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Characteristics of traffic flow at a non-signalized intersection in the framework of game theory

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HIGHLIGHTS

- We model the traffic flow at the non-signalized intersection.
- The model introduces the game theory into simulating the traffic flow of non-signalized intersection.
- We use the Weibull distribution to describe driver's behavior.
- We found one transition regime in the phase diagram.
- The existence of defectors is benefit for the capacity of intersection, but also reduces the safety of intersection.

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ABSTRACT

At a non-signalized intersection, some vehicles violate the traffic rules to pass the intersection as soon as possible. These behaviors may cause many traffic conflicts even traffic accidents. In this paper, a simulation model is proposed to research the effects of these behaviors at a non-signalized intersection. Vehicle's movement is simulated by the cellular automaton (CA) model. The game theory is introduced for simulating the intersection dynamics. Two types of driver participate the game process: cooperator (C) and defector (D). The cooperator obey the traffic rules, but the defector does not. A transition process may occur when the cooperator is waiting before the intersection. The critical value of waiting time follows the Weibull distribution. One transition regime is found in the phase diagram. The simulation results illustrate the applicability of the proposed model and reveal a number of interesting insights into the intersection management, including that the existence of defectors is benefit for the capacity of intersection, but also reduce the safety of intersection.

1. Introduction

The urban traffic system includes many different facilities such as various level roads and structure intersections. That means vehicles moving in urban traffic must face complex environment, and drivers have to make decisions from time to time to move safely as well as quickly. The unreasonable or wrong decision will make the traffic conditions worsen. The terrible traffic causes many environmental issues such as smog and noise pollution, and huge economic losses. In order to

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improve the traffic condition, it is very important to understand the form mechanism of traffic congestion in a certain traffic environment, and then identify optimal control strategies to help alleviate the traffic congestion.

The study of traffic flow theory and modeling started in 1930s, pioneered by Bruce D. Greenshields [1]. He first used photographic measurement methods to measure traffic flow, density and velocity. A model of uninterrupted traffic flow was developed to predict and explain the trends which are observed in real traffic. However, since the 1990s, the field has gained considerable attraction due to the increase of traffic demand and the advance of computing power. In order to investigate the dynamical behavior of traffic flow, many traffic models such as hydrodynamics models [2–4], gas-kinetic-based models [5,6], car-following models [7–9] and cellular automaton models [10–12] have been proposed by physicists. Among these models, cellular automaton (CA) models have been extensively applied and investigated, because they can reproduce many realistic phenomena and are easy to simulate the traffic system fast. The Nagel–Schreckenberg (NaSch) model is a basic CA model describing one-road traffic system [10]. Based on the NaSch model, many CA models have been extended to investigate the properties of the system with realistic traffic factors.

The intersection is very important facility in the urban traffic system. The capacity of intersection restricts the efficiency of the urban traffic network. In order to research the characteristics of traffic flow at the intersection, many simulation models [13–21] are proposed. These models generally include two parts: vehicles movement on the road and the dynamical behaviors at the intersection. These models deal with the conflicts at the intersection by some rules such as first come first pass, assigning the right-of-way according to the given hierarchy. In real traffic, drivers have different drive behaviors. Various vehicles at the intersection may adopt different strategies to resolve the obvious traffic conflicts. This process is very similar to the game theory including players, strategies, and payoff. The game theory [22–25] is introduced to deal with the problems that how to assign the right-of-way between the vehicles. These methods can reflect the vehicle behaviors in a more flexible and realistic way in the traffic system.

Non-signalized intersection is a very common facility in urban traffic. When the different direction vehicles meet at the non-signalized intersection, they should obey the basic traffic rule. Namely, the right-of-way can be only given one vehicle and the vehicle on the main road has the priority to get it. But if the waiting time for the right-of-way exceeds drivers' endurance limit, they will change their behaviors and disobey this rule. All the drivers can be classified into two types: cooperator (C) and defector (D). These two types of drivers adopt different behaviors when they face different situations. For example, the defector on the side road will obtain the right-of-way of intersection forcibly or conflict with the other defector at the non-signalized intersection. The effect of this behavior is not very clear at the intersection. In order to study the effect, a simulation model is established, including two parts: vehicle movement and intersection dynamics. The vehicle movement is simulated by a cellular automaton model. The Weibull distribution is used to describe the behavior change probability. The intersection dynamics is described in the framework of game theory. The remainder of the paper is organized as follows. In Section 2, the details of the simulation model are illustrated. In Section 3, the numerical simulation is carried out and the simulation results are analyzed. The concluding remarks are made and future research directions are briefly discussed in the Section 4.

