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NPV-based decision support in multi-objective design using evolutionary algorithms

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ABSTRACT

Optimum design problems are frequently formulated using a single excellence criterion (minimum mass or similar) with evolutionary algorithms engaged as decision-support tools. Alternatively, multi-objective formulations are used with a posteriori decision-making amongst the Pareto candidate solutions. The former typically introduces excessive simplification in the decision space and subjectivity, the latter leads to extensive numerical effort and postpones the compromise decision-making. In both cases, engineering excellence metrics such as minimum mass can be misleading in terms of performance of the respective design in the given operational environment. This paper presents an alternative approach to conceptual design where a compound objective function based on the Net Present Value (NPV) and Internal Rate of Return (IRR) aggregate performance metrics is developed. This formulation models the integral value delivered by the candidate designs over their respective life-cycles by applying value-based NPV discounting to all objectives. It can be incorporated as an a priori compromise and consequently viewed as a weighted sum of individual objectives corresponding to their economically faithful representation over the entire operational life-time of the designs. The multi-objective design optimization is consequently expanded from purely engineering terms to coupled engineering–financial decision support.

1. Introduction

In most technical problems, the optimality criteria cannot be expressed in terms of a single objective. Multi-objective optimization problems deal with problems where the formulation is characterized by several optimality criteria that individually do not yield coincident relative optima in the design variables space. In such problems, best-compromise formulations determine the trade-off optima that are assumed to truthfully model the combined impact of the individual excellence criteria.

In design optimization and decision-making, the constraints are typically more obvious then appropriate measures of design excellence. In many cases, objective functions express the criterion of minimum mass or some similar tangible physical metrics directly linked to the design geometry. However, this does not necessarily correspond to the notion of value that the user experiences related to the respective product. His/her impression of the product excellence is wider and includes both the initial acquisition-related terms and the operation-related terms throughout the life-cycle of the product, involving issues of performance.

The user's notion of the product excellence would in most cases be composed of a number of qualitative criteria including the acquisition terms, operational expenses, safety, reliability or failure rate, versatility and adaptivity, environmental friendliness, subjective and intangible expectations, etc. A decision-support procedure is therefore needed to model excellence in the objective function and result in the particular design which maximizes the overall life-time performance corresponding to the given distribution of operational conditions.

Those multi-objective optimization problems where all partial objectives can be represented by economic value equivalents can potentially be formulated as single-objective best-compromise problems as shown here. However, this implies that the model of the problem needs to go beyond the purely technical specification and must be expanded to the equivalent economic model for decision support that accounts for all the economic aspects as functions of the design variables. In addition, the distribution of the future operating conditions needs to be specified over the entire life-span. Appropriate value-based modeling of the excellence criteria is necessary and objectives need to be attributed corresponding valuation terms making the proposed approach to optimum design a value-based decision-support model.

Within the project performance valuation methodology in the business assessment environment, evaluation of projects typically encompasses the integral valuation of performance over the entire project life-cycle. This includes the valuation and cumulation of all recurrent elements (ongoing operation) and non-recurrent investment-related inputs. In order to accomplish the formulation

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of an aggregate indicator, all economic (value) flows are included in terms of their respective tangible value and attributed to increasing or decreasing the respective economic potential. This is done by time-based discounting and aggregation of individual elements. Such an approach leads to integral measures of excellence named Net Present Value (NPV) and Internal Rate of Return (IRR).

A measure of integral value-based design excellence simplified for the purpose of optimum design as developed in this paper is the total cost of operation (TCO) that includes all due costs. This approach proposes the measurement of design excellence expressed in terms of several optimality criteria based on the respective overall life-time value equivalents. The prerequisite for such a formulation is that all the individual optimality criteria can be expressed by corresponding value contribution terms, which have to be aggregated for the given distribution of different operational regimes during the corresponding product life-span. This provides the decision metrics for total value and/or total cost in constructing the fitness functions in optimum design. Multi-objective design optimization is consequently expanded from purely engineering terms to coupled engineering–financial decision-making by an a priori value-related compromise. In the simple form as developed in this paper, the design variables and constraints are modeled in engineering terms only, with objectives formulated in the 'expanded space' using equivalent economic values. In many cases, individual optimality criteria can be expressed equivalently by their respective economic impact during the entire respective life-span. In those cases, particular values of physical and technical design variables result in economic categories such as investments, operational expenses, revenues, etc. In those cases, a single, economically realistic, best-compromise criterion can be defined as an aggregate measure for the domain of candidate solutions. This implies that respective investment costs and all accompanying operating costs over the entire life-time of all candidate solutions (parameterized by optimization variables) need to be included.

