



Advanced in Control Engineering and Information Science

## Edge Detection Algorithm Based on Multiscale Product with Gaussian Function

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### Abstract

According to Mallat multi-resolution analysis, A new edge detection algorithm based on multiscale product is presented, which uses Gaussian function and its first-derivative as lowpass and highpass filter to enhance edge and suppress noise, then detect edge embedded noise by gradient direction and updating search method. The experiments show that this approach has advantages of detecting edge in different gray contrast, high signal-noise ratio and pixel-level location accuracy.

*Keywords: multiscale product; Gaussian function; edge detection, ;gradient direction*

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### 1 Introduction

Edge is the important characteristic of image. Edge detection technique specially address the problem of image enhancement, segmentation, recognition and registration. It is also an important research issue in computer vision and pattern recognition. Image edge is often buried by noise, so it's significant to research edge detection algorithm.

Traditional edge operators, such as Sobel、Roberts、Prewitt and canny etc, have conflict between suppressing noise and edge location because noise and edge are high frequency components. In recent years, some new theory are applied in edge detection such as morphology<sup>[1]</sup>, neural network<sup>[2]</sup> and wavelet transform<sup>[3~4]</sup>. The researching results<sup>[3~4]</sup> indicate that multiscale product of edge increase exponentially

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and that of noise decrease rapidly with increasing scale. Detecting image edge using multiscale, hence, can solve conflicts in suppressing noise and location accuracy.

Selection of wavelet basis effects edge detection result. Because Gaussian function is lowpass filter and its derivative is wavelet function, so Gaussian function and its first-derivative are served as lowpass and highpass filter to detect image edge. Many experiments demonstrate the proposed algorithm has detection result of good quality and strong robustness against noise.

## 2 Wavelet Multiscale Product

Image gray level have different performance under various resolution in the same scene<sup>[6]</sup>. The edge of large object is clearly visible, but the contour of small object is degraded under low resolution. Edge effect at single scale is not ideal, so it is necessary to detect edge at different scale .

Let  $f(x)$  is one-dimension signal,  $\psi(x)$  and  $\phi(x)$  are wavelet and scale function respectively.  $W_{2^j} f(x) = f * \psi_{2^j}(x)$ ,  $S_{2^j}(x) = f * \phi_{2^j}(x)$  (1)

Formula 1 is wavelet transform and smooth filtering to  $f(x)$  at scale  $2^j$ , where \* represents convolution. The wavelet transform of  $f(x)$  with respect to  $\psi(x)$  is considered as derivative of convolution of  $f(x)$  with smoothing function of  $\psi(x)$

Let  $f(x, y)$  is two-dimension image, similarly,  $\phi(x, y)$  is two-dimension scale function.

$$\psi^x(x, y) = \partial\phi(x, y) / \partial x, \quad \psi^y(x, y) = \partial\phi(x, y) / \partial y \quad (2)$$

Two-dimension wavelet transform are defined as follows:

$$\begin{aligned} W_{2^j}^x f &= W_{2^j}^x f(x, y) = f * \psi_{2^j}^x(x, y); \\ W_{2^j}^y f &= W_{2^j}^y f(x, y) = f * \psi_{2^j}^y(x, y) \end{aligned} \quad (3)$$

Wavelet transform to  $f(x, y)$  is carried to J stages.. Two-dimension multiscale product at point  $(x, y)$  along  $x$  and  $y$  direction are:

$$p_J^x f = \prod_{j=1}^J W_{2^j}^x f(x, y), \quad p_J^y f = \prod_{j=1}^J W_{2^j}^y f(x, y) \quad (4)$$

Property 1 of multiscale product is that multiscale product of edge point is positive while J is even, that is  $p_J$  (edgepoint) > 0.

Property 2 of multiscale product is that multiscale product of edge point far larger than that of noise with increasing scale, that is  $p_J$  (edgepoint) >>  $p_J$  (noisepoint).