2. Model formulation

In this section, a simulation model is proposed to depict a non-signalized intersection system. Fig. 1 is the sketch of the analyzed non-signalized intersection system. This traffic system includes two crossed roads. Each road is the unidirectional road and has a single lane. Road 1 is the main road and permits vehicles to move from west to east. Road 2 is the side road and permits vehicles to move from south to north. The cross point X of two roads is located at the middle of the road. For simplicity, there are no turning vehicles in this traffic system.

2.1. The vehicle movement model

Cellular automata models are wildly used in traffic simulation. According to the difference of research problems, it is necessary to establish an appropriate cellular automaton model. The NaSch model [10] is the first and simplest cellular automaton model which is actually used for traffic flow simulations. Although its rules are very simple, it can reproduce many basic phenomena in realistic traffic, such as the start–stop waves. In this paper, our research focus is the behavior of vehicles at the intersection not the movement of vehicles on the road. So the NaSch model is suitable for simulating the vehicle movement on the road. In the NaSch model, space is divided into cells and time is divided into time steps. In this intersection system, the length of each road is divided into L cells. The cross point X of two roads is the $L/2$th cell. Each cell has two conditions, occupied or empty. All cells can be occupied by only one vehicle except the $L/2$th cell. The $L/2$th cell can be occupied by two different direction vehicles in some special situations. Each vehicle has a time-dependent speed $v_i(t)$ that takes discrete values $0, 1, 2, \ldots, v_{\max}$; here $v_{\max}$ is the maximum speed. The update rules of the NaSch model in each iteration step are as follows, which are performed in parallel for all vehicles:

- Step1: deterministic acceleration:

$$v_i(t+1) = \min(v_i(t)+1, v_{\max}, \mathrm{gap}_i). \tag{1}$$

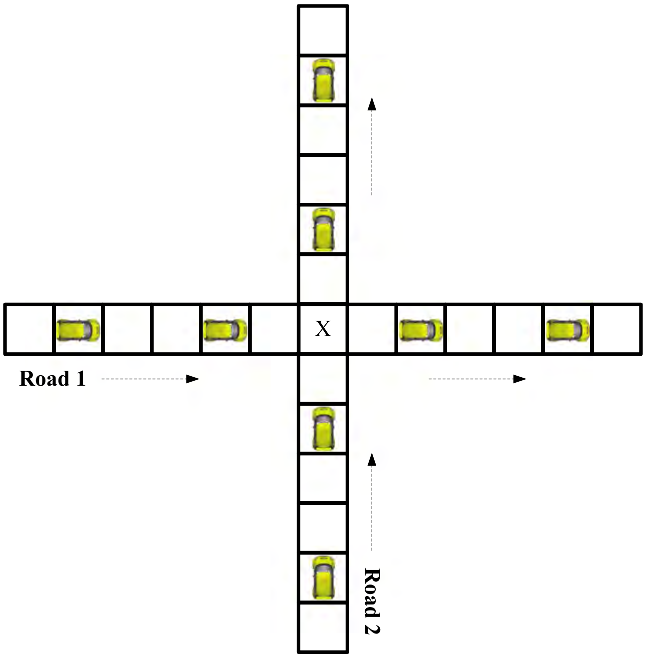

Fig. 1. Sketch of the analyzed non-signalized intersection system. This system includes two un-directional roads, road 1 and road 2. The cross point is X which is located in the middle of road.

- Step2: random behavior with a certain probability p when the vehicle is not at the cross point X,

$$v_i(t+1) = \begin{cases} \max(v_i(t+1) - 1, 0) & \text{with probability } p; \\ v_i(t+1) & \text{otherwise.} \end{cases} \tag{2}$$

- Step3: position update,

$$x_i(t+1) = x_i(t) + v_i(t+1). \tag{3}$$

Here $v_i(t)$ and $x_i(t)$ denote the speed and position of vehicle i at time step t respectively; $\text{gap}_i = x_{i-1}(t) - x_i(t) - l$ is the empty cells from the rear-bumper of leading vehicle $i-1$ to the front-bumper of following vehicle i; l is the length of the vehicle $i-1$; p is the randomization deceleration probability.