2. Value-based excellence approach to optimum design

The approach presented here is to treat multi-objective optimum design using an a priori compromise formulation based on the full life-span value aggregate indicator. This measure is intended to truthfully encompass the integral economic value associated with the product and hence be an adequate indicator of the total excellence of the candidate designs. In order to do so, non-recurrent elements (investments) and recurrent elements (various costs during operation) must be taken into account. Due to the time-dependent character of economic value, all these elements must be expressed in terms of their respective equivalent economic value at a defined moment of reference, discounted accordingly, and aggregated into an integral measure. The companion aspect with multi-criterial optimization is multiple criteria decision-making or compromise decision-making, where a number of a priori and a posteriori approaches are used.

The challenging and complex problem of handling multiple objectives and dealing with constraints in terms of general multi-criterial decision-making is dealt with by many researchers, (Anderson, 2000; Coello, 1999). Generally, full generation of the Pareto front implies extensive and expensive computation, and a priori compromise formulations offer inexpensive computation but provide less (sometimes subjective) information.

The concept of the Net Present Value (NPV) and Internal Rate of Return (IRR) aggregate measures (Behrens and Hawranek, 1991; Bendeković et al., 1993) has been applied in business project evaluations and operations research, but typically not in the framework of design optimization. The question of formulating best compromises is partly related also to problems with variable operating conditions and working regimes since the decision-making must be linked to their given distributions over the entire life-span, i.e. there is a link to robust optimization. In recent years, significant effort was dedicated to integrating business objectives into the early stage product design processes. A survey of the current status of integration of optimization techniques (Saitou et al., 2005) in product development problems also discusses mass vs. cost as excellence criteria. Several authors (Gu et al., 2002; Marston and Mistree, 1998; Wassenaar and Chen, 2001) have built on the decision-based-design approach proposed by Hazelrigg (1998), combining market-related and engineering-related attributes of products in single-criterion formulations of excellence in the form of overall economic benefit. Formulation of adequate utility functions for the design problem are a matter of ongoing research. The coordination and balancing of marketing, engineering design and manufacturing subproblems in the framework of the overall utility is discussed in Michalek et al. (2006). These papers argue that problem modeling and formulation receives little attention in research, although in many cases it has more impact than applied algorithms on finding the optimal solutions. In Hallerbach and Spronk (2002) multi-criterial financial decision-making is discussed, and several authors include the investment terms and operational expenses (such as running costs) as elements of objective functions in the conceptual design phase. Trade-offs and design strategies are discussed in Otto and Antonsson (1991) in the framework of overall design preferences, and a flexible decision support framework for design is discussed in Olewnik and Lewis (2006).

An elaboration of cost terms (Laan and Tooren, 2005) in the design of aircraft movables also lists many items that enter the corresponding feasibility evaluation. An example of applying value-based compromise decision-making is also found in Markish and Willcox (2002) for applications with aircraft design. Combining different ingredients of cost as excellence criteria in the framework of design optimization of aircraft elements (Harris, 2002) was also considered from the viewpoint of a cost/performance trade-off problem.

Performance-based and value-based designs are also compared in Peoples and Willcox (2006) using deterministic and probabilistic NPV-based measures and other, also taking risk and uncertainty into account. Maximization of profits based on NPV was utilized (Eliasson, 2000) as a design tool in the conceptual planning of hydropower stations. In Georgiopoulos et al. (2002), the decision problem based on expected value of NPV for the firm (automotive) is discussed as the objective function, which depends both on the product portfolio and the product design variables, including uncertainty. An application in industrial (automotive) R&D and design is presented in Suh et al. (2004), where product platform components are optimized for flexibility with performance- and economic objectives, using structural and economic simulation under uncertainty (NPV and Monte Carlo). Design-related issues of robust optimization (Pediroda et al., 2005) and reliability-based optimization (Papadrakakis, 2007) are increasingly being introduced in decision-making.

