A Model of the Single-jersey Loop-formation Process

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A Model of the Single-jersey Loop-formation Process

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A deterministic model of the single-jersey loop-formation process is formulated by incorporating the initial geometry of the knitting zone, the contribution of the take-down load, and the force required to move the needles. This model is based on the principle that the loop length is decided by the height beneath the sinker/verge line to which a needle is pulled up by the tension in the yarns of the loop.

A computer program, based on the model of the single-jersey loop-formation process, is developed for predicting the effect of some relevant yarn, machine, and process variables on loop length as well as the resulting yarn-tension profile inside the knitting zone. The model is validated qualitatively in terms of the loop length and yarn-tension profile inside the knitting zone as well as quantitatively only in terms of the loop length. This modelled system is then explored to determine favourable conditions for knitting on orthodox single-jersey knitting systems.

1. INTRODUCTION

During the process of loop formation in single-jersey knitting, an interaction takes place between various yarn variables (rigidities, coefficient of friction, diameter, etc.), knitting-machine variables (speed, diameter, gauge, cam shape, etc.), and knitting-process variables (yarn input tension, fabric take-down tension, cam setting, etc.). Modelling of single-jersey loop formation is aimed at determining this interaction qualitatively and quantitatively. Investigation of the mechanics of the single-jersey process has revealed that bidirectional yarn flow within the knitting zone plays an important role in determining the response variables, namely, the loop length and the yarn-tension profile within the knitting zone. In order to predict the magnitudes of these response variables, it is essential to identify the important input variables, which play a critical role during the process of loop formation.

A model of the single-jersey loop-formation process is formulated under conditions of the specific nature of yarn movement inside the knitting zone, derived on the basis of experimental work (Ghosh and Banerjee, 1990). It has been demonstrated clearly in the experimental work that:

(i) the loop length is decided beyond the knitting point, so the needle movement beyond the knitting point is very important;
(ii) there is a perceptible effect of the old loop on the newly drawn yarn; and
(iii) the resistance to needle movement also plays a decisive role.

All these factors have been taken into account in developing the model of the single-jersey loop-formation process. This model is designed to indicate the build up of yarn tension inside the knitting zone as well as the final value of the loop length going into the fabric.
2. ASSUMPTIONS AND NOTATION

In order to develop a model of the single-jersey loop-formation process, the following assumptions were made.

(i) The build up in yarn tension obeys Amontons's capstan equation of friction as the yarn passes over and under the knitting elements or across the yarn surfaces.

(ii) The coefficient of friction is constant and independent of the value of the yarn tension and the velocity of the moving yarn.

(iii) The yarn-bending stiffness (flexural rigidity) is negligible.

(iv) The needles, sinkers, and yarn are circular in cross-section at the points of contact. The inertia of the needle is negligible.

(v) The yarn tension developed inside the knitting zone is proportional to the tensile strain in the yarn.

(vi) There is no sinker movement across the needle path, i.e. the sinkers are stationary. This would be true in a verge-type machine.

The following notation is used in formulating mathematical relations (see Fig. 1).

- $E$ = Elastic coefficient of yarn, equalling the load per unit strain (relative rigidity)
- $H$ = Feeding-point height with respect to the sinker line
- $N_i$ = The $i$th needle
- $O$ = Clearing point, taken as the origin of the co-ordinate system
- $Q$ = Resistance to needle movement in the trick
- $S_i$ = The $i$th sinker
- $T(AL_i \text{ or } AT_i)$ = Tension in the leading arm ($AL_i$) or trailing arm ($AT_i$) of the loop held by needle $N_i$
- $U$ = Resultant upward force applied on needle owing to yarn tension
- $X_i$ and $Y_i$ = Co-ordinates of the hook of the $i$th needle
- $X$ = Horizontal distance between the feeding point and the $y$-axis
- $Y$ = Vertical distance between the sinker line and the $x$-axis
- $a$ = Half needle-spacing
- $d_i$ = Geometrical yarn length between sinker $S_i$ or $S_{i+1}$ and needle $N_i$, equalling the length of the line segment joining the relevant two parts
- $g$ = Machine gauge
- $h$ = Depth of stitch cam below the sinker line at the knitting point
- $l(AL_i \text{ or } AT_i)$ = Length of yarn in the leading arm ($AL_i$) or trailing arm ($AT_i$) of the $i$th needle at zero tension
- $\mu_d$ = Magnitude of the slope of the cam on the descending side.
- $\mu_s$ = Magnitude of the slope of the cam on the ascending side
- $n_r$ = Needle-crown radius
- $s_t$ = Sinker radius
- $y_r$ = Yarn radius
- $\theta_d$ and $\theta_a$ = Descending- and ascending-side stitch-cam angles
- $\theta_i$ = Half the angle of yarn wrap around needle $N_i$
- $\mu_{fm}$ = Coefficient of fabric-metal friction
- $\mu_{nm}$ = Coefficient of metal-metal friction
- $\mu_{ym}$ = Coefficient of yarn-metal friction
- $\mu_{yy}$ = Coefficient of yarn-yarn friction
A Model of the Single-jersey Loop-formation Process

3. FORMULATION OF THE MODEL

3.1 Geometry of the System

Fig. 1 shows AZ as the sinker line described by the equation $Y = Y$. After crossing the clearing point, the needle would follow the profile of the descending side of the stitch cam, i.e. OAC. After crossing the knitting point (C), the needle may follow the ascending-side profile of stitch cam CZ or a path decided by the resultant of the yarn tension and the trick-resistance forces acting on the needle.

