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Innovative Applications of O.R.

On selecting portfolio of international mutual funds using goal programming with extended factors

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ABSTRACT

This paper proposes and investigates the use of several factors for portfolio selection of international mutual funds. Three of the selected factors are specific to mutual funds, additional three factors are taken from Macroeconomics and one factor represents regional and country preferences. Each factor is treated as an objective in the multiple objective approach of goal programming. Three variants of goal programming are utilized.

Past performance of twenty mutual funds selected from ten countries in seven regions provide the data for various goal programming models used in the experiments. The resulting portfolios and their performances which seem to adequately reflect the investor's preferences are fully discussed.

The main aim of this paper is to provide a vehicle for practitioners to incorporate their preferred factors, ideal target values and aspirations into their choice of GP model to obtain their desired portfolio of international mutual funds. Another aim is to exploit the favorable findings of this paper in investigating portfolios of other financial instruments such as stocks and bonds.

1. Introduction

Mutual funds have become a popular structure for investors seeking exposure to financial markets. Gregoriou (2007) claims there are two reasons why rational investors delegate their wealth management to mutual funds. First, economies of scale which reduces wealth management costs. Second, private investors might expect that professional mutual fund managers have superior management skills, leading to positive risk-adjusted excess returns.

Portfolio management is about risk and return. Although good returns are difficult to achieve and good risk-adjusted returns can be difficult to identify. The concept of return requires no explanations other than larger returns are preferred to smaller ones. Risk is more challenging and inherently a probabilistic or statistical concept. There are various, and sometimes conflicting, notions and measures of risk. As a result, it can be difficult to measure the risk of a portfolio and determine how various investments and asset allocations affect that risk (Pearson, 2002; Travers, 2004).

The first notable work to consider risk in portfolio optimization was when Markowitz presented the well-known expected value-variance model for portfolio optimization (Gregory et al., 2011). Markowitz (1952) designed a portfolio model based on only two factors (risk and return) as they are the common ones to all investors (Markowitz, 1995). In practice, analysts use models with 'common factors', which affect all assets to a greater or lesser extent, and 'sector or regional factors', which affect only some assets within a portfolio. Identification and prediction of truly pervasive factors is an extremely difficult task. Hence, the goal should be focused on permanent and important sources of asset and portfolio risk and return, not the transitory and unimportant phenomena that occur in any given period (Sharpe, 1985).

Yu and Lee (2011) suggest that portfolio selection must consist of more criteria than only risk and return in order to provide investors with additional choices. Steuer et al. (2007) focus on investors whose purpose is to build a suitable portfolio taking additional concerns into account. Such investors would have additional stochastic and deterministic objectives that might include liquidity, number of securities in a portfolio, social responsibility, and so forth. They develop a multiple criteria Portfolio Selection formulation (Aouni, 2009, 2010; Zopounidis, 1999 and Aouni et al., 2010).

Portfolio Selection problems with risk and return optimization can be viewed as a goal programming with two objectives. However, as more realistic approach to portfolio selection problems in today's world require a number of additional factors which may include the assets specific factors, macroeconomic factors, regional preferences, etc. The use of one set of factors or another depends on the investor's attitudes and aspirations. Extra objectives representing other factors can easily be incorporated into the goal programming model.

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Goal Programming (GP) is a pragmatic tool to analyze portfolio selection problems and reach reasonable solutions in terms of the inclusion of the decision maker's preferences (Jones and Tamiz, 2010; Azmi and Tamiz, 2010). Charnes et al. (1955) developed goal programming in 1955.

The ethos of GP (a multi-objective programming technique) lies in the concept of satisficing of objectives (Tamiz et al., 1998). In fact, the two philosophical concepts that serve to best distinguish Goal Programming from conventional methods of optimization are the incorporation of flexibility in constraint functions and the adherence to the philosophy of Satisficing as oppose to Optimization. Satisficing is an old Scottish word that defines the desire to find a practical and real world solution to a problem, rather than an idealistic or optimal solution to a highly simplified model of that problem. In Goal Programming, the decision maker usually seeks a useful, practical, implementable and attainable solution rather than one satisfying the mathematician's desire (Ignizio, 1985).

Three well-known variants of GP are used in this paper, namely, Weighted, Lexicographic and MinMax GP. A brief description of these variants and their unique approach to tradeoffs between the unwanted deviations in GP is given in the next sections. The choice of these variants is purely for experimenting the new MF portfolio selection approach suggested in this paper. Other variants of GP, such as Fuzzy and Stochastic GP could also be used for further experiments.

Xidonas et al. (2011) argue that models allowing the selection of portfolios based on Multiple Criteria Decision Making (MCDM) provide the sound methodological basis to resolve the inherent multicriteria nature of portfolio selection problem. The main contribution of the MCDM framework is related to providing the potential for more realistic models to be built, by taking into account, apart of the two basic criteria of return and risk, a number of important other criteria. In addition, the MCDM framework has the advantage of taking into account the specific preferences of any particular investor, while allows for combining in a single procedure all the theoretical and practical aspects of the portfolio management theory.

This paper, therefore, investigates the incorporation of several factors (called extended factors from here on) into three variants of Goal Programming models for international portfolio of mutual funds selection and analysis. Without any loss of generality, and for the purpose of experimentation, the data is taken from several mutual funds across the globe and three well-known variants of goal programming are explored.

The remaining parts of this paper are organized as follows. Section 2 overviews the choice of extended factors for mutual funds portfolio selection as well as discussing the experimental data. Models for portfolio choice are given in Section 3. The resulting portfolios and their comparisons are provided in Section 4. The analysis of overall results is given in Section 5. Further discussion and conclusions are provided in Sections 6 and 7, respectively.

2. Extended factors choice for mutual funds portfolio selection

2.1. Factors specific to mutual funds

Three mutual fund's specific factors, namely, fund age, risk and return are used in the goal programming models developed in this paper. Carhart (1997) and Gallagher (2002) consider mutual fund age as one of the mutual funds specific factors influencing funds' performance. Whereas, Das et al. (2002) argue that mutual fund risk is one of the key factors for performance.

The notion of risk has found practical application within the science of Risk Management and Risk Control. Deciding which types of risk to mitigate is the first dilemma of a decision maker and demands considerable attention. According to Sharpe's (1966) model, the rate of return on any security is the result of two factors; a systematic component (beta) which is market related, and factors which are unique to a given security (alpha). In any application, however, concern should not only be with the alpha and beta, but with the level of uncertainty about their estimates as well.

This paper uses an equivalent risk measurement to the covariance matrix in analysing risk of each Mutual Fund and the risk of the resulting portfolios of mutual funds. Konno and Yamazaki (1991) propose an alternative to the Mean–Variance (M–V) model of Markowitz which is the Mean-Absolute Deviation (MAD) model that can be formulated and solved as a Weighted goal programming problem. The M–V model assumes normality of stock returns, while the MAD model does not make this assumption. The MAD model also minimises a measure of risk, where the measure in this case is the mean absolute deviation. Konno and Yamazaki (1991) show that L_1 risk model can be used as an alternative to Markowitz's L_2 risk model (1952).

Besides these factors, there are other ones that could easily be incorporated and investigated in the models. For example, Volkman and Wohar (1995) as well as Gallagher (2002) claim the existence of positive relations between mutual fund performance and the fund's objective (income, growth or mixed). Peterson et al. (2001) find a negative relation between fund performance and the fund's systematic risk. Other studies (example Carhart, 1997) find no relation between fund performance and one of the fund's factors like fund's size.

2.2. Factors for macroeconomics

The globalization of markets for goods, services, finance, labor and ideas reinforces the interdependence of economies and the need of their measurement on a common scale. Converting Gross Domestic Product (GDP) and its components to a common currency using Purchasing Power Parity (PPP) leads to major revisions in size and structure of world economies (IMF, 2006, 2007). A basic premise of economics is that all economic decisions are made in the face of trade-offs, due to the scarcity of resources. Investors need to know about an economy's market size, prices and productivity. The goal programming models for Mutual Fund portfolio, therefore, use three macroeconomic factors, namely, inflation, current account balance and GDP in Purchasing Power Parity as a percent of world's total. The current account balance is one of the major measures of the nature of a country's foreign trade. A current account surplus increases a country's net foreign assets by the corresponding amount, and a current account deficit does the reverse. It is called the current account because goods and services are generally consumed in the current period.

2.3. Factor for regional or country preferences

The economic indicators vary between countries and so does the viability of investment in certain countries. Previous studies (for instance, Lehmann and Modest (1987); and Otten and Schweitzer (2002)) show varied results for portfolio selection in developed countries compared to developing economies. In addition, there are some factors (such as: the volatility of markets, the size of government involvement and the extent of regulations) which distinguish mutual funds in emerging markets from their counterparts in more established markets. Therefore, the regional or country preferences influence the overall performance of a portfolio as some countries provides good investment opportunities and sustainable returns compared to others.

For the factor representing seven regional preferences in the models a score from 10 (least preferred) to 70 (most preferred) is used to reflect a preference to certain region. The decision maker (fund manager) scores each Mutual Fund's regional components on this scale to represent their desire to invest in the regions. Other scores could be assigned to reflect other preferences.

2.4. Data used in the experiments

This paper explores the Mutual Fund portfolio selection problem at an international level, where the selection involves 20 mutual funds of equities from 10 different countries representing seven regions across the world. The experiments use a constructing period of 84 weeks for portfolio selection (April 2006 to December 2007), 1680 observations, and a testing period of 16 weeks for portfolio's performance testing (January 2008 to April 2008), 320 observations. As mentioned above, seven factors are used in the models. Mutual funds specific factors data are gathered from the respective mutual fund website and financial news databases (such as those provided by the CNN finance). The macroeconomic factors data for countries are available from the World Bank and the United Nations; all of which are as of December 2007. These factors are shown in Appendix A, in columns heading RE (return), RI (risk), AG (mutual fund's age), GD (GDP in PPP), CA (current account), IN (inflation) and RG (regional preferences), respectively.

3. The goal programming models

A brief description of goal programming variants used in this paper is given in this section. The reader is referred to Jones and Tamiz (2010) for a full description of goal programming and its variants.

3.1. The general Weighted Goal Programming (WGP) model

The weighted goal programming variant allows for direct trade-offs between all unwanted deviational variables by placing them in a weighted, normalized single achievement function. Weighted goal programming is sometimes termed non-pre-emptive goal programming in the literature. If we assume linearity of the achievement function then we can represent the linear weighted goal programming by the following formulation:

$$\text{Min} \sum_{i=1}^{m} \left(\frac{\alpha_i n_i}{k_i} + \frac{\beta_i p_i}{k_i} \right)$$

$$\begin{aligned} \text{Subject to}: \; & f_i(\mathbf{x}) + n_i - p_i = b_i \quad i = 1, \ldots, m \\ & \mathbf{x} \in C_s \\ & \mathbf{x} \geqslant 0, \quad n_i, p_i \geqslant 0 \quad i = 1, \ldots, m \end{aligned}$$

where n_i is the ith negative deviational variable, α_i is the weighting factor for negative deviational variable i, p_i is the ith positive deviational variable, β_i is the weighting factor for positive deviational variable i, k_i is the normalizing factor for deviational variable i, $\mathbf{x}$ is the vector of the decision variables, $f_i(\mathbf{x})$ is the ith objective function, b_i is the ith target value, Cs is a set of hard constraints that may exist in the model.

