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Strength Reduction Factors in Performance-Based Design

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SUMMARY

Strength reduction factors that are used to reduce design forces in earthquake resistant design are discussed. Based on recent research, the paper presents the different components of the so called R factors and discusses how these can be incorporated into a performance-based earthquake resistant design. The first component discussed is the reduction in lateral strength demand produced by nonlinear behavior in the structure which takes into account the hysteretic energy dissipation capacity of the structure. The paper presents first a summary and comparison of recent statistical studies on strength reduction factors computed for single-degree-of-freedom systems undergoing different levels of inelastic deformation when subjected to a large number of recorded earthquake ground motions. Despite having used significantly different ground motions data bases, results from various studies are remarkably similar. The main parameters that affect the amplitude of strength reductions are discussed. The evaluation of the results indicates that strength reductions due to nonlinear behavior are primarily influenced by the maximum tolerable displacement ductility demand, the period of the system and the soil conditions at the site. Based on these parameters simplified expressions that can be used in codes are presented. The paper then describes how strength reduction factors derived from single-degree-of-freedom systems need to be modified in order to be used in the design of multi -degree-of-freedom systems. Reductions in design forces due to overstrength are discussed. These reductions are due to the fact that the lateral strength of a structure is typically higher and in some case much higher than the nominal strength capacity of the structure. These reductions can be divided to take into account the additional strength from the nominal strength to the formation of the first plastic hinge and the additional strength from this point to the formation a mechanism. Finally, the paper discusses how these reductions factors can be implemented in performance-based design.

INTRODUCTION

Design lateral strengths prescribed in earthquake-resistant design provisions are typically lower and in some cases much lower than the lateral strength required to maintain a structure in the elastic range in the event of severe earthquake ground motions. Strength reductions from the elastic strength demand are commonly accounted for through the use of reduction factors. In U.S. practice the reduction factors are called response modification factor, R, in the National Earthquake Hazard Reduction Program (NEHRP, pp. 35-39) or system performance factor, R, in the Uniform Building Code (UBC-1988) and the Structural Engineers Association of California (SEAOC-1988). While reduction factors prescribed in seismic codes intent to account for damping, energy dissipation capacity as well as for overstrength, the level of reduction specified in seismic codes is primarily based on the observation of the performance of different structural systems in previous strong earthquakes. Strength reduction factors are one of the most controversial aspects of current buildings codes. Several researchers have expressed their concern about the lack of rationality in current R factors and their improvement has been identified as a way to improve the reliability of present earthquake-resistant design provisions (Bertero, 1986, Uang 1991, ATC, 1995b).

COMPONENTS OF STRENGTH REDUCTION FACTORS

REDUCTIONS DUE TO NONLINEAR BEHAVIOR

One of the first and better studied components of the R factors is the reduction in strength demand due to nonlinear hysteric behavior in a structure. The component of the strength reduction factor due to nonlinear hysteretic behavior, [R subscript mu), is defined as the ratio of the elastic strength demand to the inelastic strength demand,

$$R_\mu = \frac{F_y(\mu = 1)}{F_y(\mu = \mu_i)}$$

Equation 1

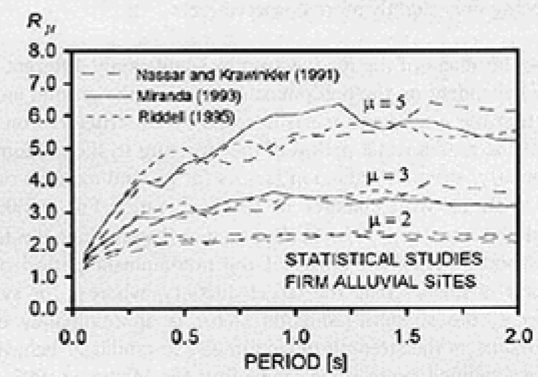
where [F subscript y (mu = 1)] is the lateral yielding strength required to maintain the system elastic; [F subscript y (mu = mu subscript i) is the lateral yielding strength required to maintain the displacement ductility demand [mu] less or equal to a predetermined maximum tolerable displacement ductility ratio [mu subscript i].

In general, for structures allowed to respond nonlinearly during earthquakes ground motions, inelastic deformations increase as the lateral yielding strength of the structure decreases (or as the design reduction factor increases). For a given ground motion and a maximum tolerable displacement ductility demand [mu subscript i], the problem is to compute the minimum lateral strength capacity [F subscript y (mu = mu subscript i)] that has to be supplied to the structure in order to avoid ductility demands larger than [mu subscript i]. Alternatively, for a given elastic design spectrum, the problem is to compute the maximum strength reduction factor that can be used in order to avoid ductility demands larger than [mu subscript i].