## 3 Proposed Algorithm

### 3.1 Multiscale product based on Gaussian function

By analysis of wavelet multiscale product above and Mallat multiresolution analysis, Gaussian function below (formula 5) is served as lowpass filter of edge detection,

$$g(u, \delta_j) = \frac{1}{\delta_j \sqrt{2\pi}} e^{-\frac{u^2}{2\delta_j^2}} \quad (5)$$

$g(u, \delta_j)$  is Gaussian function at scale  $\delta_j$ . Gaussian first-derivative (formula 6) is served as highpass filter

$$g^x(u, \delta_j) = \frac{-u}{\delta_j^2} g(u, \delta_j) \quad (6)$$

In order to reduce computation cost, edge points which coefficient sign are different at different scale are regarded as noise and discarded as properties 1 and only product of two scales need be calculated as properties 2. In addition, gradient direction is involved to obtain single-pixel edge. Because two-dimensional separable transform can be computed quickly: The procedure of multiscale product edge detection using Gaussian function is designed as follows:

- (1) Approximate Image,  $S_{2^j} f$ , is obtained by implementing lowpass filtering with formula 5 along x and y direction.
- (2) Vertical edge information,  $W_{2^j}^x f$ , can be obtained by performing lowpass filtering with formula 5 along y direction, then highpass filtering with formula 6 along x direction.
- (3) Horizontal edge information,  $W_{2^j}^y f$ , can be obtained by performing lowpass filtering with formula 5 along x direction, then highpass filtering with formula 6 along y direction.
- (4) Approximate Image  $S_{2^j} f$  come from procedure (1) is carried procedure (2) and (3) at higher scale to obtain  $W_{2^{j+1}}^x f$  and  $W_{2^{j+1}}^y f$ .
- (5) Multi-scale product is computed with formula 4 along x and y direction respectively. Along x direction,  $p^x = \text{sign}(W_{2^j}^x f) \times W_{2^j}^x f \times W_{2^{j+1}}^x f$  if  $W_{2^j}^x f$  and  $W_{2^{j+1}}^x f$  have the same sign, or so  $p^x = 0$ .  $p^y$  is computed similarly.
- (6) Gradient magnitude and direction of pixel at lower scale can be computed with formula 7~8.

$$M_{2^j} f(x, y) = \sqrt{(p^x f(x, y))^2 + (p^y f(x, y))^2} \quad (7)$$

$$A_{2^j} f(x, y) = \arctan\left(\frac{W_{2^j}^y f(x, y)}{W_{2^j}^x f(x, y)}\right) \quad (8)$$

(7) Maximum gradient magnitude of pixel is obtained with revised searching method of local maximum by gradient direction

- (8) Binary edge image is obtained by proper threshold.

### 3.2 Improved Method of Seeking Local Maximum gradient magnitude

A method of local maximum gradient magnitude is presented in [7]. Algorithm is improved on the basis of researching. If  $M_{2^j} f(x, y)$  is local maximum gradient magnitude of point (x, y) in gradient direction

$A_{2^j}f(x,y)$ , 8-neighbor of point  $(x,y)$  has 4 condition, namely,  $M_{2^j}f(x,y)$  is only local maximum gradient magnitude of three pixel in one direction which is horizontal, vertical,  $35^\circ, 135^\circ$  direction.

Because main value range of inverse tangent is  $(-\frac{\pi}{2}, \frac{\pi}{2})$ , direction angle  $A_{2^j}f(x,y) \in (-\frac{\pi}{2}, \frac{\pi}{2})$ , so

improved algorithm is as follows:

- (1) if  $|A_{2^j}f(x,y)| \leq \frac{\pi}{8}$ , comparison is made among points  $(x,y-1), (x,y), (x,y+1)$ , then proceed to (5);
- (2) if  $A_{2^j}f(x,y) > \frac{\pi}{8}$  and  $A_{2^j}f(x,y) \leq \frac{3\pi}{8}$ , comparison is made among points  $(x+1,y-1), (x,y), (x-1,y+1)$ , proceed to (5);
- (3) if  $A_{2^j}f(x,y) < -\frac{\pi}{8}$  and  $A_{2^j}f(x,y) \geq -\frac{3\pi}{8}$ , comparison is made among points  $(x-1,y-1), (x,y), (x+1,y+1)$ , proceed to (5);
- (4) if  $(|A_{2^j}f(x,y)| > \frac{3\pi}{8})$  and  $(|A_{2^j}f(x,y)| < \frac{\pi}{2} \text{ or } (W_{2^j}^1 f(x,y) = 0))$ , comparison is made among points  $(x-1,y), (x,y), (x+1,y)$ , proceed to (5);
- (5) if  $M_{2^j}f(x,y)$  is at or above magnitude of other two points and strictly above one of other two points,  $M_{2^j}f(x,y)$  is local maximum module, otherwise  $M_{2^j}f(x,y)$  is 0.