3.2 The general Lexicographic Goal Programming (LGP) model

The vast majority of the early goal programming formulations (Charnes and Cooper, 1961; Lee, 1972) used the lexicographic goal programming variant. This is also sometimes termed 'pre-emptive' goal programming in the literature. The distinguishing feature of lexicographic goal programming is the existence of a number of *priority levels*. Each priority level contains a number of unwanted deviations from a subset of the goals to be achieved.

To formulate a generic lexicographic goal programming algebraically we define the number of priority levels as q with corresponding index $l = h_1, h_2, \ldots, h_q$ Each priority level is now a function of a subset of unwanted deviational variables which is defined as $h_l(\underline{n}, \underline{p})$. This leads to the following formulation:

$$\text{Lex Min } \mathbf{a} = \left[\left(\sum_{i \in h_1} \left(\frac{\alpha_i n_i}{k_i} + \frac{\beta_i p_i}{k_i} \right) \right), \left(\sum_{i \in h_2} \left(\frac{\alpha_i n_i}{k_i} + \frac{\beta_i p_i}{k_i} \right) \right), \ldots, \left(\sum_{i \in h_q} \left(\frac{\alpha_i n_i}{k_i} + \frac{\beta_i p_i}{k_{ii}} \right) \right) \right]$$

$$\begin{aligned} \text{Subject to}: \; & f_i(\mathbf{x}) + n_i - p_i = b_i \quad i = 1, \ldots, m \\ & \mathbf{x} \in C_s \\ & \mathbf{x} \geqslant 0, \quad n_i, p_i \geqslant 0 \quad i = 1, \ldots, m \end{aligned}$$

where $\mathbf{a}$ is the ordered vector of q priority levels being lexicographically minimized. And h_1, h_2, ..., h_q represent different sets of deviational variables penalized in their respective priority levels.

3.3. The general MinMax goal programming model

This variant of goal programming was introduced by Flavell (1976). It is known as Chebyshev goal programming, because it uses the underlying Chebyshev ($L\infty$) means of measuring distance. That is, the maximal deviation from any goal, as opposed to the sum of all deviations, is minimized. For this reason Chebyshev goal programming is sometimes termed MinMax goal programming. The underlying philosophy when using the $L\infty$ distance metric is that of balance. That is the decision maker is trying to achieve a good balance between the achievement of the set of goals as opposed to the lexicographic approach which deliberately priorities some goals over others or the weighted approach which chooses the set of goals of a very poor value in one or two of the other goals.

All of the above points lead to the conclusion that MinMax goal programming has the potential to give the most appropriate solution where a balance between the level of satisfaction is needed. This should include a large number of application areas, especially those with multiple stakeholders each of which has a preference to their own subset of goals that they regard as most important. The MinMax GP is represented by the following formulation.

$$\text{Minimise } \lambda$$

$$\begin{aligned} \text{Subject to}: \; & \frac{\alpha_i n_i}{k_i} + \frac{\beta_i p_i}{k_i} \leqslant \lambda \quad i = 1, \ldots, m \\ & f_i(\mathbf{x}) + n_i - p_i = b_i \quad i = 1, \ldots, m \\ & \mathbf{x} \in C_s \\ & \mathbf{x} \geqslant 0, \quad n_i, p_i \geqslant 0 \quad i = 1, \ldots, m \end{aligned}$$

where λ represents the maximum deviation.

In general, the decision maker's preferences can be incorporated into the GP models by:

- Penalization and weights assigned to the unwanted deviational variables. These are represented by α and β in the above models.
- The target vales (b_i) set for each objective i.
- The GP variants used.

Different deviational variable weights, set of target values and GP variants are explored in the following experiments and discussions of their relevance to the decision maker's preferences are given. For the experiments reported in this paper, the following strategy for the objective functions is adopted.

The 'desired' level of achievement for each objective function and the penalization of the respective deviational variables are:

- Return (RE) more than the target value (b_{RE}): more is better; penalize negative deviational variable.
- Risk (RI) less than the target value (b_{RI}): less is better; penalize positive deviational variable.
- Mutual fund age (AG) the same as the target value (b_{AG}): exact achievement required; penalize both deviational variables.
- GDP in PPP (GD) more than the target value (b_{GD}): more is better; penalize negative deviational variable.
- Current account (CA) the same as the target value (b_{CA}): exact achievement required; penalize both deviational variables.
- Inflation (IN) less than the target value (b_{IN}): less is better; penalize positive deviational variable.
- Regional preferences score (RG) the same as the target value (b_{RG}): exact achievement required; penalize both deviational variables.

Note – The objective functions in the GP models used in this paper have target values which are strictly positive and measured in different units of measurement. Hence the GP models are ideally suited for the deployment of percentage normalization (Tamiz and Jones, 1997), whereby the normalized factor (k_i) for each objectives' unwanted deviational variables is the corresponding target value b_i (i.e. for the return objective where $i = RE$, the normalized factor $k_{RE} = b_{RE}$ is used for the unwanted (negative) deviational variable (n_{RE})). In general, techniques have been developed to enable decision makers to incorporate their preferences into goal programming and its variants (Aouni et al., 2009).

3.4. The normalized WGP model for MF

The following normalized WGP model is developed for setting up portfolios of mutual funds using the constructing period data (1680 observations), for the 20 mutual funds used in this paper:

$$\text{Min}\left(\frac{\alpha_{RE}n_{RE}}{b_{RE}}+\frac{\alpha_{AG}n_{AG}}{b_{AG}}+\frac{\alpha_{GD}n_{GD}}{b_{GD}}+\frac{\alpha_{CA}n_{CA}}{b_{CA}}+\frac{\alpha_{RG}n_{RG}}{b_{RG}}+\frac{\beta_{RI}p_{RI}}{b_{RI}}+\frac{\beta_{AG}p_{AG}}{b_{AG}}+\frac{\beta_{CA}p_{CA}}{b_{CA}}+\frac{\beta_{IN}p_{IN}}{b_{IN}}+\frac{\beta_{RG}p_{RG}}{b_{RG}}\right)$$

$$\text{Subject to}: \sum_{j=1}^{20} RE_jX_j + n_{RE} - p_{RE} = b_{RE}$$

$$\sum_{j=1}^{20} RI_jX_j + n_{RI} - p_{RI} = b_{RI}$$

$$\sum_{j=1}^{20} AG_jX_j + n_{AG} - p_{AG} = b_{AG}$$

$$\sum_{j=1}^{20} GD_jX_j + n_{GD} - p_{GD} = b_{GD}$$

$$\sum_{j=1}^{20} CA_jX_j + n_{CA} - p_{CA} = b_{CA}$$

$$\sum_{j=1}^{20} IN_jX_j + n_{IN} - p_{IN} = b_{IN}$$

$$\sum_{j=1}^{20} RG_jX_j + n_{RG} - p_{RG} = b_{RG}$$

$$\sum_{j=1}^{20} X_j = 1$$

$$X_j \geqslant 0 \quad j = 1,\ldots,20$$

$$\textit{All negative and positive deviations} \geqslant 0$$

where X_j is the proportion of funds invested in the jth mutual fund. The experiments are carried out according to the following sequence of models and data:

3.4.1. The WGP baseline model

The WGP baseline model has weights of 1 (i.e. there are no preferred weights) for each of the seven factors (objectives). The target values are reasonably set close to their averages for each factor as shown in Table 1.

3.4.2. The WGP models with different weights

The WGP models of different deviational variable weights are shown in Table 2.

In Table 2, the highlighted part of each column represents the weight changes from the baseline model. W1, W2 and W3 represent possible preferences in investment decision making based on the seven available factors. W1 gives high weights for return, risk and fund age (mutual funds specific factors), while W2 gives the high weights for the macroeconomic factors (the share of the country in the world GDP in PPP, current account balance as a percent of GDP and inflation rate). W3 gives high weighting for both risk and regional preferences.

3.4.3. The WGP models with different target values

The WGP models of different target values are shown in Table 3.

The highlighted part of each column represents the target values changes from the baseline model. W4, W5, W6 and W7 represent another type of preferences in investment decision making.

An investor might decide that the average for mutual funds risk is not good enough and hence seeks to reach a risk level which is

Table 1
The baseline WGP model parameters. The WGP baseline model attaches the value of 1 to the unwanted deviational variables weights as shown in the second column. The assigned target values of the model is shown in column number three. The average value of the 20 mutual funds for all the factors is given in the last column as a guidance on deciding the target values in the fourth column. These target values are ultimately assigned by a portfolio's decision maker using their knowledge of the global financial markets.

The extended factors	Deviational variables weights	The target values (b_i)	The averages
RE: Return (%)	$\alpha_{RE} = 1$, $\beta_{RE} = 0$	$b_{RE} = 0.015$	0.010
RI: Risk (measured by mean-absolute deviation)	$\alpha_{RI} = 0$, $\beta_{RI} = 1$	$b_{RI} = 0.023$	0.024
AG: MF Age (years)	$\alpha_{AG} = 1$, $\beta_{AG} = 1$	$b_{AG} = 13.0$	12.0
GD: GDP/PPP (% of world's total)	$\alpha_{GD} = 1$, $\beta_{GD} = 0$	$b_{GD} = 0.07$	0.06
CA: Current account balance (% of the GDP)	$\alpha_{CA} = 1$, $\beta_{CA} = 1$	$b_{CA} = 0.05$	0.04
IN: Inflation (% annual change)	$\alpha_{IN} = 0$, $\beta_{IN} = 1$	$b_{IN} = 0.03$	0.04
RG: Regional preferences (scored from 10 to 70 points)	$\alpha_{RG} = 1$, $\beta_{RG} = 1$	$b_{RG} = 35.0$	35.0

Note: The averages in the fourth column of Table 1 are the simple mean for each factor's value of the 20 mutual funds. For example the average for the risk factor is calculated as:

$$\sum_{j=1}^{20} RI_j/20$$

These averages are calculated and used as an indication for setting up the target values for each factor. Each Mutual Fund's risk, RI_j, is measured utilizing the MAD approach for the construction period of 84 weeks of data:

$$RI_j = \sum_{t=1}^{84} |r_{jt} - \bar{r}_j| j = 1,\ldots 20$$

In general, the resulting portfolios of the GP models developed in this paper are influenced by all the seven factor's target values and the penalization of their unwanted deviational variables. In the WGP Baseline model, for example, the portfolio is desired to include Mutual Funds with lower than average risk of 0.024 (2nd row, 4th column of Table 1). The resulting portfolios' return, risk and cost are then calculated for comparing their performances against each other (see Section 4).