For design purposes, [R subscript mu] corresponds to the maximum reduction in strength that can be used in order to limit the displacement ductility demand to a maximum tolerable ductility demand [mu subscript i] in a single-degree-of-freedom (SDOF) system that will have a lateral strength equal to the design strength.

For a given ground motion, computation of [F subscript y (mu = mu subscript i)] involves iteration, for each period and for each target (i.e., maximum tolerable) ductility ratio, on the lateral strength [F subscript y] until the computed ductility demand [mu] is, within a certain tolerance, the same as the target ductility ratio [mu subscript i]. For a given ground acceleration time history, a [R subscript mu] spectrum can be constructed by plotting the strength reduction factors (computed with Eq. 1) of a family of SDOF systems (with different periods of vibrations) undergoing different levels of inelastic deformation [mu subscript i] when subjected to the same ground motion.

FIGURE 1.
COMPARISON OF MEAN STRENGTH REDUCTION FACTORS FOR FIRM SITES



Miranda and Bertero (1994) recently summarized the results of 13 different studies on strength reduction factors due to nonlinear behavior carried out in the last 30 years and put them in a common format in order to facilitate their direct comparison. A comparison of mean strength--reduction factors for systems subjected to ground motions recorded on firm alluvium sites from three different studies (Nassar and Krawinkler, 1991, Miranda, 1993; Riddell, 1995) is shown in Figure 1. The curve obtained by Nassar and Krawinkler was computed with 15 ground motions recorded in firm sites in California, Miranda's curve is computed from 62 ground motions recorded on firm alluvium sites in several countries during different earthquakes, while Riddell's curve is computed from 34 ground motions recorded on firm sites in Chile primarily during the march 3, 1985 earthquake. Although these studies are based on different sets of ground motions, the similarity of the results is remarkable and suggests that the general trends in reduction factors due to nonlinear behavior do not change significantly from one seismic region to another.

Based on the results of a comprehensive statistical study on strength-reduction factors of SDOF systems undergoing different levels of inelastic deformation when subjected to 124 ground motions, Miranda (1993) proposed simplified expressions to obtain analytical estimates of the strength-reduction factors for rock, alluvium and soft soil sites. Similarly, Nassar and Krawinkler (1991) and Riddell have recommended simplified expressions. However, none of these expressions have been incorporated into code provisions. Miranda's study showed that although some differences exist between strength reduction factors for rock and firm alluvium sites, for practical applications these differences are relatively small and can be neglected. If one makes such simplification and in the absence of more specific information on site conditions one could use the following simplified expression in the design of structures built on rock or firm sites:

$$R_\mu = \mu + (1 - \mu)\exp\left(\frac{-16\,T}{\mu}\right)$$

Equation 2

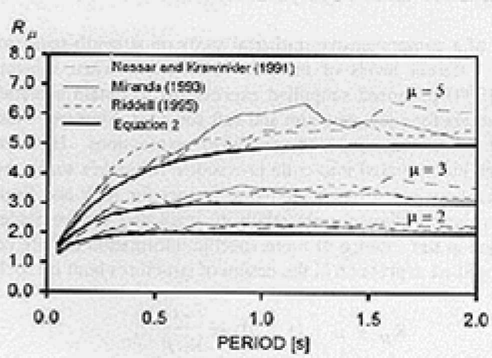
where [mu] is the displacement ductility ratio and T is the period of vibration. Equation 2 is simpler than the equations previously proposed and as shown in Figure 2 has very good agreement with the statistical studies being only slightly more conservative.

For very soft soil sites, the shape of the [R subscript mu] spectrum is significantly different to that of rock and firm sites and strongly dependent on the predominant period of the ground motion [T subscript S]. Studies by Miranda have shown that the elastic and inelastic response of structures on very soft soil sites depends on the ratio of the fundamental period of the structure to the predominant period of the soft soil site, [T/T subscript S]. Similarly, strength-reduction factors for ground motions recorded on soft-soil sites exhibit strong variations with changes in the [T/T subscript S] ratio. For periods closer than the predominant period of the site (i.e., [T/T subscript S is approximately equal to 1)] [R subscript mu] is much larger than the target ductility. For systems with periods shorter than two thirds of the predominant period of the soil site, the strength-reduction factor is smaller than the target ductility, whereas for systems with periods longer than two times [T subscript S] the strength-reduction factor is approximately equal to the target ductility. Further discussion on the strength reduction due to nonlinear behavior in structures on soft soil sites as well as simplified expressions can be found in Miranda (1993, 1996).