![Fig. 1 Diagrammatic representation of the knitting zone](image)

After a needle makes contact with the yarn, the tension in the yarn and the length of yarn controlled by this needle change continuously at every minute stage of needle movement. The model of loop formation is formulated in such a way that changes in yarn tension and yarn length can be calculated at every minute stage of needle movement inside the knitting zone.

3.2 Co-ordinates of Needle after Displacement $(X_i, Y_i)$

If the needle, just after making contact with the yarn at the point $X'$, is shifted towards the knitting point by an amount $\Delta X$, then the new co-ordinates of the needles, counted from this contact point onwards, and hence in a sense opposite to that shown in Fig. 1, will be given by:

$$X_i = X' + \Delta X + 2(i-1)a; \ i = 1, 2, 3, ....$$

$$Y_i = \mu_d X_i \text{ (descending side)}$$

and

$$Y_i = (Y + h) (\mu_a/\mu_d) + (Y + h) - \mu_a X_i \text{ (ascending side)}$$

Alternatively, on the ascending side, $Y_i$ is determined by the force, i.e. $Y_i = f(U,Q)$. 

*J. Text. Inst., 1999, 90 Part I, No. 2 © Textile Institute*
3.3 New Geometrical Yarn Length

As the needles take up new positions, their vertical distance with respect to the imaginary sinker line also changes. This would result in changes in the geometrical yarn length ($\delta$) as well as the wrap angle ($\theta$). The corresponding expressions can be stated as follows:

$$d_i = k\theta + (h_i^2 + a^2 - k^2)^{0.5}$$

where

$$\theta = \arctan \left[ \frac{k}{(h_i^2 + a^2 - k^2)^{0.5}} \right] + \arctan \left( \frac{h_i}{a} \right)$$

$$k = s_r + 2y_r + n_r$$

$$h_i = Y_i - Y$$

The change in geometrical yarn length is given by:

$$\Delta d_i = d_i - d'_i$$

where $d'_i$ is the original geometrical yarn length.

3.4 Change in Tension in Yarn Segment Due to Change in Geometrical Yarn Length

With a change in the needle depth below the sinker line, the tension in the yarn segments between the needle and the sinker would also change. If there is no yarn flow in any segment, then the original unstrained length of yarn, $l_i$, would be changed to $d_i$. Hence the change in yarn tension due to yarn strain would be given by:

$$TE_i = (d_i - l_i)/l_i$$

3.5 Change in Yarn Tension Due to Flow of Yarn

The change in tension in different yarn segments within the knitting zone may cause a flow of yarn either from the $i$th to the $(i - 1)$th segment or vice versa. Thus, when

$$T_i > T_{i-1}, \exp \{ \mu_{ym} (\theta_i + \theta_{i-1}) \}$$

the yarn would move from the $(i - 1)$th to the $i$th segment until this inequality vanished. However, if

$$T_i > T_{i-1}$$

but

$$T_i \leq T_{i-1}, \exp \{ \mu_{ym} (\theta_i + \theta_{i-1}) \}$$

there would be no yarn movement from one section to another.

3.6 Final Relaxed Length of a Yarn Segment

If a state of balance in the $i$th section is achieved through a drop in tension from $T_i$ to $\bar{T}_i$ and an increase in length from $l_i$ to $l'_i$, then we have:

$$l'_i = Ed'_i/(E + \bar{T}_i)$$

where $l_i$ and $l'_i$ are equivalent lengths at zero tension and $E$ is the relative rigidity of the yarn.
3.7 Geometry of the Knitting Zone

By studying the knitting action at an extremely low speed, it was observed that the needle came into contact with the yarn well before entering the knitting zone. After making contact with the yarn, the needle passed through different stages while moving downward, as illustrated in Figures 2(a)–2(f). The angles of wrap between yarn and sinker as well as those between yarn and needle at each stage are listed in Table I.