Table 2
The WGP models with different weightings for the unwanted deviational variables. There are three different WGP models developed besides the baseline model based on assigning different weightings for the unwanted deviational variables. In W1, high weights are attached to the mutual funds specific factors relative to other factors, where 10, 5 and 2 weightings are assigned to risk, return and mutual fund age factors, respectively. In Return (RE), the model penalizes the negative deviational variable, as the more is the better, whereas in risk (RI) the model penalizes the positive deviational variable, as the less is the better. In AG, the model penalizes both deviational variables, as exact achievement is preferred. In W2, the high weights are attached to the macroeconomic factors, where 10, 5 and 2 weightings are attached to GDP in PPP, current account and inflation, respectively. Another scenario for portfolio selection models is illustrated in W3, where high weightings are assigned to both factors of risk and regional preferences.

Extended factors	Baseline deviational variable weights	W1	W2	W3
RE	$\alpha_{RE} = 1$, $\beta_{RE} = 0$	$\alpha_{RE} = 5$, $\beta_{RE} = 0$	$\alpha_{RE} = 1$, $\beta_{RE} = 0$	$\alpha_{RE} = 1$, $\beta_{RE} = 0$
RI	$\alpha_{RI} = 0$, $\beta_{RI} = 1$	$\alpha_{RI} = 0$, $\beta_{RI} = 10$	$\alpha_{RI} = 0$, $\beta_{RI} = 1$	$\alpha_{RI} = 0$, $\beta_{RI} = 5$
AG	$\alpha_{AG} = 1$, $\beta_{AG} = 1$	$\alpha_{AG} = 2$, $\beta_{AG} = 2$	$\alpha_{AG} = 1$, $\beta_{AG} = 1$	$\alpha_{AG} = 1$, $\beta_{AG} = 1$
GD	$\alpha_{GD} = 1$, $\beta_{GD} = 0$	$\alpha_{GD} = 1$, $\beta_{GD} = 0$	$\alpha_{GD} = 10$, $\beta_{GD} = 0$	$\alpha_{GD} = 1$, $\beta_{GD} = 0$
CA	$\alpha_{CA} = 1$, $\beta_{CA} = 1$	$\alpha_{CA} = 1$, $\beta_{CA} = 1$	$\alpha_{CA} = 5$, $\beta_{CA} = 5$	$\alpha_{CA} = 1$, $\beta_{CA} = 1$
IN	$\alpha_{IN} = 0$, $\beta_{IN} = 1$	$\alpha_{IN} = 0$, $\beta_{IN} = 1$	$\alpha_{IN} = 0$, $\beta_{IN} = 2$	$\alpha_{IN} = 0$, $\beta_{IN} = 1$
RG	$\alpha_{RG} = 1$, $\beta_{RG} = 1$	$\alpha_{RG} = 1$, $\beta_{RG} = 1$	$\alpha_{RG} = 1$, $\beta_{RG} = 1$	$\alpha_{RG} = 5$, $\beta_{RG} = 5$

Table 3
The WGP models with changed target values for some factors. This table illustrates different WGP models based on changing the target values for some factors. W4 represents the scenario where an investor is focusing on the risk factor and needs to achieve a challenging target value, given the normal values for such a factor. W5 focuses on GDP in PPP. W6 and W7 focus on regional preferences factor, where a target value representing the score of a certain region is assigned to reflect the preference towards that region.

Factors' target values	Baseline target values	W4	W5	W6	W7
b_{RE}=	0.015	0.015	0.015	0.015	0.015
b_{RI}=	0.03	0.017	0.023	0.023	0.023
b_{AG}=	13.0	13.0	13.0	13.0	13.0
b_{GD}=	0.07	0.07	0.20	0.07	0.07
b_{CA}=	0.05	0.05	0.05	0.05	0.05
b_{IN}=	0.03	0.03	0.03	0.03	0.03
b_{RG}=	35	35	35	70	10

significantly lower, say 0.017 instead of 0.023, which is the case for W4. In W5 the target value for a country's share in the world GDP in PPP is changed from 0.07 to 0.20. This might be the case for an investor who decides on investing in a country (through its mutual funds) based on its share in the world GDP and seeks the highest share across the globe.

Assuming that an investor is interested in specific region according to his/her own analysis and risk tolerance, models W6 and W7 are set up to represent two different regional preferences relative to the baseline case of 35. The aim for W6 is to invest in the North America region, which has a score of 70, while the aim in W7 is to invest in the Middle East and North Africa region, which has the score of 10.

3.5. The LGP model for MF

The achievement function of the LGP model for this section consists of 2 priority levels which are lexicographically minimized subject to the same objectives of the WGP model. The weights for the unwanted deviational variables in LGP models are kept the same as in WGP models for the purpose of comparing the performance of the resulting portfolios. The unwanted deviational variables are further grouped in two priority levels, indicating that achieving the objectives in the first priority level is infinitely more important than achieving the objectives in the second priority level. The LGP experiments are carried out according to the following sequence of models:

3.5.1. The baseline model

Table 4 illustrates the parameters for the LGP baseline model:

In Table 4, the highlighted part of each column represents the priority levels with the weights on their deviational variables.

The LGP for the baseline model is as follows:

$$\text{Lex Min } a = \left[\frac{n_{RE}}{0.015}, \frac{p_{RI}}{0.023}, \frac{n_{AG}+p_{AG}}{13}\right], \left[\frac{n_{GD}}{0.07}, \frac{n_{CA}+p_{CA}}{0.05}, \frac{p_{IN}}{0.03}, \frac{n_{RG}+p_{RG}}{35}\right]$$

$$\begin{aligned} \text{Subject to}: &\sum_{j=1}^{20} RE_j X_j + n_{RE} - p_{RE} = 0.015 \\ &\sum_{j=1}^{20} RI_j X_j + n_{RI} - p_{RI} = 0.023 \\ &\sum_{j=1}^{20} AG_j X_j + n_{AG} - p_{AG} = 13 \\ &\sum_{j=1}^{20} GD_j X_j + n_{GD} - p_{GD} = 0.07 \\ &\sum_{j=1}^{20} CA_j X_j + n_{CA} - p_{CA} = 0.05 \\ &\sum_{j=1}^{20} IN_j X_j + n_{IN} - p_{IN} = 0.03 \\ &\sum_{j=1}^{20} RG_j X_j + n_{RG} - p_{RG} = 35 \\ &\sum_{j=1}^{20} X_j = 1 \\ &X_j \geqslant 0 \quad j = 1, \ldots, 20 \\ &\textit{All negative and positive deviations} \geqslant 0 \end{aligned}$$

3.5.2. The LGP models with different weights and priority levels

Three experiments are conducted with different weightings for the unwanted deviational variables and different priority levels as shown in Table 5.

Table 4
The baseline LGP model The LGP baseline model in this paper assigns the priority level 1 to the mutual funds specific factors, however, with weightings of 1 for each of their unwanted deviational variables. Whereas, priority level two is assigned to the macroeconomic factors and regional preferences (with weightings of 1 assigned to their unwanted deviational variables).

The extended factors	Priority levels of the baseline LGP model		The baseline target values
	Priority level 1 [h_1]	Priority level 2 [h_2]	
RE	$\alpha_{RE} = 1$, $\beta_{RE} = 0$		$b_{RE} = 0.015$
RI	$\alpha_{RI} = 0$, $\beta_{RI} = 1$		$b_{RI} = 0.023$
AG	$\alpha_{AG} = 1$, $\beta_{AG} = 1$		$b_{AG} = 13.0$
GD		$\alpha_{GD} = 1$, $\beta_{GD} = 0$	$b_{GD} = 0.07$
CA		$\alpha_{CA} = 1$, $\beta_{CA} = 1$	$b_{CA} = 0.05$
IN		$\alpha_{IN} = 0$, $\beta_{IN} = 1$	$b_{IN} = 0.03$
RG		$\alpha_{RG} = 1$, $\beta_{RG} = 1$	$b_{RG} = 35.0$

The priority levels and the weightings of the unwanted deviational variables for LGP models L1, L2 and L3 are shown in Table 5. These priority levels and weightings are set out to reflect the decision maker preferences. For instance, L1 gives higher weights than the baseline model for return, risk and fund age (mutual funds specific factors) within priority level 1.

3.5.3. The LGP models with different target values

The LGP models of different factor's target values are shown in Table 6.

L4, L5, L6 and L7 represent the cases where an investor might be interested in higher or lower target values for some of the factors within the priority level set in the baseline model. L4 and L5 change the target value for RI and GD, while L6 and L7 represent two different regional preferences relative to the baseline case.

3.6. The MinMax goal programming model for MF

The following MinMax GP model is used in this paper:

$$
\begin{aligned}
&\text{Minimize } \lambda\\
&\text{Subject to: } \frac{\alpha_{RE}n_{RE}}{b_{RE}} - \lambda \leqslant 0\\
&\frac{\beta_{RI}p_{RI}}{b_{RI}} - \lambda \leqslant 0\\
&\frac{\alpha_{AG}n_{AG}}{b_{AG}} + \frac{\beta_{AG}p_{AG}}{b_{AG}} - \lambda \leqslant 0\\
&\frac{\alpha_{GD}n_{GD}}{b_{GD}} - \lambda \leqslant 0\\
&\frac{\alpha_{CA}n_{CA}}{b_{CA}} + \frac{\beta_{CA}p_{CA}}{b_{CA}} - \lambda \leqslant 0\\
&\frac{\beta_{IN}p_{IN}}{b_{IN}} - \lambda \leqslant 0\\
&\frac{\alpha_{RG}n_{RG}}{b_{RG}} + \frac{\beta_{RG}p_{RG}}{b_{RG}} - \lambda \leqslant 0\\
&\sum_{j=1}^{20} RE_jX_j + n_{RE} - p_{RE} = b_{RE}\\
&\sum_{j=1}^{20} RI_jX_j + n_{RI} - p_{RI} = b_{RI}\\
&\sum_{j=1}^{20} AG_jX_j + n_{AG} - p_{AG} = b_{AG}\\
&\sum_{j=1}^{20} GD_jX_j + n_{GD} - p_{GD} = b_{GD}\\
&\sum_{j=1}^{20} CA_jX_j + n_{CA} - p_{CA} = b_{CA}\\
&\sum_{j=1}^{20} IN_jX_j + n_{IN} - p_{IN} = b_{IN}\\
&\sum_{j=1}^{20} RG_jX_j + n_{RG} - p_{RG} = b_{RG}\\
&\sum_{j=1}^{20} X_j = 1\\
&X_j \geqslant 0 \quad j = 1, \ldots, 20\\
&\textit{All negative and positive deviations and } \lambda \geqslant 0
\end{aligned}
$$

The MinMax GP experiments are carried out according to the scenarios shown in Table 7:

Table 7 shows the different MinMax GP models, where for comparison purposes the weights for the unwanted deviational variables in M1, M2 and M3 are the same as in W1, W2 and W3, while the target values in M4, M5, M6 and M7 are the same as the target values in W4, W5, W6 and W7, respectively.