The dispersion on strength-reduction factors have been recently studied (Miranda, 1993 1-Riddell, 1995). These studies have concluded that with the exception of very short periods (T < 0.2 s), the coefficient of variation (COV) of [R subscript mu] is approximately period independent and that the dispersion increases with increasing displacement ductility ratio. COV's vary from 0.2 for ductility ratios of 2 to 0.5 for ductility ratios of 6.

Nassar and Krawinkler (1991) and Miranda (1993) studied the influence of earthquake magnitude and epicentral distance on the strength-reduction factors. Both studies concluded that the effect of both parameters is negligible on [R subscript mu].

FIGURE 2.
RESULTS OF STATISTICAL STUDIES COMPARED WITH THE PROPOSED EQUATION.



Miranda (1996) has shown that the use of approximate reduction Pictors like those computed with equation 2 combined with the use of smoothed linear elastic response spectra (SLERS) can lead to very good estimates inelastic strength demands (i.e., lateral strength required to control displacement ductility demands).

MODIFICATIONS FOR MDOF SYSTEMS

The [R subscript mu] factors previously discussed can be used for the design of structures which can be approximately modeled like a SDOF system. However, most structures need to be modeled as multi-degree-of-freedom (MDOF) systems and have a much more complex behavior than SDOF systems, particularly in the nonlinear range. Thus, the [R subscript mu] factors for SDOF systems need to be modified for the design of MDOF structures. It is proposed that the [R subscript mu] factor be multiplied by a [R subscript M] modifying factor that takes into account the possible concentration of displacement ductility demands in certain floors, thus the product of [R subscript mu] and [R subscript M] represents the maximum strength reduction factor that will produce an adequate control of story displacement ductility demands in structures that have a strength equal to the design strength. The [R subscript M] factor is defined as follows

$$R_M = \frac{R_{MDOF}}{R_{SDOF}} = \frac{R_{MDOF}}{R_\mu} = \frac{F_{y\,SDOF}}{F_{y\,MDOF}}$$

Equation 3

where [R subscript MDOF] is the ratio of the lateral yielding strength required in the MDOF structure to remain elastic to [F subscript yMDOF] which is the lateral yielding strength required in the MDOF structure to avoid story displacement ductility demands larger than the maximum tolerable story displacement ductility ratio [mu subscript i]; and [R subscript SDOF] is equal to the previously defined [R. subscript mu] factor.

A study of the [R subscript M] factor is currently being conducted by the author. As part of this study, three reinforced-concrete SMRSF 8, 12 and 16 stories high were designed according to a strong column-weak beam philosophy and were subjected to three ground motions with a variable amplitude until maximum story displacement ductilities of 3, 4 and 5 were produced and until the buildings remain totally elastic. Strength reduction factors for equivalent SDOF models of the buildings undergoing the same levels of displacement ductility demands when subjected to the same records were also computed. The equivalent SDOF systems had a period of vibration equal to the fundamental period of vibration of the MDOF structures. Table I shows the [R subscript M] factor computed for story displacement ductility ratios of 3, 4 and 5 when subjected to the three ground motions. Although these results are only preliminary, two general trends can be observed: (a) [R subscript M] decreases with increasing story displacement ductility ratio (design base shear in MDOF structures increases with respect to SDOF structures with increasing ductility ratio); (b) [R subscript M] decreases with increasing number of stories (design base shear in MDOF structures increases with respect to SDOF structures with increasing number of stories).

Based on these results and other limited results presented by Nassar and Krawinkler (1991), the following preliminary equation is proposed for [R subscript M]

$$R_M = \left[1 + 0.15\,T^2 \cdot \mathrm{Ln}(\mu)\right]^{-1}$$

Equation 4

TABLE 1.
MODIFICATION OF STRENGTH REDUCTION FACTORS FOR MDOF STRUCTURES

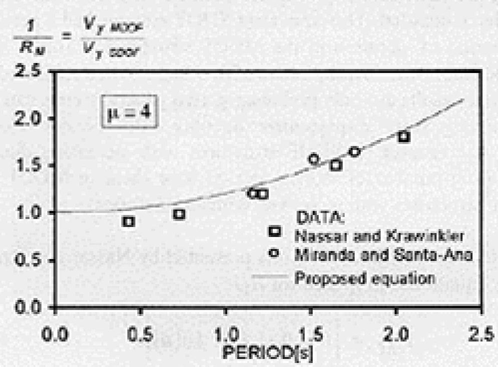
NUMBER OF STORIES	PERIOD OF VIBRATION	STORY DUCTILITY	R_M SCT19.EW	R_M 0625.EW	R_M 5625.NS	$\overline{R}_M$
8	1.16 s	3	0.85	0.83	0.88	0.85
		4	0.84	0.80	0.85	0.83
		5	0.82	0.79	0.84	0.82
12	1.52 s	3	0.67	0.71	0.78	0.72
		4	0.56	0.68	0.68	0.64
		5	0.49	0.60	0.60	0.56
16	1.76 s	3	0.61	0.72	0.67	0.67
		4	0.61	0.61	0.62	0.61
		5	0.60	0.60	0.64	0.61