2.2. The intersection dynamics model

In urban traffic, the intersection is formed by the crossed roads. The signal control is not necessary for the intersection when the traffic flow is small. In order to reduce the conflicts between the different direction vehicles, there is a basic rule which is accepted by the drivers at the non-signalize intersection. Namely, the right-of-way can be only given one vehicle and the vehicle on the main road has the priority to get it. At first, all vehicles obey this basic rule. But with the increase of the waiting time before the intersection, some drivers may change their behaviors and disobey the basic rule. Therefore, the drivers can be classified into two types: cooperator (C) and defector (D). The problem between the vehicles at the intersection is how to assign the right-of-way. Through the game process between vehicles, it can resolve this problem and confirm the ownership of right-of-way.

In order to discuss the influence of these drive behaviors, an intersection dynamics model is established based on the game theory. In this model, there are two types of driver: cooperator (C) and defector (D). When the vehicles meet at the intersection, the result that which one passes the intersection depends on the behavior of vehicles. The details of the model that imitates what happens at real intersection are as follows:

First, the model contains three elements: one basic rule and two types of drivers.

- Basic rule: the right-of-way can be only given one vehicle and the vehicle on the main road has the priority to get it compared to the vehicle on the side road.
- Cooperator: driver who obeys the basic rule.
- Defector: driver who disobeys the basic rule and tries to enter the intersection aggressively.

Second, the model contains how to generate the driver behavior. At first, all drivers are cooperators and obey this basic rule. But when the waiting time exceeds a critical value that a driver can endure before the intersection, the driver may

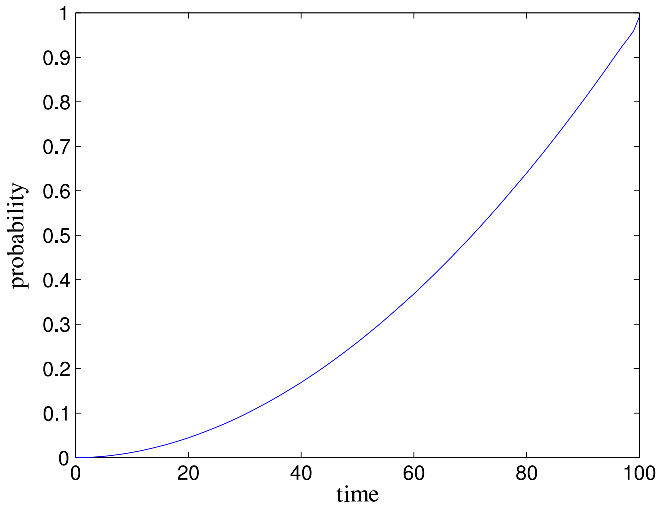

Fig. 2. Diagram of hazard rate function with $a = 30$ and $b = 2.92$.

change his behavior and becomes a defector. This critical value is not constant for each vehicle. It is similar to the lifetime of object. The Weibull distribution is widely used to describe the lifetime of object. So it is assumed that the critical value obeys the Weibull distribution. With strictly positive values of the shape parameter b and scale parameter a, the function of Weibull distribution is

$$F(x) = \begin{cases} 1 - \exp\left\{-\left(\frac{x}{a}\right)^b\right\} & x > 0; \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

and the probability density function of Weibull distribution is

$$f(x) = \begin{cases} \frac{b}{a}\left(\frac{x}{a}\right)^{b-1} \exp\left\{-\left(\frac{x}{a}\right)^b\right\} & x > 0; \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

According to the definition of the hazard rate (instantaneous failure rate), the function of the hazard rate is

$$h(x) = \frac{f(x)}{1 - F(x)}. \tag{6}$$

Fig. 2 is the diagram of the hazard rate function. The parameters are set as $a = 30$ and $b = 2.92$. This figure reflects the change probability of driver behavior.

When the vehicle begins to wait before the intersection, its behavior pattern will change with one probability, which is formulated by the function of the hazard rate in each waiting time step. This will be ended when the cooperator changes into the defector or the vehicle can pass the intersection. After the vehicle passed the intersection, its drive behavior changes back to cooperate.

Third, the model contains four strategies (I, II, III and IV) when the vehicles meet at the intersection. The vehicles nearest to the intersection are identified as c1 and c2 (in road 1 and road 2 respectively).