The simplest definition of the compromise optimality criterion $f(\boldsymbol{x})$ is obtained by introducing weight factors w_k for a total of K objectives that belong to individual (k) partial objective functions $f_k(\boldsymbol{x})$

$$f(x) = \sum_{k=1}^{K} w_k f_k(x), \quad \sum_{k=1}^{K} w_k = 1, \ w_k > 0 \tag{1}$$

and prescribe the fraction of relative impact of an individual criterion in the overall excellence of the design (parameterized by the vector of design variables $\boldsymbol{x}$). The choice of the weight factors, in many cases rather arbitrary and subjective, essentially determines the compromise optimum.

The concept of Pareto-optimality is the standard approach in looking at multi-objective problems. A point $\boldsymbol{x}^{opt}$ is Pareto-optimal if there is no point $\boldsymbol{x}$ such that

$$f_k(x) < f_k(x^{opt}) \text{ for at least one } k \in K \tag{2}$$

$$f_k(x) \leq f_k(x^{opt}) \text{ for all } k \in K \tag{3}$$

i.e. if there is no other point in which at least one of the objective functions is reduced without increasing any other.

The final selection of the particular optimum design from the Pareto set involves a decision on the best compromise criterion. One possibility is to select that particular Pareto point where the maximum offset of the values of the normalized individual criteria from the respective 1D optima has the minimum value.

$$\min\left\{\max_k\left(\frac{|f_k(\mathbf{x}) - f_k^{\min}|}{f_k^{\min}}\right)\right\} \tag{4}$$

This is, however, numerically expensive since it implies solving for K single-objective minima or their approximations.

In this paper, a formulation of the best trade-off is developed based on total project impact valuation as used in project feasibility assessments and appraisal (Behrens and Hawranek, 1991; Bendeković, 1993). This approach provides aggregate metrics and a framework to include cumulated discounted values of all individual economic flows. This approach is well-founded in the sense that it removes subjective judgment of the decision maker in the best compromise formulation and also provides decision support based on total lifetime performance of the product. It allows for both a priori compromise definition and a posteriori decision-making.

In some generic example, let the overall design excellence be a composition of the following qualitative criteria:

1. Low investment price
2. Low operational expenses
3. High revenue generation (performance-related efficiency)
4. High safety in operation
5. Low failure rate and down-time
6. High adaptivity to variable operational conditions, low loss in efficiency beyond nominal design-operating regimes
7. Environmental friendliness
8. Satisfaction of non-technical intangible user's expectations
9. Other excellence criteria

The objectives 1, 2, and 7 typically directly (deterministically) depend on the values of the technical design variables (Eqs. (1)–(4), vector $\boldsymbol{x}$) of candidate designs, therefore they can be valuated by some measure and aggregated into NPV or IRR. The objective 3 also directly depends on the design variables and also enters the NPV directly after discounting.

The criteria 4 and 5 involve uncertainty and can generally also be related to the design variables and indirectly incorporated in the NPV since they have probabilistic and statistically predictable impact on NPV. The criterion 6 is related to the operational efficiency in changing operational conditions and to the concept of robust optimization. Its impact on the overall design excellence can also be quantified in the NPV measure if the frequency distribution of the operational regimes over the life-span can be specified. In such a case, the impact of criterion 6 can be measured in terms of the metrics of item 3 and the NPV decision criterion in fluctuating conditions essentially results in the corresponding robust optimum.

The impact of reliability-related criteria 4–5 can be quantified in the design phase by applying Monte Carlo based simulations for given fluctuations of parameters (for example Papadrakakis, 2007) and consequently be attributed value terms to contribute to the overall NPV accordingly (Fig. 1).

The criteria 8 and 9 can generally not be directly attributed corresponding equivalent value in a deterministic way, as they, in addition to the design variables, also depend on non-technical terms and the project environment conditions as well as interactions of the project and external circumstances. In some cases, however, these criteria can also be related to the design variables and modeled by empirical methods.