where T and [mu] are the period of vibration and the maximum tolerable story displacement ductility demand in the NMOF structure, respectively. A comparison of available data with the results of Eq. 4 is shown in Figure 3. Caution should be exercised in the use of Eq. 4 for design purposes as it is based on only few results. Furthermore, it is intended only for regular buildings in plan and in elevation and designed with strong columns-weak beams, so the use of Eq. 4 for other situations can lead to unconservative results.

REDUCTIONS DUE TO STRUCTURAL OVERSTRENGTH

For design purposes [R subscript mu] times [R subscript M] corresponds to the maximum reduction in strength that can be used in order to limit the maximum story displacement ductility demand to a maximum tolerable limit the pre-determined target ductility [mu subscript i] in a structure that will have a lateral strength equal to

FIGURE 3.
MODIFICATION FOR STRENGTH REDUCTION FACTORS FOR MDOF STRUCTURES.



the design lateral strength. An additional strength reduction can be considered in the design of a structure to take into account the fact that structures usually have a lateral strength higher than the design strength. These additional reductions can be divided into reductions due to element overstrength [R subscript SE] which accounts for the increase the lateral strength of the structure from the design strength to the strength associated to the formation of the first plastic hinge and reductions due to redundancy, strain hardening and other factors [R subscript SS] which increase the lateral strength of the structure from the strength associated to the formation of the first plastic hinge to the strength associated to the formation of a mechanism. Thus the suggested reduction factor to be used in design would be given by:

$$R = R_{\mu} \cdot R_M \cdot R_{SE} \cdot R_{SS}$$

Equation 5

For a more detailed discussion on strength reductions due to overstrength the reader is referred to Miranda (1991) or ATC (1995a).

IMPLEMENTATION OF R FACTORS IN PERFORMANCE-BASED DESIGN

In performance-based design an adequate design is produced when a structure is dimensioned and detailed in such a way that the local deformation demands are smaller than their corresponding maximum tolerable limits for each performance level. Ideally, the deformation demands and deformation capacities must be checked at the critical region of all members (i.e., at all plastic hinges) by checking the maximum strain, the maximum strain ductility ratio [mu subscript epsilon] the maximum curvature, the maximum curvature ductility ratio [mu subscript phi], the maximum rotation or the maximum rotation ductility [mu subscript Theta] with their corresponding limits, however in the preliminary design of a structure the final sizing and detailing is not known, and other parameters at a more global level are more suitable. For preliminary design purposes the author believes that, with the information known to date, the best parameters to achieve an implementation of performance--based design are the story displacement ductility demand and the interstory drift demand, which are related to each other by the story yield displacement. While these parameters do not take into account for cumulative damage in structural members and may have other disadvantages, they have several important advantages: (a) are very simple parameters; (b) structural engineers are familiar with them; (c) most experimental research is based in these parameters, so with a careful calibration in the maximum tolerable limits they can provide an adequate damage control for different performance levels.

The limits in story ductility demands, as well as the limits in interstory drift, vary with the structural system and with the performance level. For example, the maximum tolerable story ductility demand in a steel special-moment-resisting-space frame (SMRSF) is larger than for a concentrically-braced steel frame. Similarly, for the steel SMRSF the maximum tolerable demands will be different for example in the Life Safe performance level and for the Near Collapse performance level. Thus, during the preliminary design of a structure there is a need to estimate the lateral strength (lateral load capacity) of the structure that is required in order to limit the global (structure) displacement ductility demand and the global drift demand to a certain limit which results in the adequate control of local (i.e. story) ductility demands and interstory drifts. If the elastic design spectra are known for each earthquake design level, the R factors permit an estimation of such required lateral strength, particularly for the life safe, near collapse and collapse performance levels. Implementation of R factors in performance-based design requires the specification of such maximum tolerable story ductility demands and maximum tolerable interstory drift demands for each structural system and for each performance level. An important contribution to presently proposed performance-based design methodologies would be the specification and calibration of such limits.

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