- Strategy I: if c2 is driven by the cooperator, c2 gives the right-of-way to c1 whether it is driven by the cooperator or the defector. It means that c1 can pass the intersection without changing its velocity, while c2 should decrease and not be allowed to enter the intersection.
- Strategy II: if c1 is driven by the cooperator and c2 is driven by the defector, c1 gives the right-of-way to c2. The detail of this strategy is that c1 should decrease and not be allowed to enter the intersection, while c2 can pass the intersection.
- Strategy III: if c1 is driven by the defector and c2 is driven by the defector, both of them may try to pass the intersection at the same time, namely, no one will give up the right-of-way. In this situation, both of them should decrease and stop at the intersection at this time step due to their illegal behaviors. At next time step, c1 can start to go and pass the intersection and c2 should stop at the intersection until next time step.
- Strategy IV: if one vehicle is already entering the intersection, the other road vehicle will adjust its velocity exactly to enter the intersection if and only if its driver is a defector.

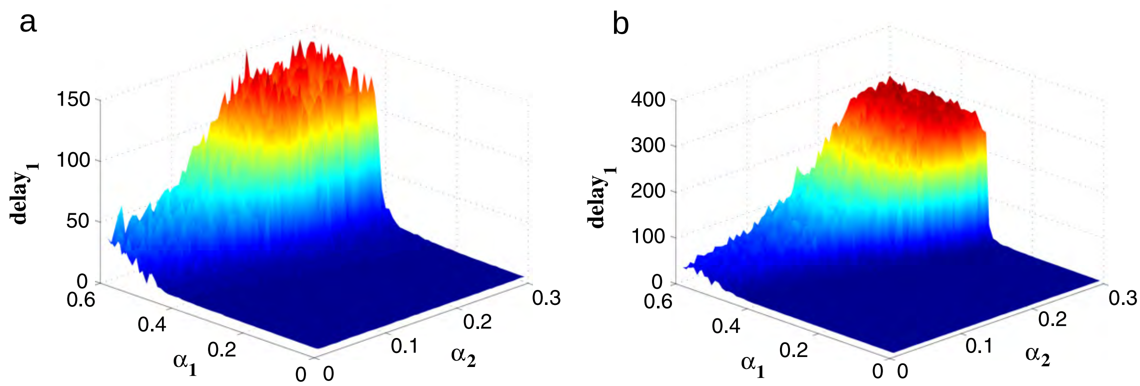

Fig. 3. Delay of road 1 with (a) no defector and (b) defectors in space (α_1, α_2).

2.3. The boundary conditions

In this paper, two boundary conditions are used to analyze the different cases. One is the open boundary condition. In this condition, we check the position of the last vehicle on the road at each time step. The position is represented as x^{last}. If $x^{last} > v_{max}$, a new vehicle with the maximum velocity v_{max} is injected with inflow rate α at the position $\min(v_{max}, x^{last} - v_{max})$. If the position of first vehicle on the road is larger than the length of road L, the vehicle will be removed and the following vehicle will become the new leading vehicle. Another one is the periodic boundary condition. In this condition, the number of vehicles N on the road is set at first. Then, these vehicles are located one by one from the start cell of road. At each time step, we check the position of the last vehicle on the road. If the position of the leading vehicle on the road is larger than the length of road L, the vehicle will reenter the system as the last vehicle and the following vehicle will become the new leading vehicle.

3. Simulation and discussion

3.1. Influence of the defector on the intersection

In this part, we use the open boundary condition and set the parameters $L = 500$, $l = 1$, $p = 0.3$, $v_{max} = 5$. Each cell corresponds to 5 m; thus the length of a vehicle is 5 m. The first 50 000 time steps are discarded to avoid the transient behaviors. The time detectors are placed at the 10th and 350th cell to better capture the delays of each movement at the intersection. The actual time that the vehicle passes the two detectors is represented as t_r. The free time that the vehicle passes the two detectors with no disturbance is represented as t_f. The delay of the vehicle equals t_r minus t_f. Delay data are extracted for the 20 000 time step. delay_1 and delay_2 denote the average delay of each vehicle on roads 1 and 2, respectively. The average delay of each vehicle on all roads is denoted as $\text{delay}_{\text{All}}$. α_1 and α_2 denote the inflow rates of roads 1 and 2, respectively.