Table I

<table>
<thead>
<tr>
<th>Stage</th>
<th>Angle of Wrap around Needle N,</th>
<th>Angle of Wrap around Sinker S₁</th>
<th>Angle of Wrap around Sinker S₂</th>
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<tr>
<td>Stage V</td>
<td>α + δ</td>
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<tr>
<td>Stage VI</td>
<td>2δ</td>
<td>δ + θ</td>
<td>δ − ω</td>
</tr>
</tbody>
</table>

A Model of the Single-jersey Loop-formation Process

Fig. 2 Needle and sinker positions at different stages

Fig. 3 Initial yarn-needle contact point

Fig. 4 Initial yarn-sinker contact point

The angles $\alpha$, $\beta$, $\theta$, $\delta$, and $\omega$ are transitional in character. Thus their values keep on changing as the system passes through various stages. The expressions for these angles are listed in Appendix A.

### 3.8 Initial Yarn–Needle and Yarn–Sinker Contact Points

Let it be assumed that a needle is just in contact with the feeding yarn when it is positioned at a distance $X$ from the clearing point. It is observed from Figures 2(a) and 3 that, at the first yarn–needle contact point, angle $\alpha$ is equal to angle $\beta$. Thus, the expressions for $\alpha$ and $\beta$ can be equated at the initial yarn–needle contact point and can be solved for values of $X$ and $Y$ by employing the process of iteration, and hence the initial yarn–needle contact point can be established.

It is similarly observed from Figures 2(d) and 4 that the yarn comes into contact with the incoming sinker $S$ when the angle $\alpha$ is equal to the angle $\omega$. The expressions for $\alpha$ and $\omega$ can thus be equated at the first yarn–sinker contact point and can be solved for values of $X$ and $Y$ by employing the process of iteration and hence the initial yarn–sinker contact point can be established.

### 3.9 Role of the Cast-off Loop

#### 3.9.1 Contribution of Take-down Load

A cast-off loop and the yarn newly drawn in through this loop interact with each other during the process of loop formation, and the take-down load is transmitted through the cast-off loop into the newly formed loop. Knapton and Munden (1966) assumed that the minimum force acting on a newly formed loop equals the take-down load/wale line ($F_A$). It has been correctly mentioned by Peat and Spicer (1973) that the contribution of the take-down load is not straightforward as suggested by Knapton and Munden. But they did not suggest any relationship between $F_A$ and the resultant tension in the newly formed loop. A method of deriving this is proposed in the following treatment.

#### 3.9.2 Expression for Force Components $q_1$ and $q_n$ Due to Take-down Load

From Fig. 5(a), it is observed that the knitted fabric moves in the direction of $F_A'$. If $F_A'$ is the tension in the cast-off loop in contact with the newly formed loop, then we have:

$$F_A = F_A' \cdot \exp \{(\mu_{\text{im}} (\varepsilon + \lambda_r))$$

where $\lambda_r$ is the angle made by the cast-off loop with the horizontal plane and $\varepsilon = \arctan(Y_0/X_0)$. The values of $X_0$ and $Y_0$ are determined by the design of the relevant machine element.

If $q$ is the tension per arm of the cast-off loop, then we have:

$$F_A' = 2q$$

and hence

$$q = 0.5 \frac{F_A}{\exp \{(\mu_{\text{im}} (\varepsilon + \lambda_r))}$$

(1)

With the help of Figures 5(b) and 5(c), the expression for angle $\lambda_r$ can be worked out as:

$$\lambda_r = \arctan \left(2h \frac{y_r}{(b - (n_r + 2y_r) / \sin \theta)} \{3y_r + c\} \right) + \{y_r / (3y_r + c)\}$$
Fig. 6 shows that the old loop is lying in an inclined plane ABCD, which makes an angle $\lambda$ with the plane EFCD. The force $q$ acting along the arms of the old loop and the force $q_0$ along its crown are different in magnitude and direction. The resultant of these two forces is $q_r$, also in the plane ABCD, given by:

$$q_r = (q_0^2 + q^2)^{0.5}$$

When the needle moves downward, the distance between two arms of the newly drawn yarn along any horizontal line is expected to increase (as shown in Fig. 7(a)). To accommodate the new girth of the new loop, the yarn in the old loop should flow from its
arms to the crown. This flow is only possible when

\[ q_0 = q \cdot \exp (\mu_{yy} \cdot \pi/2) \]

and hence

\[ q_r = q(1 + \exp (\mu_{yy} \cdot \pi))^{0.5} \]  \hspace{1cm} (2)

Fig. 6 Geometry of the cast-off loop

When the needle moves up (Fig. 7(b)), the resultant force \( q_r \) will be given by:

\[ q_r = q[1 + \{1/ \exp (\mu_{yy} \cdot \pi)\}]^{0.5} \] \hspace{1cm} (3)

Resolving the force \( q_r \) into components \( q_i \) along the axis of the leading arm of the newly drawn yarn and \( q_n \) along the plane EFCD, perpendicular to the leading arm, we obtain:

\[ q_i = q_r \sin \lambda \] \hspace{1cm} (4)

\[ q_n = q_r \cos \lambda \] \hspace{1cm} (5)

where \( \lambda \) is the angle made by \( q_r \) with the EFCD plane.
A Model of the Single-jersey Loop-formation Process

The expressions for \( q_i \) and \( q_n \) for both the upward and downward movements of the needle can subsequently be worked out in terms of \( F_L \) and other known and measurable variables by substituting Equations (1)–(3) into Equations (4) and (5).