4. The resulting portfolios and their comparisons

The various GP models utilized in this paper have selected interesting portfolios in terms of the factors considered. In order to compare and analyze the constructed portfolios, the following basic investment criteria are computed for each selected portfolio:

- Return (the average return).
- Risk (utilizing MAD model).
- Cost: there is not enough data to calculate the actual costs involved when deciding to invest in a certain mutual fund located in a country. However, generally, the more mutual funds included in a portfolio, the higher the cost both for setting up and rebalancing. This is even more compelling for investments in different countries and regions (due to differences in financial, technological, political, economic, exchange rates, social and legal conditions across countries), which is the case for this paper.

Furthermore, the models are analyzed in terms of their ability to achieve the targets for all the factors considered.

Note: The portfolio's risk is calculated for the testing period of 16 weeks utilizing the MAD approach as follows:

$$\sum_{t=1}^{16}\sum_{j=1}^{20}|r_{jt} - \bar{r}_j|X_j$$

4.1. Results of WGP models

The results are shown in details in Appendix B for the WGP-constructed portfolios, namely; return, risk, selected mutual funds (name, country and proportions) as well as percentage of achievement for each target value of the seven factors.

There are eight WGP models examined in this paper. The WGP baseline model selects a portfolio of six mutual funds from five countries, with a return of −0.003 (the negative return here is indicative of the general downward trend of the equities market during the experimentation period) and risk of 0.393. The model has 100% achievement for 5 of the factors involved, while the percentage achievements of the remaining factors are 90% for RE and 102% for IN.

W1 selects six mutual funds from five countries and its portfolio has a return and risk of −0.003 and 0.405 respectively. W1 produces the same return compared to the baseline model but with higher risk. The percentage achievement for 5 of the factors involved is 100%, while its 114% for GD and 115% for RG. Although higher weights were assigned to return and risk in W1, the model's portfolio was not able to have the same achievement rates for some of the factors and did not generate higher return or lower risk.

W2 selects exactly the same portfolios picked by the baseline model with similar achievement rates as well as risk and return levels. This result implies that even though W2 assigns higher weighting for macroeconomic factors, its model could not improve the portfolio selection process in terms of risk, return and achievement levels for other factors. It could be implied that the macroeconomic factors were already taken care of in the baseline model (although with equal weights) and there was no room for further improvements.

W3 assigns higher weighting for risk and regional preferences, in which it selects seven mutual funds from five countries and its portfolio has a return and risk of −0.003 and 0.405 respectively.

Table 5
The LGP models with different weightings for the unwanted deviational variables. Three LGP models are illustrated, where L1 attaches priority level one as well as higher weightings for the mutual funds specific factors (RE, RI, AG). L2 assigns priority level one and higher weights for the macroeconomic and regional preferences factors (GD, CA, IN, RG). The risk and regional preferences factors (RI, RG) are assigned the first priority level and the higher weightings in model L3.

The extended factors	Priority levels for L1		Priority levels for L2		Priority levels for L3	
	Priority Level 1 [h_1]	Priority Level 2 [h_2]	Priority Level 1 [h_1]	Priority Level 2 [h_2]	Priority Level 1 [h_1]	Priority Level 2 [h_2]
RE	$\alpha_{RE} = 5,\ \beta_{RE} = 0$			$\alpha_{RE} = 1,\ \beta_{RE} = 0$		$\alpha_{RE} = 1,\ \beta_{RE} = 0$
RI	$\alpha_{RI} = 0,\ \beta_{RI} = 10$			$\alpha_{RI} = 0,\ \beta_{RI} = 1$	$\alpha_{RI} = 0,\ \beta_{RI} = 5$	
AG	$\alpha_{AG} = 2,\ \beta_{AG} = 2$			$\alpha_{AG} = 1,\ \beta_{AG} = 1$		$\alpha_{AG} = 1,\ \beta_{AG} = 1$
GD		$\alpha_{GD} = 1,\ \beta_{GD} = 0$	$\alpha_{GD} = 10,\ \beta_{GD} = 0$			$\alpha_{GD} = 1,\ \beta_{GD} = 0$
CA		$\alpha_{CA} = 1,\ \beta_{CA} = 1$	$\alpha_{CA} = 5,\ \beta_{CA} = 5$			$\alpha_{CA} = 1,\ \beta_{CA} = 1$
IN		$\alpha_{IN} = 0,\ \beta_{IN} = 1$	$\alpha_{IN} = 0,\ \beta_{IN} = 2$			$\alpha_{IN} = 0,\ \beta_{IN} = 1$
RG		$\alpha_{RG} = 1,\ \beta_{RG} = 1$	$\alpha_{RG} = 1,\ \beta_{RG} = 1$		$\alpha_{RG} = 5,\ \beta_{RG} = 5$	

Table 6
The LGP models of changed target values for some factors. L4, L5, L6 and L7 represent the LGP models where the target values for some of the factors are changed to challenge the respective models to overperform their averages set in the baseline model.

The extended factors	Baseline LGP model			L4	L5	L6	L7
	Priority Level 1 [h_1]	Priority Level 2 [h_2]	b_i				
RE	$\alpha_{RE} = 1,\ \beta_{RE} = 0$		$b_{RE} = 0.015$	$b_{RE} = 0.015$	$b_{RE} = 0.015$	$b_{RE} = 0.015$	$b_{RE} = 0.015$
RI	$\alpha_{RI} = 0,\ \beta_{RI} = 1$		$b_{RI} = 0.03$	$b_{RI} = 0.017$	$b_{RI} = 0.023$	$b_{RI} = 0.023$	$b_{RI} = 0.023$
AG	$\alpha_{AG} = 1,\ \beta_{AG} = 1$		$b_{AG} = 13.0$	$b_{AG} = 13.0$	$b_{AG} = 13.0$	$b_{AG} = 13.0$	$b_{AG} = 13.0$
GD		$\alpha_{GD} = 1,\ \beta_{GD} = 0$	$b_{GD} = 0.07$	$b_{GD} = 0.07$	$b_{GD} = 0.20$	$b_{GD} = 0.07$	$b_{GD} = 0.07$
CA		$\alpha_{CA} = 1,\ \beta_{CA} = 1$	$b_{CA} = 0.05$	$b_{CA} = 0.05$	$b_{CA} = 0.05$	$b_{CA} = 0.05$	$b_{CA} = 0.05$
IN		$\alpha_{IN} = 0,\ \beta_{IN} = 1$	$b_{IN} = 0.03$	$b_{IN} = 0.03$	$b_{IN} = 0.03$	$b_{IN} = 0.03$	$b_{IN} = 0.03$
RG		$\alpha_{RG} = 1,\ \beta_{RG} = 1$	$b_{RG} = 35.0$	$b_{RG} = 35.0$	$b_{RG} = 35.0$	$b_{RG} = 70.0$	$b_{RG} = 10.0$

Table 7
The MinMax GP experimentations' models. This table shows the group of MinMax GP models with different weightings for the unwanted deviational variables (M1, M2 and M3) as well as the group of models with different target values (M4, M5, M6 and M7).

The extended factors	**First:** The baseline MinMax GP model		**Second:** The MinMax GP models after changing some deviational variable weights			**Third:** The MinMax GP models after changing some factors' target values				
	Deviational variable weights	(b_i)	Deviational variable weights			b_i	Target values			
			M1	M2	M3		M4	M5	M6	M7
RE	$\alpha_{RE} = 1,\ \beta_{RE} = 0$	$b_{RE} = 0.015$	$\alpha_{RE} = 5,\ \beta_{RE} = 0$	$\alpha_{RE} = 1,\ \beta_{RE} = 0$	$\alpha_{RE} = 1,\ \beta_{RE} = 0$	b_{RE}	0.015	0.015	0.015	0.015
RI	$\alpha_{RI} = 0,\ \beta_{RI} = 1$	$b_{RI} = 0.023$	$\alpha_{RI} = 0,\ \beta_{RI} = 10$	$\alpha_{RI} = 0,\ \beta_{RI} = 1$	$\alpha_{RI} = 0,\ \beta_{RI} = 5$	b_{RI}	0.017	0.023	0.023	0.023
AG	$\alpha_{AG} = 1,\ \beta_{AG} = 1$	$b_{AG} = 13$	$\alpha_{AG} = 2,\ \beta_{AG} = 2$	$\alpha_{AG} = 1, \beta_{AG} = 1$	$\alpha_{AG} = 1,\ \beta_{AG} = 1$	b_{AG}	13.0	13.0	13.0	13.0
GD	$\alpha_{GD} = 1,\ \beta_{GD} = 0$	$b_{GD} = 0.07$	$\alpha_{GD} = 1,\ \beta_{GD} = 0$	$\alpha_{GD} = 10, \beta_{GD} = 0$	$\alpha_{GD} = 1,\ \beta_{GD} = 0$	b_{GD}	0.07	0.20	0.07	0.07
CA	$\alpha_{CA} = 1,\ \beta_{CA} = 1$	$b_{CA} = 0.05$	$\alpha_{CA} = 1,\ \beta_{CA} = 1$	$\alpha_{CA} = 5, \beta_{CA} = 5$	$\alpha_{CA} = 1,\ \beta_{CA} = 1$	b_{CA}	0.05	0.05	0.05	0.05
IN	$\alpha_{IN} = 0,\ \beta_{IN} = 1$	$b_{IN} = 0.03$	$\alpha_{IN} = 0,\ \beta_{IN} = 1$	$\alpha_{IN} = 0,\ \beta_{IN} = 2$	$\alpha_{IN} = 0,\ \beta_{IN} = 1$	b_{IN}	0.03	0.03	0.03	0.03
RG	$\alpha_{RG} = 1,\ \beta_{RG} = 1$	$b_{RG} = 35$	$\alpha_{RG} = 1,\ \beta_{RG} = 1$	$\alpha_{RG} = 1, \beta_{RG} = 1$	$\alpha_{RG} = 5,\ \beta_{RG} = 5$	b_{RG}	35	35	70	10

W3 produces the same return compared to the baseline model but with higher risk. The percentage achievements are 100% for 6 of the factors involved and 88% for RE.

The above results mean also that the target values set for each of the seven factors are realistic and achievable to the extent that there was no room for further improvement. Other possibilities, that an investor might be interested to explore, are setting challenging target values to generate portfolios that could beat the averages. Such possibilities are explored with W4, W5, W6 and W7 experiments.