First, the influence of defector is investigated by the average delay. Fig. 3 shows the delay of road 1 with (a) no defector and (b) defector in space (α_1, α_2). In Fig. 3(a), the delay of road 1 increases with α_2 when α_1 is larger than the critical value α_1^c. The critical value α_1^c is the line between the low delay region and the high delay region. In Fig. 3(b), the delay has the same regularity as that in Fig. 3(a). But the critical value α_1^c is less than that in Fig. 3(a) and the high delay value is larger than that in Fig. 3(a). Compared Fig. 3(a) with Fig. 3(b), it can be seen that the existence of defectors can increase the delay of road 1 when α_1 and α_2 are large enough and make the critical value α_1^c become small. That means the existence of defectors has negative influence on the traffic condition of road 1. Fig. 4 shows the delay of road 2 with (a) no defector and (b) defectors in space (α_1, α_2). In Fig. 4(a), the delay increases sharply when $\alpha_1 > 0.27$ and $\alpha_2 > 0.16$. It forms a high platform. In Fig. 4(b), it also forms a platform. But the platform is lower and narrower than that in Fig. 4(a). It means that the existence of defectors can reduce the delay of road 2 and make the delay keep low value in a wide inflow rate region. Fig. 5 shows the delay of intersection with (a) no defector and (b) defectors in space (α_1, α_2). On comparing Fig. 5(a) and (b), we can found that the existence of defectors can reduce the delay of intersection in the particular region. That means the existence of defectors can enhance the capability of the intersection system in this region. Fig. 6 shows the spatiotemporal of road 1 and road 2 with no defector and defectors. The inflow rates are set as $\alpha_1 = 0.5$ and $\alpha_2 = 0.18$. This figure can directly reflect the traffic conditions of each road. It can be seen very clearly that the traffic condition of road 1 is much worse than that without defectors. But the traffic condition of road 2 is much better than that without defectors. The defect behavior of road 2 reduces the waiting time before the intersection and shortens the length of the queue. But it may reduce the safety of intersection and cause more conflicts. These phenomena are corresponding to Figs. 4 and 5.

Second, the influence of the defector is investigated by the conflict between vehicles. The existence of defectors can improve the capacity of intersection but it also causes many conflicts. Sometimes, conflicts may cause the traffic accident. One traffic accident may cause the intersection does not work for a while or even longer. In this paper, we define that if the

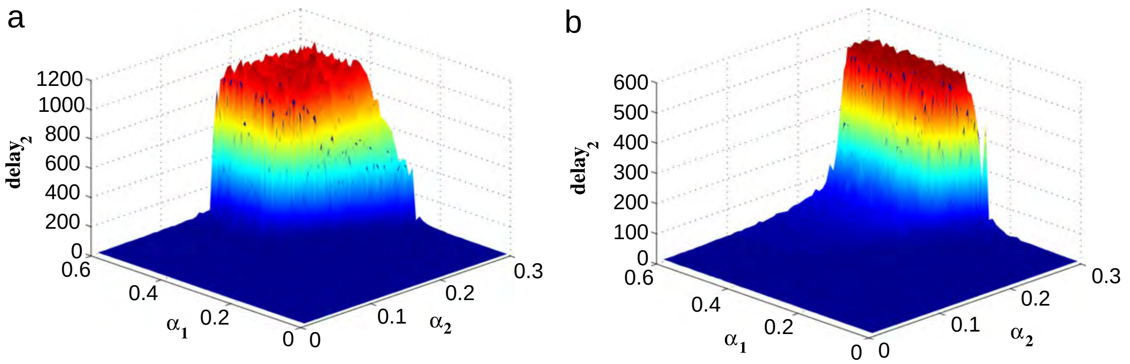

Fig. 4. Delay of road 2 with (a) no defector and (b) defectors in space (α_1, α_2).