The expression for \( \lambda \) follows similarly from Equations (1)–(4) as well as from the observation that:

\[
\frac{\sin \lambda_i}{\sin \lambda_n} = \frac{q_i}{q_n}
\]

It is therefore possible to calculate \( q_i \) and \( q_n \) in terms of the yarn and needle radii, the coefficients of friction (\( \mu_m, \mu_y \)), the instantaneous depth of the needle below the sinker line, the semi-wrap angle, the take-down load, and the machine parameter (\( e \)).

The force \( q_i \) will always act along the leading and trailing arms of the newly drawn yarn, thus adding to the tension in them. But, whenever there is a flow of yarn to either of these arms, the force \( q_i \) will either assist or resist the flow. Moreover, in the event of yarn flow, the force \( q_n \) acting normal to the yarn will cause frictional resistance, which also has to be overcome. This will result in a tension difference between the yarn segments in a newly drawn yarn across the point of contact with the cast-off loop.

3.10 Forces Opposing Needle Movement beyond the Knitting Point

During the process of knitting, the needle butt is constrained to follow the descending side profile of the stitch cam. But, beyond the knitting point, the exact path followed by the needle butt would depend on (a) the upward force \( U \) acting on the needle as a result of yarn tension, (b) the forces resisting needle movement \( Q \), and (c) inertial forces. In this analysis, the inertial forces are ignored. As long as the upward force acting on the needle is higher than the opposing forces, the needle butt will follow the ascending profile of the stitch cam (Fig. 8).

![Fig. 8 Forces acting on the needle beyond the knitting point](image-url)
On equating the vertical forces acting on the needle during its movement along the ascending profile of the stitch cam, we obtain:

$$U = Q + R \cos \theta + \mu_{\text{mm}} R \sin \theta$$

When $U = Q, R = 0$, i.e. the needle butt loses its contact with the ascending profile of the stitch cam. Beyond this point, the needle would be expected to travel parallel to the sinker line until it strikes the upthrow cam.

4. METHOD OF DETERMINATION OF THE FINAL LOOP LENGTH AND TENSION PROFILE

This method involves, in the first cycle of iteration, moving a needle $N_1$ from its position of initial contact with the yarn through the knitting zone in small discrete steps without permitting it to rob any yarn. The relaxed lengths of the trailing and leading arms corresponding to each needle as well as the tensions in them are computed until the needle $N_1$ reaches the knitting point. Upon a further displacement, the needle $N_1$ will cross the knitting point and move up along the ascending profile of the stitch cam. The tension in the yarn held by $N_1$ will start falling owing to the recovery from extension. Subsequently, the yarn is capable of flowing from this needle in the reverse direction by satisfying the corresponding equations given in Appendix B. This reverse flow will affect the equilibrium of the length and tension profiles in the yarn segments controlled by the needles on the descending side of the stitch cam. Upon establishing the new equilibrium, the needle $N_1$ is moved further, and the next equilibrium condition is worked out. This process continues until an eventuality arises when the trailing yarn held by the needle $N_1$ does not flow across the verge, and the magnitude of the upward tension on the needle falls to such a level that the needle $N_1$ does not move up any more. This would mean that this needle $N_1$ had reached the loop-forming point (LP). The amount of yarn contained between the verges neighbouring the needle $N_1$ is the final loop length. The relaxed lengths of the leading and trailing arms of the final loop and the tensions in them are stored, and the iteration is continued with a fresh movement of needle $N_1$ after initialising it as $N_1$, $N_2$ as $N_1$, $N_3$ as $N_1$, and so on. For each cycle of this and succeeding iterations centred around needles $N_2, N_3, N_4$, etc., sets of values of the final loop length as well as the tensions in the leading and trailing arms will be obtained. The iteration process is terminated when the difference between the new and stored values of length and tension is less than a predetermined small quantity.

5. VALIDATION OF THE MODEL

5.1 Variables and Coding Method

A computer program based on the model of single-jersey loop formation was developed for studying the effects of some relevant yarn (relative rigidity and coefficient of friction), knitting-machine (gauge, cam shape, forces opposing movement of needle, verge/sinker, and needle dimensions), and knitting-process variables (cam setting, input tension, and fabric take-down load) on the loop length and yarn-tension profile inside the knitting zone.