W4 selects five mutual funds (from five countries) with an average return of −0.004 and risk level of 0.339, while W5 provides −0.004 return associated with 0.443 risk in a portfolio of six mutual funds. When compared to the baseline model, W4 generates a lower risk but with lower return too. W5 has a lower return with higher risk, compared to the baseline portfolio.

W6 selects three mutual funds with a return of −0.007 (less than the baseline portfolio return) and a risk of 0.368 (slightly lower than the risk of the baseline portfolio). The aim in W6 is to increase investing in the North America region but that preference seems to adversely impact the return of the selected portfolio.

W7, in contrary to W6, has a return of +0.001 and a risk of 0.317 which are both better than the baseline portfolio. W7's aim is to invest more in the Middle East and North Africa region. The resulting portfolio generates better return and risk levels, but this selection adversely affects the achievement levels for some factors. Specifically, W7 has 100% achievement for 3 of the factors involved, while the percentage achievements of the remaining factors are 43% for RE, 26% for GD, 124% for IN, and 135% for RG.

W7 appears to be the only experiment from the WGP experiments that has a major underachievement in some factors. It appears that a specific preference to one region could mean sacrificing some factors. The decision maker should decide in such case about the importance and the priority of each factor in order to be able to sacrifice some factors if they are less important than others. In fact, an investor could decide that the achievement of some factors is infinitely more important than achieving the others. Such decisions are investigated using LGP models.1

4.2. Results of LGP models

The results of LGP models' experiments are shown in details in Appendix C. There are eight LGP models, where the baseline model generates −0.003 return and 0.405 risk through selecting six mutual funds from five countries. The baseline's percentage achievement is 100% for all of the three factors (mutual funds specific factors) included in priority level 1.

L1 selects the same mutual funds as in the baseline portfolio, with the same risk and return. This result is expected since L1 has exactly the same priority levels as in the baseline, but with higher weights for the unwanted deviational variables in priority level 1. L2 selects a portfolio of seven mutual funds and achieves similar return (−0.003) and similar risk (0.405) compared to the baseline portfolio return and risk. However, it has higher achieving levels for most of the factors. The percentage achievement is 100% for six factors compared to five factors with 100% achievement in the baseline model. This is interesting since the baseline model gives the priority level 1 to mutual funds specific factors, while L2 model gives the priority level 1 and higher weights to macroeconomics.

L3 could not improve in terms of return and risk compared to L2, although the priority levels are different. L4 selects three mutual funds with a lower return (−0.006) compared to the baseline return, but it is associated with a lower risk too compared to the baseline model and at expense of lower percentage of achievements for some factors.

L5 produces a portfolio of six mutual funds, generating lower return and higher risk compared to the baseline portfolio. L6 produces a smaller portfolio (compared to the baseline) of three mutual funds. L7 generates exactly the same return and risk levels as in baseline model with the same percentage achievements for 6 of the factors. The percentage achievement changes only for the factor whose target value is changed.

4.3. Results of MinMax GP models

The results for 8 MinMax GP models are illustrated in details in Appendix D, where the baseline model selects 6 mutual funds with a portfolio return of −0.003 and risk of 0.339. This portfolio gives a reasonable balance to achieving the seven factors involved with percentage achievements of 104% for RI, IN and RG, 96% for RE, AG and CA, and 103% for GD.

M1 generates the same return with higher risk (with a portfolio of 6 mutual funds) even though it has higher weights for risk and return than baseline model. M2 selects a portfolio of the same mutual funds as in the baseline portfolio, however with different proportions as well as with higher risk for the same level of return.

In M3, higher weights for risk and regional preferences are assigned and it provides a portfolio that is characterized with slightly higher risk at a slightly higher return level (consisting of six mutual funds), compared to the baseline model.

Furthermore, setting a challenging target value for the risk in M4 provides a portfolio that has an interestingly higher risk with the same level of return (however with a constituent of five mutual funds) compared to the baseline portfolio. This is at the expense of percentage achievement for other macroeconomic factors.

In M5, the target is to achieve more in one of the macroeconomic factors, namely GD, which produces a portfolio with lower return and higher risk compared to the baseline portfolio. M6 gives a particular preference for the North America region where the model selects three mutual funds, one is from the preferred region, as the model tries to put together a portfolio that achieves all the targeted values of different factors. This implies that the model could not find suitable mutual funds in one region to select from and complements the choice with other regions' mutual funds to achieve the overall balance between factors with reasonable return and risk.

M7, on the other hand, selects four mutual funds in which 3 of them are strictly from the targeted region (the MENA region) with an average return of −0.001 and a risk of 0.317, which are significantly better than the baseline results.

5. Analysis of overall results

5.1. Relevant portfolios' comparison

Table 8 illustrates the relevant portfolios' comparison. It gives the return, risk, and the number of mutual funds for all the models experimented in this paper. The MinMax GP baseline portfolio has the lowest risk compared to the LGP and WGP baseline portfolios. M1 has slightly better risk than W1 and L1, while it has similar return.

Comparing W1, L1 and M1 (which are the experiments that give higher weight or priority to mutual funds specific factors) with their respective baseline models, there are no improvements in returns, which are associated with higher risks.

The models that give more weights or higher priority level to the macroeconomic factors are W2, L2 and M2, where the return is almost the same amongst the three of them, while W2 has the lowest risk compared to L2 and M2. Model M3 selects a portfolio that has a better return and risk compared with W3 and L3.

W4, L4 and M4 have a target value of 0.017 for risk which is lower than the average in this experiment. The W4 and L4 models succeed in reaching the lowest risk for the selected portfolio compared to most of the experimented models in this paper. M4, however, has not produced a lower risk compared to its baseline and most other MinMax GP models.

W5, L5 and M5 give a high target value for the GD factor and select a portfolio from 3 to 6 mutual funds. M5 portfolio has a lower return compared with W5 and L5, but with the lowest risk amongst the three of them.

W6, L6 and M6 try to select their portfolios based on a high preference to the North America region. M6 has the highest return, while W6 and L6 have the same return (although lower than the M6's return) with the lowest risk in both W6 and L6 compared to M6. The models' result and the aggregated comparison seem to imply that the preference to such a region does not generate reasonable performance for the selected portfolios.

W7 and M7, on the other hand, generate better returns and risk compared to their respective baseline models and in fact compared with many of the other models. However, L7 produces the same return and risk of its baseline model. This result seems to imply that the preference given to that region is effective as the models could find a good fit for all the factors, while selecting a portfolio that generates better return and risk levels (or at least the same level as the respective baseline, as in L7).

Finally, comparing all the portfolios selected in this paper with a hypothetical portfolio (consisting of 20 mutual funds equally weighted, and called EW in Table 8) shows that using any GP model for selecting international mutual funds is significantly better. GP models select fewer numbers of mutual funds for the portfolios. The GP portfolios perform better not only in terms of risk adjusted returns, but also in terms of the transaction and investment costs.

5.2. Redundancy issues in LGP

Redundancy of goals in Lexicographic goal programming is not only a theoretical possibility but a practical problem. Usually redundancy occurs due to one or more of the following reasons (Tamiz et al., 1998):

- Fixing the target values equal to or close to the ideal values (optimistic values in this context).

Table 8
The return, risk and number of mutual funds in Goal Programming models' portfolios. All the portfolios selected in this paper's experiments are compared to each other using the last three columns in this table. The various goal programming variants used are illustrated in the same table, namely, weighted, lexicographic and MinMax goal programming variants. Each GP variant has 8 models, which are the baseline model besides three models with changed weightings and four models with changes in target values.

Goal programming models	Experiments		Portfolios analysis criteria		
			Return	Risk (MAD)	Number of MFs
Weighted GP models	Baseline model		−**0.003**	**0.393**	**6**
	Models changing the deviational variable weights	W1: High weights for RE, RI & AG	−**0.003**	**0.405**	**6**
		W2: High weights for GD, CA & IN	−**0.003**	**0.393**	**6**
		W3: High weights for RI & RG	−**0.003**	**0.405**	**7**
	Models changing some factor's target values	W4: Changed target for RI	−**0.004**	**0.339**	**5**
		W5: Changed target for GD	−**0.004**	**0.443**	**6**
		W6: Changed target for RG(70)	−**0.007**	**0.368**	**3**
		W7: Changed target for RG(10)	**0.001**	**0.317**	**4**
Lexicographic GP models	Baseline model		−**0.003**	**0.405**	**6**
	Models changing the deviational variable weights	L1 RE, RI & AG are in the 1st priority level	−**0.003**	**0.405**	**6**
		L2 GD, CA & IN are in the 1st priority level	−**0.003**	**0.405**	**7**
		L3 RI & RG in the 1st priority level	−**0.003**	**0.405**	**7**
	Models changing some factor's target values	L4 Changed target for RI	−**0.006**	**0.353**	**3**
		L5 Changed target for GD	−**0.004**	**0.443**	**6**
		L6 Changed target for RG (70)	−**0.007**	**0.368**	**3**
		L7 Changed target for RG (10)	−**0.003**	**0.405**	**6**
MinMax (Chebyshev) GP models	Baseline model		−**0.003**	**0.399**	**6**
	Models changing the deviational variable weights	M1 High weights for RE, RI & AG	−**0.003**	**0.391**	**6**
		M2 high weights for GD, CA & IN	−**0.003**	**0.404**	**6**
		M3 high weights for RI & RG	−**0.002**	**0.397**	**6**
	Models changing some factor's target values	M4 change the target for RI	−**0.003**	**0.414**	**5**
		M5 change the target for GD	−**0.007**	**0.426**	**3**
		M6 change the target for RG (70)	−**0.005**	**0.484**	**3**
		M7 change the target for RG (10)	−**0.001**	**0.317**	**4**
–	EW (hypothetical equally-weighted portfolio)		−**0.004**	**0.020**	**20**

- Setting an excessive number of priority levels, especially compared to the number of goals.
- Including many two-sided goals in the achievement function. These are goals in which both deviational variables (negative and positive) are penalized.

The redundancy problem does not occur, however, in this paper's experiments of LGP as the priority levels, target values and penalization of the unwanted deviational variables, are all set realistically and reasonably. However, if investors or decision makers want to set optimistic target values or assign many priority levels based on their preferences, the redundancy issue may occur and should be tested for (Tamiz et al., 1998; Jones and Tamiz, 2010).

In fact, upon carrying further experiments with the target value for RE set at 0.02 (which is 100% higher than the average and 50% higher than the target value for experiments reported in this paper); the resulting solutions indicate a redundancy issue with priority level 2 in some models, i.e. the objectives in priority level 2 of these models are ignored completely (this in turn means that exactly the same results are obtained if the associated models are run with the priority level one objectives only). Thus, the results do not reflect the decision maker's preferences as represented by the two priority levels. Appendix E illustrates the resulting portfolios return, risk and constituents after re-running all the LGP models with RE = 0.02.