#### 4 Experiment Analysis

The entropy of random variable which meets normal distribution is maximum in terms of information theory. Because Gaussian noise meets normal distribution, the image is embedded Gaussian noise which variance is 0.01 in order to analyze ability of suppressing noise. Two frames images are carried edge detection based multi-scale product at scale  $\delta_j=1$  and 2 respectively. Figure 1~2 show edge detection results using proposed algorithm compared to canny operator.

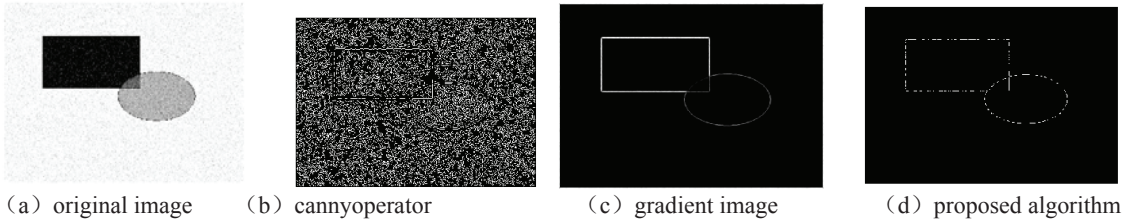


Figure 1 rectangle & circle

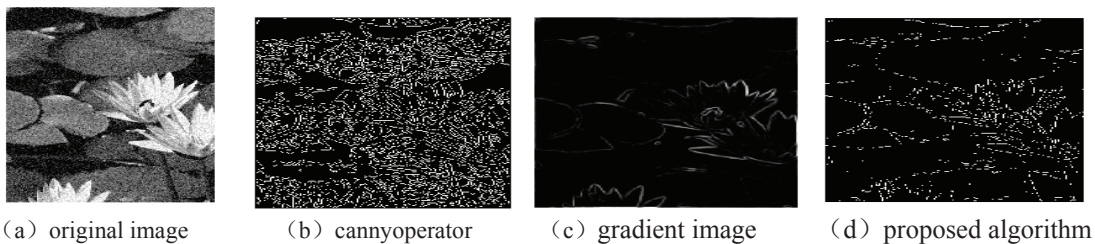


Figure 2 edge detection of lotus

Figure 1~2 (b) show edge with canny operator is buried by noise. Figure (c) illustrate gradient magnitude image without gradient direction. Although gradient images detect continuous edge, the edge is wider than single-pixel, so location of edge is not accurate. On the basis of figure (c), using revised

method of seeking local maximum gradient magnitude, we can detect all edges from high contrast to low contrast, such as lotus and lotus leaves, and edge image has advantages of pixel-level location accuracy and .

$$SNR = 10 \lg \left[ \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - \bar{f}]^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\hat{f}(x, y) - f(x, y)]^2} \right] \quad (9)$$

Performance of edge detection algorithm is assessed by location accuracy and signal-noise ratio. signal-noise ratio of edge image is computed with formula 9. The results are showed in sheet1.

Sheet 1 Signal-Noise Ratio(snr) of proposed algorithm and canny operator in different image

	snr of proposed algorithm	snr of canny operator
figure 2	-3.0119db	-54.5688db
figure3	-1.3377db	-49.2046db

## 5 conclusion

The basic task of edge detection is to solve conflicts of location accuracy and suppressing noise. According to Mallat multi-resolution Analysis, Gaussian function and its first-derivative are served as lowpass and highpass filter, respectively, to enhance edge and suppress noise by computing multi-scale product at different scale along x and y direction, then detect edge buried by noise using gradient direction. The experiments show that this approach has advantages of detecting edge in different gray contrast, high signal-noise ratio and pixel-level location accuracy. Next step is edge linking and subpixel-level location accuracy.

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