In order to identify the different yarn, machine, and process variables, a coding method was employed (Table II). A code consists of nine digits indicating a particular combination of yarn, machine, and process variables. The first two digits of a code indicate yarn variables (relative rigidity and coefficient of friction), and the next four digits represent machine variables, namely, gauge, descending- and ascending-side cam angles, and resistance to...
needle movement. The last three digits represent process variables, namely, take-down load, cam setting, and yarn input tension. A higher code indicates a larger magnitude of the corresponding variable. Each variable is represented by a single-digit code, except for the yarn input tension because seventeen different values of this needed to be coded. Hence a double-digit coding was employed, requiring the corresponding number(s) to be put within parentheses. Some of the codes in Table II belong to the experimental combination for quantitative validation of the model.

### Table II

**Coding of Variables**

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<th>Yarn Relative Rigidity (cN)</th>
<th>Coefficient of Friction</th>
<th>Gauge</th>
<th>Descending Angle of Cam (deg)</th>
<th>Ascending Angle of Cam (deg)</th>
<th>Resistance to Needle Movement (cN)</th>
<th>Tension in Old Loop (cN)</th>
<th>Cam Setting (mm)</th>
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### 5.2 Qualitative Validation

#### 5.2.1 Effect of Relevant Variables on Loop Length

Loop lengths were calculated over a range of cam settings (from 3.0 to 4.9 mm) and input tensions (from 2.5 to 20 cN), all other variables being kept constant at the coded values 1831411 (—). Some selected values are plotted in Figures 9 and 10. It is observed from Fig. 9 that, at a constant value of input tension, an increase in the stitch-cam setting always results in a nearly linear increase in loop length. Fig. 10 shows the variation of loop length with increase in input tension at four different cam settings along with an experimental curve reproduced from the published work of Knapton and Munden (1966). The shapes of these five curves are very similar. It is also observed that, for higher cam settings, the curves shift upwards, the shape remaining the same. In general, the rate of increase in loop length with an increase in the input tension decreases for higher cam settings. This is similar to the observations of several workers (Aisaka et al., 1969; Henshaw, 1968; Knapton and Munden, 1966; Nutting, 1960; Oinuma, 1986; Peat and Spicer, 1973).

Calculation of loop lengths was next carried out with the cam setting kept constant at the value of 4.3 mm and the cam angles and input tension changed. The effects of the input tension on the loop length at different combinations of descending and ascending angles of the stitch cam are shown in Fig. 11. It is noted that the cam shape 36 (descending angle 3, i.e. 55°, and ascending angle 6, i.e. 55°) gives the highest and lowest values of loop length for the lowest and highest values of the yarn input tension. It is interesting to note that, at an input tension of 6.5 cN, there is no effect of the cam shape on loop length. These observations are very similar to those by Black and Munden (1970) in their experimental work with different cam shapes.

Fig. 9 Effect of cam setting on the loop length

Fig. 10 Effect of input tension on the loop length
When the take-down load per wale line is changed from 2 cN to 25 cN, there is only a marginal increase in the loop length (Table III). This has also been reflected in the results of experiments (Ghosh and Banerjee, 1990) as well as in reports published by Henshaw (1968) and Oinuma (1986) on their work with spun yarns.

![Figure 11](image)

**Table III**

<table>
<thead>
<tr>
<th>S No.</th>
<th>Code No.</th>
<th>Loop Length (mm)</th>
<th>Peak Needle Force (cN)</th>
<th>% Robbing Back</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>173</td>
<td>6.105</td>
<td>73.52</td>
<td>26.28</td>
</tr>
<tr>
<td>2</td>
<td>173</td>
<td>5.159</td>
<td>83.70</td>
<td>25.63</td>
</tr>
<tr>
<td>3</td>
<td>173</td>
<td>6.246</td>
<td>96.10</td>
<td>24.58</td>
</tr>
<tr>
<td>4</td>
<td>173</td>
<td>6.277</td>
<td>104.32</td>
<td>24.21</td>
</tr>
</tbody>
</table>

The effect of yarn friction on loop length is demonstrated by the values listed in Table IV. It is noted that the loop length decreases with an increase in the coefficient of friction of yarn. Similar results have been reported by Nutting (1960) and Oinuma (1986).

**Table IV**

<table>
<thead>
<tr>
<th>S No.</th>
<th>Code No.</th>
<th>Loop Length (mm)</th>
<th>Peak Needle Force (cN)</th>
<th>% Robbing Back</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>156</td>
<td>6.243</td>
<td>45.18</td>
<td>24.62</td>
</tr>
<tr>
<td>2</td>
<td>156</td>
<td>6.171</td>
<td>56.78</td>
<td>25.49</td>
</tr>
<tr>
<td>3</td>
<td>156</td>
<td>6.105</td>
<td>73.52</td>
<td>26.28</td>
</tr>
<tr>
<td>4</td>
<td>156</td>
<td>6.016</td>
<td>93.39</td>
<td>27.36</td>
</tr>
</tbody>
</table>
The data in Table V indicate the effect of a change in the resistance to needle movement on the loop length. The loop length increases with an increase in the resistance to needle movement. Kowalski (1986) has demonstrated that there is about 20\% change in loop length owing to the change in the needle path beyond the knitting point.