6. Further discussion

The results manifest the power of goal programming models for selecting an investor's portfolio and they seem to support the underlying philosophy of the goal programming variants quite reasonably. In particular, WGP is found to be suitable for reflecting preferences of investors by setting appropriate weights on the unwanted deviational variables. LGP is more relevant for investors who wish to further prioritize the achievement of unwanted deviational variables. The fairness of the solution of MinMax GP is also supported as it serves as a compromise in terms of achieving several factors with different weights in one model.

Goals (extended factors) their target values and GP variants should be clearly defined in order to obtain the most desired portfolio. In particular:

- The choice of appropriate deviational variables to penalize to represent more is better, less is better or neither.
- Consider extended factors concurrently; preferring some factors over others by assigning suitable weights (WGP); having a clear distinction in factors' importance by setting priorities (LGP); or simply requiring obtaining a fair solution (MinMax GP).
- Define the relevant target values for each objective. Keeney and Raiffa (1993) discuss how values are more fundamental to a decision problem than are alternatives. They rank from important principles that must be upheld to guidelines for preferences among choices. Periodically, the target values should be examined to try to improve the portfolio's performance.

Although this paper applies various goal programming portfolio selection models to international mutual funds, in the opinion of the authors there is no compelling reason why they cannot be applied to any other financial instrument (stocks, bonds, ETFs, etc.). The ultimate aim is for a decision maker to decide on the set of factors of relevance to his/her portfolio selection problem and then to use the scientific framework established in this paper to reach a practical and effective investment solution.

Overall, the WGP, LGP and MinMax GP models experimented with extended factors are found to be practical approaches in reflecting the investor's various preferences in the resulting portfolios. For instance, experiments W4, L4 and M4 have a target value of 0.017 for risk which is lower than the average for each individual mutual fund; challenging the models to achieve portfolios with lower risk. The three models succeed in reaching the lowest risk for the selected portfolios at least compared to their respective baseline models, except for M4. However, there are some differences within each model's result, most notably in percentage achievements of factors and the number of mutual funds selected.

Furthermore, many researchers recommend investing a portion of investors' portfolios in global financial assets in order to obtain better performing portfolios through diversification (for instance, Redman et al., 2000). However, investing in international assets requires significant time and efforts to select amongst them. Therefore, the success of an international portfolio depends partly on the ability of the total portfolio to generate risk-adjusted returns equal to or greater than the domestic stock market index. Also, the ability of the total portfolio to generate returns better than those of domestic assets.

Since this paper investigates international mutual funds portfolio selection from 10 countries, investors from each of the 10 countries could compare the performance of the portfolios selected in this paper against the performance of their domestic market index or domestic mutual funds for sound investing decisions. For instance, if an investor is based in the UK, the comparison should be with UK's market index, i.e. FTSE 100, where the return during the testing time period is −0.005 with a total risk of 0.027. While the return on UK's mutual funds (the two included in this paper's experiments) is −0.005 with a total risk of 0.029. Both comparisons are favorable to many of portfolios selected in this paper as they produce better returns with reasonable risks.

The results obtained in this paper, although promising, are not globally conclusive as they are based on certain factors, financial instruments, target values, priority levels, time periods and a specific set of penalized unwanted deviational variables. One way to validate the results obtained from the experiments is to include other factors, other financial instruments, different target values, other priority levels, different time periods and other sets of penalized unwanted deviational variables.

This paper has used the three major types of goal programming variant but further work could be to extend to other variants such as fuzzy GP, extended GP, meta GP, or multi-choice GP (Jones and Tamiz, 2010). These GP variants offer different types of tradeoffs between various objectives, target values and penalization of unwanted deviational variables that may result in even better and diverse portfolios.

It is worth emphasizing that the data used for the portfolio selection models in the paper are publicly and freely available. However, practitioners (portfolio decision makers) usually have access to extra wealth of information and more accurate data on their investments. Furthermore, the mean for each factor's value of the mutual funds are calculated as indicators for setting up the target values. The practitioners normally have a good idea (either gut feeling or some sort of scientific approach) to the values they wish to set for these target values. The experiments hence can be repeated utilizing such additional information with potentials of further improvements in their results.

7. Conclusion

This paper provides practitioners and academics with a scientific approach to portfolio selection using goal programming that is shown to be capable of achieving a required set of preferences. A potential investor could benefit from the results obtained by selecting portfolios that incorporate factors that characterize today's world to impact their investment performance favorably.

It is hoped that the issues discussed and their outcome will provide an added-value for practitioners in complementing their financial expertise with sound scientific decision making framework, besides starting a new avenue of research into the application of goal programming to portfolio selection and the use of suitable extended factors. In any case, the usefulness and flexibility of the use of goal programming models for portfolio selection remain promising. The paper manifested that the Goal Programming models for Portfolio Selection are characterized by simplicity of form and practicality of approach.

Appendix A. Summary of the data used in this paper's experiments

The first column shows the seven regions, the second column shows the 10 countries where the mutual funds are selected from and the third column gives each mutual fund's name (these are the names' abbreviations, where their long forms are shown in Appendix F). Three categories of factors are shown in the table, in columns heading mutual fund specific factors, macroeconomic factors and regional preferences, respectively. RE stands for return factor, which is measured by the average return during the experiments' time period. RI is the risk as measured by Mean-Absolute Deviation (MAD) model. AG is the mutual fund's age measured in years during the time period. GD stands for the GDP in PPP as a percent of world's total, while CA is the current account balance as a percent of the GDP. IN is the annual inflation rate in percentage terms during the same time period, while RG is a factor for regional preferences where a score from 10 to 70 is assigned and used to reflect a preference to certain country or region.

Region	Country	Mutual fund	Extended factors						
			Mutual funds specific factors			Macroeconomic factors			Regional preferences
			RE average return (%)	RI risk (measured by MAD model)	AG fund age (years)	GD GDP-PPP (%world's total)	CA current account (%GDP)	IN annual inflation (%)	RG regional preferences (10–70 scores)
Middle East & North Africa – MENA	Egypt	E-E	0.88	2.84	13	0.53	0.8	4.2	10
	Egypt	E-B	0.21	0.97	15	0.53	0.8	4.2	10
	KSA	K-C	−0.11	3.80	14	0.60	27.4	2.2	10
	KSA	K-S	−0.27	3.63	2	0.60	27.4	2.2	10

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Appendix A (*continued*)

Region	Country	Mutual fund	Extended factors						
			Mutual funds specific factors			Macroeconomic factors			Regional preferences
			RE average return (%)	RI risk (measured by MAD model)	AG fund age (years)	GD GDP-PPP (%world's total)	CA current account (%GDP)	IN annual inflation (%)	RG regional preferences (10–70 scores)
Asia Pacific	Japan	J-S	−0.19	1.51	7	6.30	3.9	0.3	50
	Japan	J-B	−0.52	1.85	21	6.30	3.9	0.3	50
	China	C-A	2.47	2.78	3	15.08	9.4	1.5	50
	China	C-I	2.11	2.50	6	15.08	9.4	1.5	50
Central Asia	India	I-R	0.94	2.43	12	6.28	−1.1	6.1	40
	India	I-B	1.03	2.67	5	6.28	−1.1	6.1	40
Western Europe	UK	U-A	0.38	1.64	6	3.20	−3.2	2.3	60
	UK	U-S	0.23	1.40	37	3.20	−3.2	2.3	60
	Italy	T-I	0.32	1.29	16	2.70	−2.4	2.2	60
	Italy	T-F	0.30	1.19	12	2.70	−2.4	2.2	60
Eastern Europe	Russia	R-S	3.18	5.21	2	2.61	9.7	9.7	30
	Russia	R-U	0.75	1.99	18	2.61	9.7	9.7	30
North America	USA	S-B	0.22	1.10	31	19.66	−6.2	3.2	70
	USA	S-A	0.17	1.21	14	19.66	−6.2	3.2	70
Latin America	Brazil	B-R	0.84	2.86	8	2.57	1.2	4.2	20
	Brazil	B-F	1.00	2.98	2	2.57	1.2	4.2	20

Appendix B. The results of the weighted goal programming models

WGP models		Return	Risk	The selected mutual funds		% of Achievement for each target value for the seven factors		
				MFs (country)	Proportions (%)	b_i	Achieved	%
First: The baseline model		**−0.003**	**0.393**	E-E (Egypt)	10.59	$b_{RE} = 0.015$	**0.013**	**90**
				E-B (Egypt)	23.60	$b_{RI} = 0.023$	**0.023**	**100**
				K-C (KSA)	3.41	$b_{AG} = 13.0$	**13.0**	**100**
				C-A (China)	40.69	$b_{GD} = 0.07$	**0.07**	**100**
				U-S (UK)	16.59	$b_{CA} = 0.05$	**0.05**	**100**
				<u>R-U (Russia)</u>	5.13	$b_{IN} = 0.03$	**0.03**	**102**
				Total: 6 MFs		$b_{RG} = 35.0$	**35.0**	**100**
Second: The models changing deviational variable weights	W1	**−0.003**	**0.405**	E-E (Egypt)	0.81	$b_{RE} = 0.015$	**0.015**	**100**
				E-B (Egypt)	23.40	$b_{RI} = 0.023$	**0.023**	**100**
				K-C (KSA)	1.18	$b_{AG} = 13.0$	**13.0**	**100**
				C-A (China)	46.09	$b_{GD} = 0.07$	**0.08**	**114**
				U-S (UK)	20.29	$b_{CA} = 0.05$	**0.05**	**100**
				<u>R-U (Russia)</u>	8.24	$b_{IN} = 0.03$	**0.03**	**100**
				Total: 6 MFs		$b_{RG} = 35.0$	**40.2**	**115**
	W2	**−0.003**	**0.393**	E-E (Egypt)	10.59	$b_{RE} = 0.015$	**0.013**	**90**
				E-B (Egypt)	23.60	$b_{RI} = 0.023$	**0.023**	**100**
				K-C (KSA)	3.41	$b_{AG} = 13.0$	**13.0**	**100**
				C-A (China)	40.69	$b_{GD} = 0.07$	**0.07**	**100**
				U-S (UK)	16.59	$b_{CA} = 0.05$	**0.05**	**100**

Appendix B (*continued*)

