b H)} \tag{A3}$$

$$\frac{\mu_b k_b R v\pi}{\mu k\pi a} \sim \frac{\omega^2 M_b}{\left(\frac{2\omega^2 M_s}{\pi k^2 a^2}\right)(k_b H)} = \left(\frac{ka}{2}\right)\left(\frac{M_b}{M_s}\right)\left(\frac{1}{k_b H}\right) \tag{A4}$$

$$H_o^{\wedge}(k_b R_1, k_b R) = \frac{H_o^{(1)}(k_b R) H_1^{(2)}(k_b R_1) - H_1^{(1)}(k_b R_1) H_o^{(2)}(k_b R)}{H_1^{(2)}(k_b R_1)} \quad \text{and} \tag{A5}$$

$$H_o^{\wedge\prime}(k_b R_1, k_b R) = \frac{H_o^{(1)\prime}(k_b R) H_1^{(2)}(k_b R_1) - H_1^{(1)}(k_b R_1) H_o^{(2)\prime}(k_b R)}{H_1^{(2)}(k_b R_1)}. \tag{A6}$$

Note that $H_o^{\wedge\prime}(k_b R_1, k_b R)$ with respect to R.

$$H_1^{\wedge}(k_b R_1, k_b R) = \frac{-H_1^{(1)}(k_b R) H_1^{(2)}(k_b R_1) + H_1^{(1)}(k_b R_1) H_1^{(2)}(k_b R)}{H_1^{(2)}(k_b R_1)} \tag{A7}$$

$$\frac{H_1^{\wedge}(k_b R_1, k_b R)}{H_o^{\wedge}(k_b R_1, k_b R)} = \frac{-H_1^{(1)}(k_b R) H_1^{(2)}(k_b R_1) + H_1^{(1)}(k_b R_1) H_1^{(2)}(k_b R)}{H_o^{(1)}(k_b R) H_1^{(2)}(k_b R_1) - H_1^{(1)}(k_b R_1) H_o^{(2)}(k_b R)}, \tag{A8}$$

where

$$H_o^{(1)}(x) = \sqrt{\frac{2}{\pi x}} e^{i\left(x - \frac{\pi}{4}\right)} \tag{A9}$$

$$H_o^{(2)}(x) = \sqrt{\frac{2}{\pi x}} e^{-i\left(x - \frac{\pi}{4}\right)} \tag{A10}$$

$$H_1^{(1)}(x) = \sqrt{\frac{2}{\pi x}} e^{i\left(x - \frac{3\pi}{4}\right)} \tag{A11}$$

$$H_1^{(2)}(x) = \sqrt{\frac{2}{\pi x}} e^{-i\left(x - \frac{3\pi}{4}\right)} \tag{A12}$$

$$\frac{H_1^{\wedge}(k_b R_1, k_b R)}{H_o^{\wedge}(k_b R_1, k_b R)} \sim \frac{-e^{i\left(k_b R - \frac{3\pi}{4}\right)} e^{i\left(k_b R_1 - \frac{3\pi}{4}\right)} + e^{i\left(k_b R_1 - \frac{3\pi}{4}\right)} e^{-i\left(k_b R - \frac{3\pi}{4}\right)}}{e^{i\left(k_b R - \frac{\pi}{4}\right)} e^{-i\left(k_b R_1 - \frac{3\pi}{4}\right)} - e^{i\left(k_b R_1 - \frac{3\pi}{4}\right)} e^{-i\left(k_b R - \frac{\pi}{4}\right)}} \tag{A13}$$

$$\frac{H_1^{\wedge}(k_b R_1, k_b R)}{H_o^{\wedge}(k_b R_1, k_b R)} \sim \frac{-e^{ik_b(R-R_1)} + e^{-ik_b(R-R_1)}}{ie^{ik_b(R-R_1)} - (-i)e^{-ik_b(R-R_1)}} \tag{A14}$$

$$\frac{H_1^{\wedge}(k_b R_1,\ k_b R)}{H_o^{\wedge}(k_b R_1,\ k_b R)} \sim -\frac{e^{ik_b(R-R_1)} - e^{-ik_b(R-R_1)}}{i\left[e^{ik_b(R-R_1)} + e^{-ik_b(R-R_1)}\right]}, \tag{A15}$$

where $H = R - R_1$. Eq. (A15) can be simplified to

$$\frac{H_1^{\wedge}(k_b R_1, k_b R)}{H_o^{\wedge}(k_b R_1, k_b R)} \sim -\frac{e^{ik_b H} - e^{-ik_b H}}{i\left[e^{ik_b H} + e^{-ik_b H}\right]} = -\tan(k_b H) \tag{A16}$$

The displacement of the foundation, Δ, Eq. (29a,b) can be expressed to Eq. (1) (Trifunac [2]), as follows:

$$\Delta = \frac{\left[J_1(ka) - \left(\frac{J_o(ka)}{H_o^{(1)}(ka)}\right) H_1^{(1)}(ka)\right] a_o}{\frac{ka}{2}\left[\frac{M_f}{M_s} + \frac{M_b}{M_s}\left(\frac{\tan(k_b H)}{k_b H}\right)\right] - \frac{H_1^{(1)}(ka)}{H_o^{(1)}(ka)}} \tag{A17}$$

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Glossary

a: radius of the semi-circular rigid foundation;
$\overline{a}$: radius of the semi-circular flexible foundation;
A_n: complex constants;
B: width of building;
C_β: shear wave velocity in the soil;
C_{β_b}: shear wave velocity in the building;
f_s: force per unit length due to scattered waves;
f_b: force of the shear wall acting on the foundation per unit;
H: height of the building;
$H_n^{(1)}(x)$: Hankel function of the first kind with argument x and order n;
$H_n^{(2)}(x)$: Hankel function of the second kind with argument x and order n;
i: imaginary unit;
n: subscripts used for sequence number;
$J_n(x)$: Bessel function of the first kind with argument x and order n;
k: wave number in the soil, $k=\omega^2/C_\beta$;
k_b: wave number in the building, $k_b=\omega^2/C_{\beta_b}$;
M_b: mass of shear wall per unit length;
M_f: mass of rigid foundation per unit length;
M_s: mass per unit length of soil to be replaced by the rigid foundation;
γ: angle of incidence of SH-waves;
Δ: amplitude of the displacement of the foundation;
w: amplitude of the displacement of the total wave field in the soil;
$w^{(ff)}$: amplitude of the displacement of the free-field wave in the half-space;
$w^{(B)}$: amplitude of the displacement of the wave field in the building;
$w^{(R)}$: amplitude of the displacement of the wave field in the rigid foundation;
$w^{(S)}$: amplitude of the displacement of the scattered wave field in the soil;
w^i: amplitude of the displacement of the incident plane wave in the soil;
w^r: amplitude of the displacement of the reflected plane wave in the soil;
μ: chear modulus of the soils;
μ_b: chear modulus of the shear wall;
ρ: density of the soils;
ρ_b: density of the shear wall;
ω: circular frequency of the incident *SH* waves;
δ_n: Dirac-delta function.