<table>
<thead>
<tr>
<th>No.</th>
<th>Code No.</th>
<th>Loop Length (mm)</th>
<th>Peak Needle Force (cN)</th>
<th>% Robbing Back</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 7 3 1 4 1 1 2 (5)</td>
<td>6.105</td>
<td>73.52</td>
<td>26.28</td>
</tr>
<tr>
<td>2</td>
<td>1 1 7 3 1 4 2 1 2 (5)</td>
<td>6.282</td>
<td>96.76</td>
<td>24.15</td>
</tr>
<tr>
<td>3</td>
<td>1 1 7 3 1 4 3 1 2 (5)</td>
<td>6.393</td>
<td>120.36</td>
<td>22.75</td>
</tr>
<tr>
<td>4</td>
<td>1 1 7 3 1 4 4 1 2 (5)</td>
<td>6.474</td>
<td>146.43</td>
<td>21.83</td>
</tr>
</tbody>
</table>

Table V

<table>
<thead>
<tr>
<th>Code No.</th>
<th>Loop Length (mm)</th>
<th>Peak Needle Force (cN)</th>
<th>% Robbing Back</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 7 3 1 4 1 1 2 (5)</td>
<td>6.105</td>
<td>73.52</td>
<td>26.28</td>
</tr>
<tr>
<td>1 1 7 3 1 4 2 1 2 (5)</td>
<td>6.282</td>
<td>96.76</td>
<td>24.15</td>
</tr>
<tr>
<td>1 1 7 3 1 4 3 1 2 (5)</td>
<td>6.393</td>
<td>120.36</td>
<td>22.75</td>
</tr>
<tr>
<td>1 1 7 3 1 4 4 1 2 (5)</td>
<td>6.474</td>
<td>146.43</td>
<td>21.83</td>
</tr>
</tbody>
</table>

5.2.2 The Effect of Relevant Variables on the Needle Force in the Knitting Zone

Tension in the leading and trailing arms of yarn held by needles in the knitting zone and the resultant forces acting on the needle crown as a result of yarn tension (needle force) were recorded against the needle position for various combinations of variables. Fig. 12 is such a trace at a 3.3-mm cam setting. The trace of needle force gives two distinct peaks. This observation is very similar to the experimental observations of Henshaw (1968), Peat and Spicer (1973), and Wray and Burns (1976).

![Fig. 12 Yarn tension and needle force in the knitting zone](image-url)
A Model of the Single-jersey Loop-formation Process

The prominence and amplitude of the peaks in a trace of needle force depend on the combination of variables as exemplified by the traces shown in Figures 12–15.

![Graph](Image1)

**Fig. 13** Yarn tension and needle force in the knitting zone

![Graph](Image2)

**Fig. 14** Yarn tension and needle force in the knitting zone
However, in all cases, peaks always occur before the knitting point (KP). From Figures 13 and 15, it is inferred that a high stitch-cam setting as well as a higher input tension causes the peak to move towards the feeding point and the loop-forming point to move away from the knitting point. This observation was also made by Wray and Burns (1976).

An increase in the fabric take-down load as illustrated in Fig. 14 (25 cN per wale line) pushes the second peak of the needle force upwards and marginally depresses the first peak. This was also observed by Wray and Burns (1976).

5.3 Quantitative Validation

Geometrical analysis of the weft-knitted loop indicates that its dimensions in the relaxed state are largely governed by the length of yarn contained in one stitch and that this length is firmly established during the knitting process itself, and the subsequent process of relaxation does not cause any significant change in this parameter (Munden, 1959). Thus the loop length, which is decided during the process of knitting, itself forms a crucial response of the loop-forming system and hence can be used for validating the model of single-jersey loop formation.

In order to validate quantitatively the model developed for single-jersey loop formation in terms of the actual loop-length values, samples were experimentally generated for four different yarns (24.08, 31.03, 40.09, and 60.69 tex), knitted at three different cam settings (4.1, 4.3, and 4.6 mm) by using different yarn-tension values. The yarn-feeding was regulated by using a negative-storage feeding system. The samples were knitted on a...
A Model of the Single-jersey Loop-formation Process

3.5-in. × 112, 12-gg verge-type machine, equipped with a stitch cam of θ1 = θ2 = 45° and needles having a radius of 0.205 mm at the crown of the hook and being subjected to a resistance (Q) of 18.2 cN against movement in the bed.

The data of the theoretically calculated loop length at the loop-forming point and the peak-needle-force values computed by using the program developed and also those of unroved loop-length values for the corresponding samples are presented in Table VI.