WGP models		Return	Risk	The selected mutual funds		% of Achievement for each target value for the seven factors		
				MFs (country)	Proportions (%)	b_i	Achieved	%
				R-U (Russia)	5.13	b_{IN} = 0.03	**0.03**	**102**
				Total: 6 MFs		b_{RG} = 35.0	**35.0**	**100**
	W3	−**0.003**	**0.405**	E-E (Egypt)	15.70	b_{RE} = 0.015	**0.013**	**88**
				E-B (Egypt)	19.99	b_{RI} = 0.023	**0.023**	**100**
				K-C (KSA)	3.25	b_{AG} = 13.0	**13.0**	**100**
				C-A (China)	40.98	b_{GD} = 0.07	**0.07**	**100**
				U-S (UK)	15.31	b_{CA} = 0.05	**0.05**	**100**
				R-S (Russia)	2.44	b_{IN} = 0.03	**0.03**	**100**
				R-U (Russia)	2.33	b_{RG} = 35.0	**35.0**	**100**
				Total: 7 MFs				
Third: The models changing some factors' target values	W4	−**0.004**	**0.339**	E-B (Egypt)	37.07	b_{RE} = 0.015	**0.011**	**75**
				K-C (KSA)	0.55	b_{RI} = 0.017	**0.018**	**106**
				C-I (China)	46.84	b_{AG} = 13.0	**13.0**	**100**
				U-S (UK)	10.53	b_{GD} = 0.07	**0.08**	**110**
				R-U (Russia)	5.01	b_{CA} = 0.05	**0.05**	**100**
				Total: 5 MFs		b_{IN} = 0.03	**0.03**	**100**
						b_{RG} = 35.0	**35.0**	**100**
	W5	−**0.004**	**0.443**	E-E (Egypt)	9.55	b_{RE} = 0.015	**0.015**	**100**
				E-B (Egypt)	1.04	b_{RI} = 0.023	**0.023**	**100**
				K-C (KSA)	2.39	b_{AG} = 13.0	**13.0**	**100**
				C-A (China)	52.35	b_{GD} = 0.20	**0.13**	**66**
				R-U (Russia)	9.35	b_{CA} = 0.05	**0.05**	**100**
				S-B (USA)	25.31	b_{IN} = 0.03	**0.03**	**100**
				Total: 6 MFs		b_{RG} = 35.0	**47.9**	**137**
	W6	−**0.007**	**0.368**	K-C (KSA)	0.01	b_{RE} = 0.015	**0.015**	**103**
				C-I (China)	70.40	b_{RI} = 0.023	**0.021**	**92**
				S-B (USA)	28.95	b_{AG} = 13.0	**13.0**	**100**
				Total: 3 MFs		b_{GD} = 0.07	**0.16**	**233**
						b_{CA} = 0.05	**0.05**	**100**
						b_{IN} = 0.03	**0.02**	**67**
						b_{RG} = 70.0	**55.5**	**79**
	W7	**0.001**	**0.317**	E-E (Egypt)	42.59	b_{RE} = 0.015	**0.006**	**43**
				E-B (Egypt)	35.75	b_{RI} = 0.023	**0.023**	**100**
				K-C (KSA)	12.98	b_{AG} = 13.0	**13.0**	**100**
				C-A (China)	8.68	b_{GD} = 0.07	**0.02**	**26**
				Total: 4 MFs		b_{CA} = 0.05	**0.05**	**100**
						b_{IN} = 0.03	**0.04**	**124**
						b_{RG} = 10.0	**13.5**	**135**

Appendix C. The results of lexicographic goal programming models

LGP models	Return	Risk	The selected mutual funds		% of Achievement for each target value for the seven factors		
			MFs (country)	Proportions (%)	b_i	Achieved	%
First: The baseline model	−**0.003**	**0.405**	E-E (Egypt)	0.81	b_{RZ} = 0.015	**0.015**	**100**
			E-B (Egypt)	23.4	b_{RI} = 0.023	**0.023**	**100**
			K-C (KSA)	1.18	b_{AG} = 13.0	**13**	**100**
			C-A (China)	46.09	b_{GD} = 0.07	**0.08**	**114**
			U-S (UK)	20.29	b_{CA} = 0.05	**0.05**	**100**

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Appendix C (*continued*)

LGP models		Return	Risk	The selected mutual funds		% of Achievement for each target value for the seven factors		
				MFs (country)	Proportions (%)	b_i	Achieved	%
				R-S (Russia)	8.24	$b_{IN} = 0.03$	**0.03**	**100**
				Total: 6 MFs		$b_{RG} = 35.0$	**40.2**	**115**
Second: The models changing deviational variable weights	**L1**	–**0.003**	**0.405**	E-E (Egypt)	0.81	$b_{RZ} = 0.015$	**0.015**	**100**
				E-B (Egypt)	23.4	$b_{RI} = 0.023$	**0.023**	**100**
				K-C (KSA)	1.18	$b_{AG} = 13.0$	**13**	**100**
				C-A (China)	46.09	$b_{SS} = 0.07$	**0.08**	**114**
				U-S (UK)	20.29	$b_{CA} = 0.05$	**0.05**	**100**
				R-S (Russia)	8.24	$b_{IN} = 0.03$	**0.03**	**100**
				Total: 6 MFs		$b_{RG} = 35.0$	**40.2**	**115**
	L2	–**0.003**	**0.405**	E-E (Egypt)	15.7	$b_{RZ} = 0.015$	**0.013**	**88**
				E-B (Egypt)	19.99	$b_{RI} = 0.023$	**0.023**	**100**
				K-C (KSA)	3.25	$b_{AG} = 13.0$	**13**	**100**
				C-A (China)	40.98	$b_{SS} = 0.07$	**0.07**	**100**
				U-S (UK)	15.31	$b_{CA} = 0.05$	**0.05**	**100**
				R-S (Russia)	2.44	$b_{IN} = 0.03$	**0.03**	**100**
				R-U (Russia)	2.33	$b_{RG} = 35.0$	**35**	**100**
				Total: 7 MFs				
	L3	–**0.003**	**0.405**	E-E (Egypt)	15.7	$b_{RZ} = 0.015$	**0.013**	**88**
				E-B (Egypt)	19.99	$b_{RI} = 0.023$	**0.023**	**100**
				K-C (KSA)	3.25	$b_{AG} = 13.0$	**13**	**100**
				C-A (China)	40.98	$b_{CD} = 0.07$	**0.07**	**100**
				U-S (UK)	15.31	$b_{CA} = 0.05$	**0.05**	**100**
				R-S (Russia)	2.44	$b_{IN} = 0.03$	**0.03**	**100**
				R-U (Russia)	2.33	$b_{RG} = 35.0$	**35**	**100**
				Total: 7 MFs				
Third: The models changing some factors' target values	**L4**	–**0.006**	**0.353**	E-B (Egypt)	4.89	$b_{RZ} = 0.015$	**0.015**	**100**
				C-I (China)	67.68	$b_{RI} = \mathbf{0.017}$	**0.02**	**120**
				S-B (USA)	27.4	$b_{AG} = 13.0$	**13**	**100**
				Total: 3 MF		$b_{CD} = 0.07$	**0.15**	**223**
						$b_{CA} = 0.05$	**0.04**	**94**
						$b_{IN} = 0.03$	**0.02**	**70**
						$b_{RG} = 35.0$	**53.5**	**153**
	L5	–**0.004**	**0.443**	E-E (Egypt)	9.55	$b_{RZ} = 0.015$	**0.015**	**100**
				E-B (Egypt)	1.04	$b_{RI} = 0.023$	**0.023**	**100**
				K-C (KSA)	2.39	$b_{AG} = 13.0$	**13**	**100**
				C-A (China)	52.35	$b_{CD} = \mathbf{0.20}$	**0.13**	**66**
				R-U (Russia)	9.35	$b_{CA} = 0.05$	**0.05**	**100**
				S-B (USA)	25.31	$b_{IN} = 0.03$	**0.03**	**100**
				Total: 6 MFs		$b_{RG} = 35.0$	**48.1**	**137**
	L6	–**0.007**	**0.368**	K-C (KSA)	0.65	$b_{RZ} = 0.015$	**0.015**	**103**
				C-1 (China)	70.4	$b_{RI} = 0.023$	**0.021**	**92**
				S-B (USA)	28.95	$b_{AG} = 13.0$	**13**	**100**
				Total: 3 MFs		$b_{CD} = 0.07$	**0.16**	**233**
						$b_{CA} = 0.05$	**0.05**	**100**
						$b_{IN} = 0.03$	**0.02**	**67**
						$b_{RG} = \mathbf{70.0}$	**55.5**	**79**
	L7	–**0.003**	**0.405**	E-E (Egypt)	0.81	$b_{RZ} = 0.015$	**0.015**	**100**
				E-B (Egypt)	23.4	$b_{RI} = 0.023$	**0.023**	**100**
				K-C (KSA)	1.18	$b_{AG} = 13.0$	**13**	**100**
				C-A (China)	46.09	$b_{CD} = 0.07$	**0.08**	**114**

Appendix C (*continued*)

LGP models		Return	Risk	The selected mutual funds		% of Achievement for each target value for the seven factors		
				MFs (country)	Proportions (%)	b_i	Achieved	%
				U-S (UK)	20.29	b_{CA} = 0.05	**0.05**	**100**
				R-S (Russia)	8.24	b_{IN} = 0.03	**0.03**	**100**
				Total: 6 MFs		b_{RG} = **10.0**	**40.2**	**402**