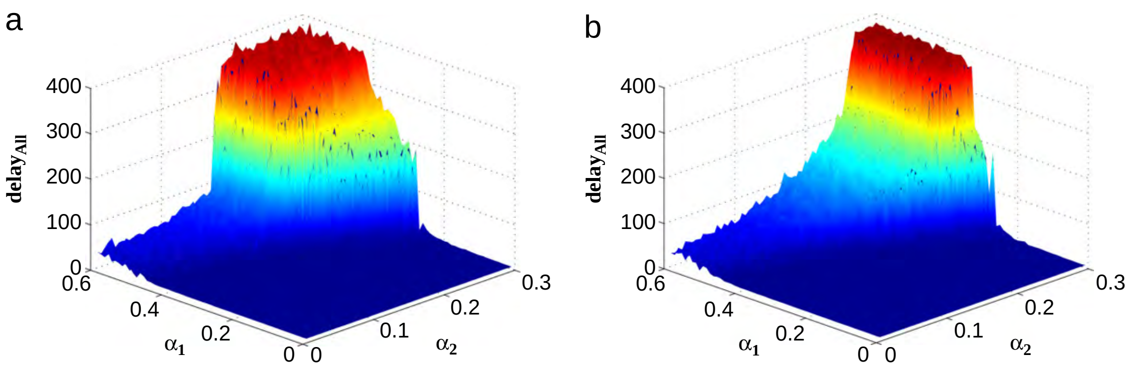

Fig. 5. Delay of intersection with (a) no defector and (b) defectors in space (α_1, α_2).

cross point X exists two vehicles at the same time, one conflict occurs. The number of conflicts in one hour is denoted as C. Fig. 7 shows the number of conflicts at the intersection in space (α_1, α_2). We can clearly see that the number of conflicts is large when α_1 and α_2 are high values. This is because that more defectors appear under this situation. These vehicles do not want to give up the right-of-way and further cause more conflicts. Fig. 8 shows the number of defectors per second on each road. A denotes the number of defectors per second. In Fig. 8, it can be observed that the number of defectors on road 2 is more than that on road 1 when the inflow rates are large enough. That means vehicles on road 2 usually wait for a longer time than that of vehicles on road 1 before the intersection. And this leads to more defectors. On comparing Fig. 7 with Fig. 8, it can be found that more defectors mean more conflicts.

3.2. Phase diagram in consideration of game theory

In this part, we use the periodic boundary condition and set the parameters $L = 500$, $l = 1$, $p = 0.3$, $v_{\max} = 5$. Each cell corresponds to 5 m; thus the length of a vehicle is 5 m. The statistic variables are formulated as follows: density, $\rho = N/L$; mean velocity, $V = \sum_{i=1}^{N} v_i(t)/N = \sum_{i=1}^{N} \sum_{t=t_0}^{T+t_0-1} v_i(t)/(NT)$; average flux, $f = \rho * V$. In the process of simulating, the first 50 000 time steps are discarded to avoid the transient behaviors. Velocity data are extracted for the $T = 20\,000$ time steps from the time $t_0 = 50\,001$. The subscripts 1 and 2 are used to denote the density, mean velocity and average flux on roads 1 and 2. In order to eliminate the effect of the random condition, the average results of 10 different initial distributions are adopted.

Now, we research the phase diagram of intersection. The division of the phase is according to the intersection status. The intersection status is the qualitative description. That may be not accurate. So the flux of road is used to reflect the intersection status as the quantitative method. According to the preceding formulation, the flux is determined by the density of each road which is set in the initial simulation. So the phase diagram of intersection is presented in space (ρ_1, ρ_2) in Fig. 9. It can be seen that the traffic status consists eight phases. In region I, the intersection status is in the free-flow regime and each vehicle moves freely. In region II, one road flux reaches a plateau, but the other road status is in the free-flow regime. In region III, one road status is in the jam regime and the flux is decreasing with the density of this road, whereas the other one is in the free-flow regime. In region IV, one road status is in the transition regime. It means that the flux is neither increasing linearly nor in a plateau. It is increasing nonlinearly. The other road is in the jamming regime. In region V, one road status is in the plateau regime, whereas the other one is in the jamming regime. In region VI, the main road is in the plateau regime and the minor road is in the transition regime. In region VII, both roads are in the plateau regime. In region VIII, both roads