Table VI

<table>
<thead>
<tr>
<th>S No.</th>
<th>Code No.</th>
<th>Loop Length (mm)</th>
<th>Error (%)</th>
<th>Peak Needle Force (cN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Actual</td>
<td>Unroved</td>
<td>Theoretical</td>
</tr>
<tr>
<td></td>
<td></td>
<td>lP</td>
<td>lP</td>
<td>lP</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.23</td>
<td>-3.55</td>
<td>43.43</td>
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<tr>
<td></td>
<td></td>
<td>8.32</td>
<td>-0.08</td>
<td>49.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.86</td>
<td>0.21</td>
<td>52.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.52</td>
<td>-1.31</td>
<td>45.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.11</td>
<td>-3.51</td>
<td>45.93</td>
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<tr>
<td></td>
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<td>8.66</td>
<td>-1.72</td>
<td>36.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.04</td>
<td>0.79</td>
<td>48.39</td>
</tr>
<tr>
<td></td>
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<td>1.05</td>
<td>59.18</td>
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<tr>
<td></td>
<td></td>
<td>8.86</td>
<td>-1.58</td>
<td>38.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.81</td>
<td>-4.46</td>
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<tr>
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<td></td>
<td>9.30</td>
<td>-3.04</td>
<td>63.85</td>
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<tr>
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<td></td>
<td>8.94</td>
<td>-5.31</td>
<td>42.11</td>
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<tr>
<td></td>
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<tr>
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<td></td>
<td>8.68</td>
<td>-4.10</td>
<td>51.65</td>
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<tr>
<td></td>
<td></td>
<td>8.92</td>
<td>-4.18</td>
<td>53.53</td>
</tr>
<tr>
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<td></td>
<td>8.35</td>
<td>-3.78</td>
<td>54.47</td>
</tr>
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<td></td>
<td></td>
<td>8.37</td>
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<td>8.39</td>
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<td>57.77</td>
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<td>8.11</td>
<td>-5.80</td>
<td>54.85</td>
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<td></td>
<td></td>
<td>8.17</td>
<td>-6.42</td>
<td>56.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.27</td>
<td>-7.26</td>
<td>62.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.59</td>
<td>-3.12</td>
<td>54.90</td>
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<td>7.66</td>
<td>-3.91</td>
<td>57.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.79</td>
<td>-5.40</td>
<td>64.00</td>
</tr>
</tbody>
</table>

It is observed from the data in Table VI that the experimental values of unroved loop length are never equal to the predicted values of loop length, although a reasonably good agreement is reflected, as illustrated by the column showing values of percentage error. It is also observed that there is a clear trend in growth in percentage error with an increase in yarn linear density. The average values of this error corresponding to yarns of 24.08, 31.03, 40.09, and 60.69 tex are -0.29, -0.46, -2.95, and -4.34%, respectively. The model is apparently overestimating the loop length, and the extent of overestimation increases with the yarn linear density. This dependence of percentage error on yarn linear density indicates a role being played by some critical yarn properties affecting the loop-formation system progressively with the yarn linear density but ignored in the model. To verify this point, yarn-bending properties were measured, and it was observed that the yarns showed a rapid increase in values of bending rigidity and bending hysteresis with a rise in linear density.
The rise in bending rigidity means that, other variables remaining the same, the needles have to do more work in forming a loop with thicker yarn. The reaction generated in the thicker yarn would also be correspondingly higher, subjecting the needles to a greater upward force. Hence the loop-forming point of a thicker yarn would be moved up on the ascending side of the stitch cam, closer to the sinker line than for that of finer yarn, resulting in smaller actual loops for yarns of higher linear density. Hence the systematically increasing overestimation of loop length for yarns of higher linear density may be attributed to the lack of provision of bending properties of yarn in the present model.

6. ANALYSIS OF THE MODELLED SYSTEM

In view of the reasonable qualitative as well as quantitative accuracy of the model, it was decided to explore the system for determining the most favourable conditions for single-jersey knitting. Under actual knitting conditions, many input variables, such as properties of the yarn and yarn input tension, as well as the fabric take-down tension, would exhibit a dispersion. The knitter would nevertheless like to achieve the minimum variation in loop length accompanied by the lowest possible value of peak needle force. In other words, it is important for the knitter to operate within the least sensitive band of the system. In order to identify this, all input variables were changed systematically one at a time and the responses for 231 different combinations studied. As a result, it was found that:

• a minimum variation of loop length is obtained at the highest values of cam setting and yarn input tension, along with the lowest values of descending and ascending angles of the stitch cam; variation in take-down tension does not have a significant effect on this response; and

• the minimum value of peak needle force is obtained at the lowest values of cam setting, yarn input tension, and fabric take-down tension, as well as the highest values of stitch-cam angles.

7. DISCUSSION

There exists a direct contradiction in terms of the effects of some critical input variables on the two response variables. After separate consideration of the effects of the descending and ascending angles of the stitch cam on these responses, it is suggested that a high descending angle followed by a low angle of ascent of the stitch cam will provide the optimum combination. Similarly, in keeping with the yarn properties, one has to employ the highest-possible input tension and choose the gauge properly so as to have a moderately high cam setting. This naturally would also result in a moderately high yarn tension and robbing back. Hence the yarn should be well lubricated and moderately strong.