Appendix D. The results of the MinMax goal programming models

MinMax GP models		Return	Risk	The selected mutual funds		% of Achievement for each target value for the seven factors		
				MFs (country)	Proportions (%)	b_i	Achieved	%
First: The baseline model		−**0.003**	**0.339**	E-E (Egypt)	12.46	b_{RE} = 0.015	**0.014**	**96**
				E-B (Egypt)	20.73	b_{RI} = 0.023	**0.024**	**104**
				K-C (KSA)	1.64	b_{AG} = 13.0	**12.5**	**96**
				C-A (China)	41.93	b_{GD} = 0.07	**0.07**	**103**
				U-S (UK)	16.25	b_{CA} = 0.05	**0.04**	**96**
				R-U (Russia)	6.99	b_{IN} = 0.03	**0.03**	**104**
				Total: 6 MFs		b_{RG} = 35.0	**36.3**	**104**
Second: The models changing deviational variable weights	M1	−**0.003**	**0.391**	E-E (Egypt)	5.34	b_{RE} = 0.015	**0.014**	**99**
				E-B (Egypt)	25.49	b_{RI} = 0.023	**0.023**	**101**
				K-C (KSA)	0.23	b_{AG} = 13.0	**12.5**	**96**
				C-A (China)	42.65	b_{GD} = 0.07	**0.07**	**105**
				U-S (UK)	17.25	b_{CA} = 0.05	**0.04**	**93**
				R-U (Russia)	9.04	b_{IN} = 0.03	**0.03**	**107**
				Total: 6 MFs		b_{RG} = 35.0	**37.5**	**107**
	M2	−**0.003**	**0.404**	E-E (Egypt)	12.58	b_{RE} = 0.015	**0.014**	**96**
				E-B (Egypt)	20.14	b_{RI} = 0.023	**0.024**	**104**
				K-C (KSA)	2.16	b_{AG} = 13.0	**12.5**	**96**
				C-A (China)	42.50	b_{GD} = 0.07	**0.07**	**104**
				U-S (UK)	16.15	b_{CA} = 0.05	**0.05**	**99**
				R-U (Russia)	6.48	b_{IN} = 0.03	**0.03**	**102**
				Total: 6 MFs		b_{RG} = 35.0	**26.4**	**104**
	M3	−**0.002**	**0.397**	E-E (Egypt)	10.58	b_{RE} = 0.015	**0.014**	**95**
				E-B (Egypt)	24.89	b_{RI} = 0.023	**0.023**	**101**
				K-C (KSA)	1.31	b_{AG} = 13.0	**12.3**	**95**
				C-A (China)	41.37	b_{GD} = 0.07	**0.07**	**101**
				U-S (UK)	14.85	b_{CA} = 0.05	**0.05**	**95**
				R-U (Russia)	7.00	b_{IN} = 0.03	**0.03**	**105**
				Total: 6 MFs		b_{RG} = 35.0	**37.0**	**101**
Third: The models changing some factors' target values	M4	−**0.003**	**0.414**	E-B (Egypt)	33.85	b_{RE} = 0.015	**0.013**	**89**
				C-A (China)	33.64	b_{RI} = 0.017	**0.019**	**111**
				C-I (China)	18.45	b_{AG} = 13.0	**11.5**	**89**
				U-S (UK)	1.93	b_{GD} = 0.07	**0.10**	**149**
				S-B (USA)	11.63	b_{CA} = 0.05	**0.04**	**89**
				Total: 5 MFs		b_{IN} = 0.03	**0.02**	**88**
						b_{RG} = 35.0	**38.9**	**111**
	M5	−**0.007**	**0.426**	E-B (Egypt)	14.04	b_{RE} = 0.015	**0.017**	**110**
				C-I (China)	75.92	b_{RI} = 0.023	**0.021**	**94**
				S-B (USA)	10.03	b_{AG} = 13.0	**9.4**	**73**

(*continued on next page*)

Appendix D (*continued*)

MinMax GP models	Return	Risk	The selected mutual funds		% of Achievement for each target value for the seven factors		
			MFs (country)	Proportions (%)	b_i	Achieved	%
			Total: 3 MFs		$b_{GD} = 0.20$	**0.13**	**67**
					$b_{CA} = 0.05$	**0.07**	**133**
					$b_{IN} = 0.03$	**0.02**	**68**
					$b_{RG} = 35.0$	**46.4**	**133**
M6	−**0.005**	**0.484**	C-A (China)	57.02	$b_{RE} = 0.015$	**0.017**	**111**
			C-I (China)	8.75	$b_{RI} = 0.023$	**0.022**	**95**
			S-B (USA)	34.23	$b_{AG} = 13.0$	**13.0**	**100**
			Total: 3 MFs		$b_{GD} = 0.07$	**0.17**	**238**
					$b_{CA} = 0.05$	**0.04**	**81**
					$b_{IN} = 0.03$	**0.02**	**69**
					$b_{RG} = 70.0$	**56.8**	**81**
M7	−**0.001**	**0.317**	E-E (Egypt)	24.59	$b_{RE} = 0.015$	**0.006**	**39**
			E-B (Egypt)	37.94	$b_{RI} = 0.023$	**0.023**	**100**
			K-C (KSA)	22.29	$b_{AG} = 13.0$	**12.9**	**99**
			C-A (China)	15.18	$b_{GD} = 0.07$	**0.03**	**39**
			Total: 4 MFs		$b_{CA} = 0.05$	**0.08**	**161**
					$b_{IN} = 0.03$	**0.03**	**111**
					$b_{RG} = 10.0$	**16.1**	**161**

Appendix E. Redundancy test: the resulting portfolios of re-running LGP models with RE = 0.02

This table shows the results for the LGP models with RE's target value set at an optimistic level of 0.02. Redundancy reported for the models LL1, LL2, LL6, LL7 and LL8 in priority level 2; i.e., the objectives in priority level 2 can be ignored completely. This means that the LL1, LL2, LL6, LL7 and LL8 portfolios reported are the result of the objectives in priority level 1 only. Thus, the results do not reflect the decision maker's preferences as represented by the two priority levels.

LGP models with RE = **0.02** *(The LGP models are called LLk* (*where k* = 1, . . ., 8) *which correspond to the 8 LGP experiment shown in* Appendix B)	Return	Risk	The selected mutual funds		% of Achievement for each target value for the seven factors		
	MFs (country)	Proportions (%)	b_i	Achieved	%		
LL1	−**0.006**	**0.522**	C-A (China)	68.29	$b_{RE} = 0.020$	0.017	88
			U-S (UK)	14.49	$b_{RI} = 0.023$	0.023	100
			S-B (USA)	17.22	$b_{AG} = 13.0$	13.0	100
			Total: 3 MFs		$b_{GD} = 0.07$	0.14	202
					$b_{CA} = 0.05$	0.05	98
					$b_{IN} = 0.03$	0.02	64
					$b_{RG} = 35.0$	55.0	157
LL2	−**0.006**	**0.522**	C-A (China)	68.29	$b_{RE} = 0.020$	0.017	88
			U-S (UK)	14.49	$b_{RI} = 0.023$	0.023	100
			S-B (USA)	17.22	$b_{AG} = 13.0$	13.0	100
			Total: 3 MFs		$b_{GD} = 0.07$	0.14	202
					$b_{CA} = 0.05$	0.05	98
					$b_{IN} = 0.03$	0.02	64
					$b_{RG} = 35.0$	55.0	157
LL3	−**0.003**	**0.405**	E-E (Egypt)	15.70	$b_{RE} = 0.020$	0.013	66
			E-B (Egypt)	19.99	$b_{RI} = 0.023$	0.023	100
			K-C (KSA)	3.25	$b_{AG} = 13.0$	13.0	100
			C-A (China)	40.98	$b_{GD} = 0.07$	0.07	100
			U-S (UK)	15.31	$b_{CA} = 0.05$	0.05	100
			R-S (Russia)	2.44	$b_{IN} = 0.03$	0.03	100
			R-U (Russia)	2.33	$b_{RG} = 35.0$	35.0	100

Appendix E (*continued*)

LGP models with RE = **0.02** *(The LGP models are called LLk* (*where k* = 1, . . ., 8) *which correspond to the 8 LGP experiment shown in Appendix B*)	Return	Risk	The selected mutual funds		% of Achievement for each target value for the seven factors		
			MFs (country)	Proportions (%)	b_i	Achieved	%
			Total: 7 MFs				
LL4	−**0.003**	**0.405**	E-E (Egypt)	15.70	b_{RE} = 0.020	0.013	66
			E-B (Egypt)	19.99	b_{RI} = 0.023	0.023	100
			K-C (KSA)	3.25	b_{AG} = 13.0	13.0	100
			C-A (China)	40.98	b_{GD} = 0.07	0.07	100
			U-S (UK)	15.31	b_{CA} = 0.05	0.05	100
			R-S (Russia)	2.44	b_{IN} = 0.03	0.03	100
			R-U (Russia)	2.33	b_{RG} = 35.0	35.0	100
			Total: 7 MFs				
LL5	−**0.003**	**0.298**	E-B (Egypt)	38.37	b_{RE} = 0.020	0.011	55
			C-I (China)	46.41	b_{RI} = 0.017	0.017	100
			S-B (USA)	15.22	b_{AG} = 13.0	13.0	100
			Total: 3 MF		b_{GD} = 0.07	0.1	146
					b_{CA} = 0.05	0.03	75
					b_{IN} = 0.03	0.02	93
					b_{RG} = 35.0	37.7	108
LL6	−**0.006**	**0.522**	C-A (China)	68.29	b_{RE} = 0.020	0.017	88
			U-S (UK)	14.49	b_{RI} = 0.023	0.023	100
			S-B (USA)	17.22	b_{AG} = 13.0	13.0	100
			Total: 3 MFs		b_{GD} = 0.20	0.14	71
					b_{CA} = 0.05	0.05	98
					b_{IN} = 0.03	0.02	64
					b_{RG} = 35.0	55.0	157
LL7	−**0.006**	**0.522**	C-A (China)	68.29	b_{RE} = 0.020	0.017	88
			U-S (UK)	14.49	b_{RI} = 0.023	0.023	100
			S-B (USA)	17.22	b_{AG} = 13.0	13.0	100
			Total: 3 MFs		b_{GD} = 0.07	0.14	202
					b_{CA} = 0.05	0.05	98
					b_{IN} = 0.03	0.02	64
					b_{RG} = 70.0	54.9	78
LL8	−**0.006**	**0.522**	C-A (China)	68.29	b_{RE} = 0.020	0.017	88
			U-S (UK)	14.49	b_{RI} = 0.023	0.023	100
			S-B (USA)	17.22	b_{AG} = 13.0	13.0	100
			Total: 3 MFs		b_{GD} = 0.07	0.14	202
					b_{CA} = 0.05	0.05	98
					b_{IN} = 0.03	0.02	64
					b_{RG} = 10.0	54.9	549

Appendix F. The names of the mutual funds used in the experiments reported in this paper

Region	Country	Mutual fund abbreviation	**Mutual fund name**
Middle East & North Africa (MENA)	Egypt	E-E	**EFG-Hermes-Bank Alexandria MF 1**
	Egypt	E-B	**Banque Misr 1**
	KSA	K-C	**CAAM Saudi Fransi-Saudi Istithmar Equity Fund**
	KSA	K-S	**Saudi Hollandi Bank-Saudi Equity Trading Fund**
Asia Pacific	Japan	J-S	**SG Target Japan Fund**
	Japan	J-B	**BlackRock Japan Small Cap**
	China	C-A	**China AMC large-cap Select Fund**
	China	C-I	**China International Alpha Equity Fund**

(*continued on next page*)

Appendix F (*continued*)

Region	Country	Mutual fund abbreviation	**Mutual fund name**
Central Asia	India	I-R	**Reliance Vision Fund**
	India	I-B	**Birla Sun Life Equity Fund**
Western Europe	UK	U-A	**Allianz RCM UK Equity Fund**
	UK	U-S	**Scottish Widows UK Equity Income Fund**
	Italy	T-I	**Imi-Italy**
	Italy	T-F	**Fondersel Italia**
Eastern Europe	Russia	R-S	**Solid Index MICEX-Open-End Fund**
	Russia	R-U	**Univer-Equity Fund Investments Fund**
North America	USA	S-B	**BlackRock Exchange Portfolio-Open-End Fund**
	USA	S-A	**Barclays S& P 500 Stock Fund**
Latin America	Brazil	B-R	**Real FIA Dividendos**
	Brazil	B-F	**FIA Mistyque-Open-End Fund**

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