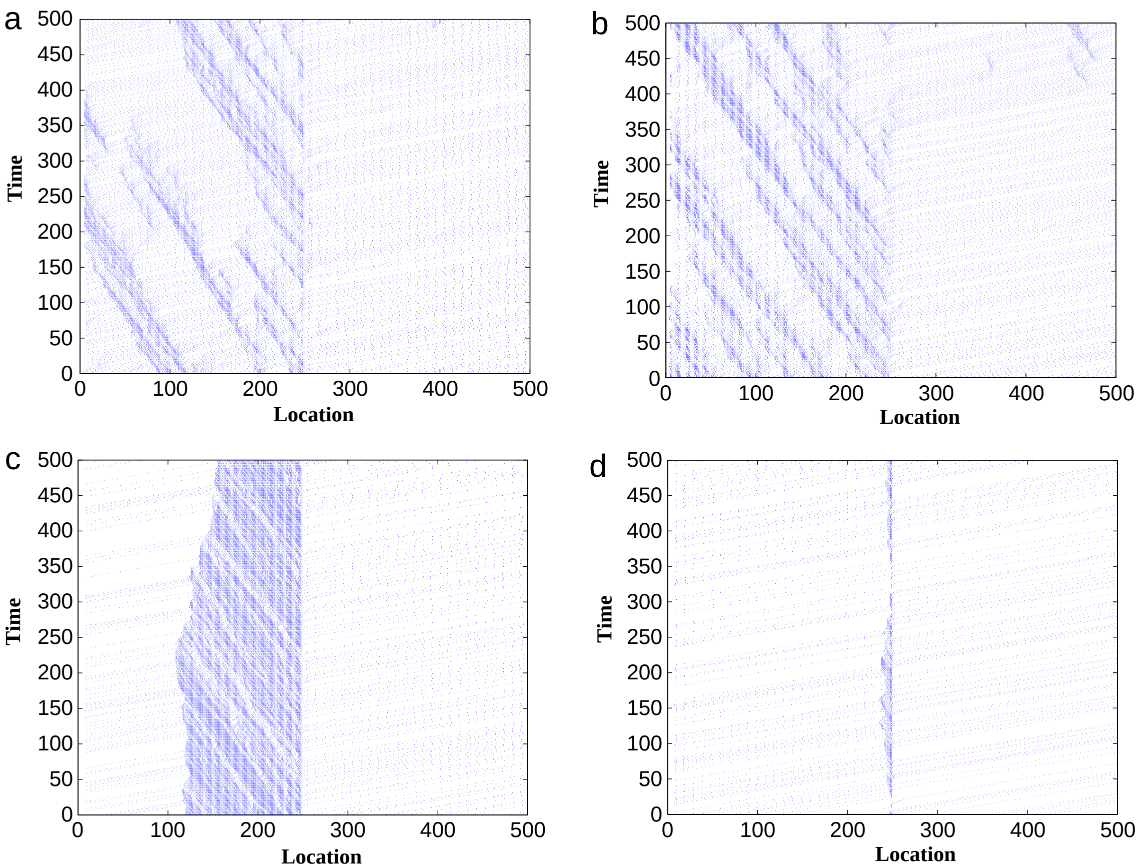

Fig. 6. Spatiotemporal diagram of road 1 and road 2 with no defector and defectors: (a) road 1 with no defector; (b) road 1 with defectors; (c) road 2 with no defector; (d) road 2 with defectors. The inflow rates are set as $\alpha_1 = 0.5$ and $\alpha_2 = 0.18$.

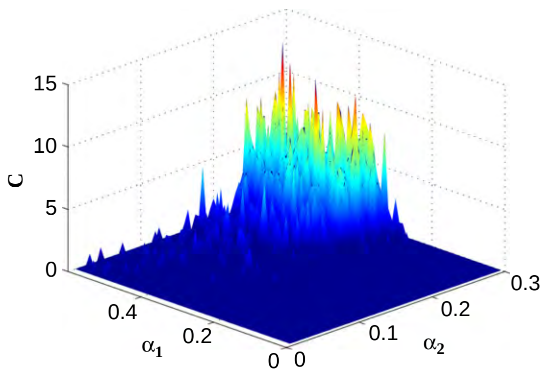

Fig. 7. Number of conflicts at the intersection in space (α_1, α_2).

are in the jamming regime. This phase diagram is different from the former research [20,21]. It consists a transition regime which may be caused by the games between the vehicles.

4. Conclusion

In this paper, the effect of defectors at non-signalized intersection is investigated by establishing a simulation model. This model is used to reproduce the intersection operation. It includes two parts: vehicle movement and intersection dynamics. The vehicle movement is described by the NaSch model which is one of cellular automaton models. In the intersection dynamics, there are two types participators: cooperator (C) and defector (D). Cooperators obey traffic rules