In the case of linear cams, a high angle will result in a high jerk on the needles, and hence the peak needle force may increase. Since the inertia of the needles was ignored in this model, it is safe to conclude that this analysis points to cam shapes with low angles of ascent and descent.

Lower cam angles would, however, result in a lower value of feeder density, violating one of the basic tenets of the development of modern circular-knitting machines. Another disturbing feature of the modern knitting process is over-reliance on positive feeding. It is clear that, unless the input tension and cam setting are synchronised with the feeding system, a large variation in loop length may occur, although the average value may remain within acceptable limits.
8. CONCLUSIONS

As a result of the limited analysis of the modelled single-jersey loop-formation process, it is observed that knitting on a conventional single-jersey system for the production of highly uniform fabric, even involving positive feeders, calls for a careful selection of the cam shape, as well as critical adjustment of the yarn input tension and stitch-cam setting in conjunction with the properties of the yarn to be knitted. The requirements of a low yarn tension within the knitting zone and a low variation in loop length being diametrically opposite, a compromise solution involving moderately high values of the stitch-cam setting as well as a yarn input tension associated with a stitch-cam profile exhibiting a high angle of descent and low angle of ascent is prescribed for average fabric quality. Very high-quality fabrics can only be produced with yarns that are very smooth, even, and strong, to be knitted only under conditions satisfying a low variation in loop length. The advantages of employing machines of high feeder density need to be viewed against the possibility of greater variation in loop length as well as a drop in production efficiency. Any machinery development that modifies the knitting zone, such as by vertical as well as horizontal movements of the sinker (e.g. in Relanit machines) or by the introduction of compound needles, calls for a thorough analysis before a rational approach can be worked out to exploit it.

REFERENCES


APPENDIX A

\[
\begin{align*}
\alpha &= \arctan \left( \frac{A}{B} \right) + \arctan \left( \frac{k_1}{k_2} \frac{(A^2 + B^2 - k_2^2)^{0.5}}{k_2} \right) \\
\beta &= \arctan \left( \frac{(C^2 + a^2 - k_2^2)^{0.5}}{k_2} \right) - \arctan \left( \frac{a}{C} \right) \\
\theta &= \arctan \left( \frac{k_2}{(C^2 + a^2 - k_2^2)^{0.5}} \right) - \arctan \left( \frac{C}{a} \right) \\
\omega &= \arctan \left( \frac{(D^2 + (B - a)^2 - k_2^2)^{0.5}}{k_2} \right) - \arctan \left( \frac{(B - a)}{D} \right),
\end{align*}
\]

where

\[
\begin{align*}
A &= H - (Y - Y) \\
B &= X - X \\
C &= Y - Y + S_t \\
D &= H + S_t \\
k_1 &= n_x + y_t \\
k_2 &= n_x + 2y_t + S_t \\
k_3 &= y_t + S_t
\end{align*}
\]

The expression for \( \delta \) is similar to that for \( \theta \).
APPENDIX B

B.1 Definition of Forward and Backward Flow

During the process of loop formation in the knitting zone, yarn may flow across the needles as well as the sinkers. This flow may take place in the direction away from the origin of the co-ordinate system employed in Fig. 1 (forward flow) or towards it (backward flow).

B.2 Forward Flow

Condition of flow across needle \( N_i \): \( TAL_i > TAT_i \exp (2 \mu_{yi} \theta_i) \)

Condition of flow across sinker \( S_{i+1} \):

\[
TAT_i > [TAL_{i+1} - q_{(i+1)} \mu_{yy} q_{n(i+1)}], \exp \{\mu_{yi} (\theta_i + \theta_{(i+1)})\} + q_{li} \exp \{\mu_{yy} q_{n(i+1)}\}
\]

Relaxed yarn length: \( l (\text{AL or AT})_i = \frac{E.d_i}{E + T (\text{AL or AT})_i} \)

Tension in yarn segment: \( T (\text{AL or AT})_i = \frac{E(d_i - l(\text{AL or AT})_i)}{l (\text{AL or AT})_i} \)

B.3 Backward Flow

Condition of flow across needle \( N_i \): \( TAT_i > TAL_i \exp (2 \mu_{yi} \theta_i) \)

Condition of flow across sinker \( S_{i+1} \):

\[
TAL_{i+1} > \{TAT_i - q_{i(i)} + \mu_{yy} q_{n(i)}\}, \exp \{\mu_{yi} (\theta_i + \theta_{(i+1)})\} + q_{l(i+1)} + \mu_{yy} q_{n(i+1)}\}
\]

Relaxed yarn length: the same as in forward flow

Tension in yarn segment: the same as in forward flow