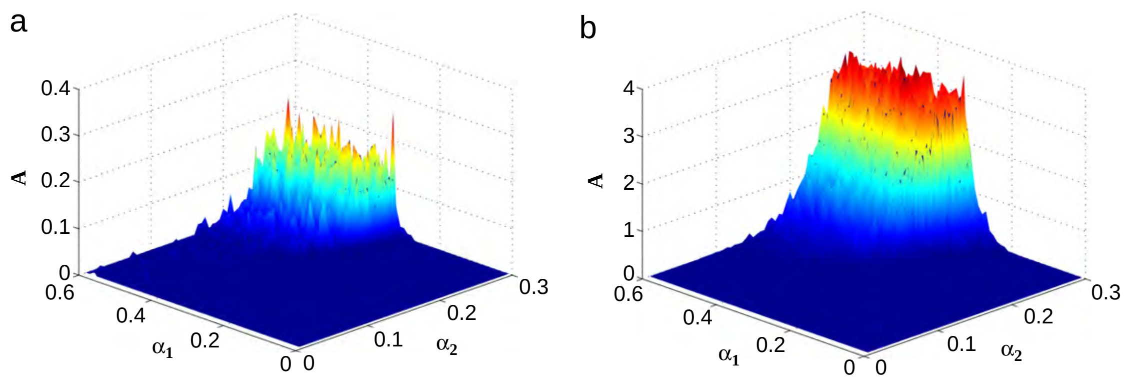

Fig. 8. Number of defectors per second in (a) road 1 and (b) road 2 in space (α_1, α_2).

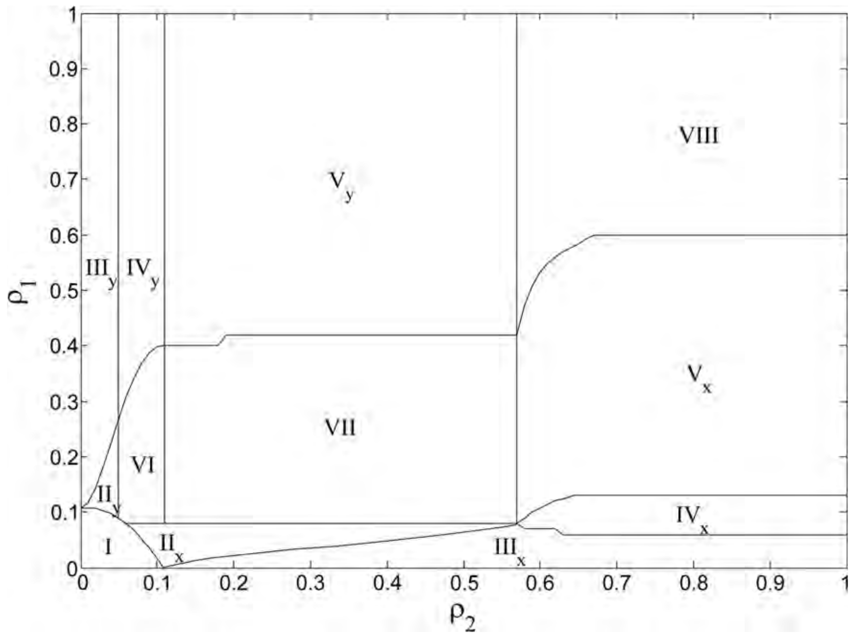

Fig. 9. Phase diagram of intersection in space (ρ_1, ρ_2).

of intersection but the defectors do not. Because the defector is not inherent existence, the Weibull distribution is used to describe the generation of defectors when vehicles are waiting before the intersection. Four strategies are used to deal with the assignation of right-of-way when different vehicles meet at intersection. The influence of defectors at the intersection is found out by the analysis of delay.

Through the simulation of the non-signalized intersection system, some results are obtained. First, the existence of the defector can make the traffic conditions of the main road worse. But it improves the traffic conditions of the side road. For the intersection, the range of delay has no obvious change. But in the special region of the inflow rate, it can reduce the delay of intersection and enhance the capacity of intersection system. Second, the existence of the defector leads to more conflicts. These conflicts sometimes may lead to traffic accidents which makes the intersection does not work for a while or even longer. In general, the existence of the defector is benefit for the capacity of intersection, especially for the side road. But the existence of the defector also reduces the safety of intersection due to generating more conflicts. Third, the phase diagram of intersection is obtained in the framework of games theory. It finds a transition regime which is different from former research. In this region, the road flux increases nonlinearly. This may help to understand the effect of vehicle behaviors.

However, this study still has several limitations, which requires improvements in future study. The failure rate function should be calibrated by the empirical data. It will make the behavior change process more reality. And the road can be expanded to two or more lanes. That will make the drivers consider more vehicles to pass the intersection.

Acknowledgments

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