3. The multi-objective best-compromise decision criterion based on NPV and IRR

The net profit or loss P_N (Bendeković et al., 1993) for a single successive time period (i) can be expressed as

$$P_N(i) = I(i) - M_C(i) - A(i) - W_C(i) - F(i) - T(i) \tag{5}$$

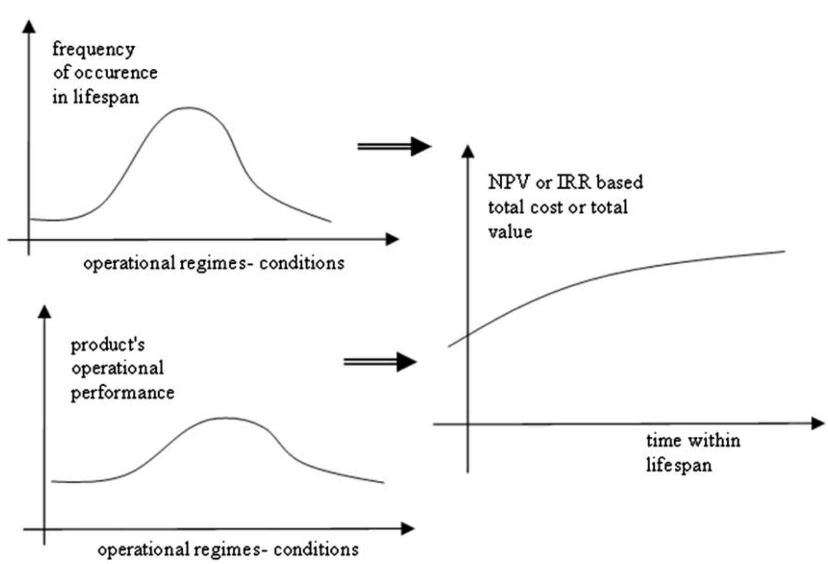

Fig. 1. NPV and IRR for variable operating conditions.

where I is the income, M_C the material costs (material, energy, other costs), A the corresponding amortization, W_C the cost of work, F the financial expenses, and T the taxes. The net economic flows (E_N), which cumulatively give an integral measure of the economic potential of the project, can be calculated as

$$E_N(i) = I(i) + R(i) - IN(i) - M_C(i) - W_C(i) - T(i) \tag{6}$$

where R is the project's released or residual value, IN incremental investment into fixed assets and working capital. The economic flows are analogous to the financial flows but do not include the financial flows (such as inflows, F, other) from and towards external financial sources since they are (in their total impact) neutral in changing the economic potential of the project (Fig. 2). Alternatively:

$$E_N(i) = P_N(i) + A(i) + R(i) - IN(i) \tag{7}$$

Based on the time-line in Figs. 2 and 7, the individual investment (it) and operating (ot) expenses can be combined into the NPV indicator as

$$NPV = \sum_{i=0}^{n} \frac{E_N(i)}{(1+D)^i} \tag{8}$$

where D is the rate of discounting and n the number of successive periods. The IRR indicator is defined as that particular rate of discounting (D^*) that adjusts the NPV value (Eq. (8)) to zero, which is evaluated iteratively:

$$IRR = D^* \Rightarrow NPV(D^*) = \sum_{i=0}^{n} \frac{E_N(i)}{(1+D^*)^i} = 0 \tag{9}$$

The NPV is interpreted as the equivalent time-zero cumulative value of all discounted E_N elements within the life-cycle, or the zero-point value of the total economic potential of discounted individual contributions (Eqs. (6) and (7)). The NPV and/or IRR indicators can therefore also be used as the best compromise criteria for the objective function in the sense of the total cost of operation (TCO) criterion. Alternatively, the TCO criterion based on Eq. (8) or (9) can be viewed as a way to determine the economically realistic trade-off values of the weight factors w_k (best compromise set) in the composed objective function in Eq. (1).

In terms of the corresponding decision support, Eqs. (8) and (9) imply that beyond the technical design variables the excellence criteria also depend on several parameters of the interaction between the project and its environment, such as the life-span or the discounting rate. Based on Eqs. (8) and (9), the total-value or total-cost optimality measures also introduce additional degrees of nonlinearity into the standard engineering optimization model. This is due to several sources in the valuation process such as the discounting process in NPV/IRR, costs of financing, stepwise taxation schemes, stepwise availability and cost structures of equipment, material, work and other resources, etc. In the general form, the NPV/IRR best compromise formulation of the objective function provides an economically trustworthy measure of optimality and a corresponding objective decision-support algorithm. However, it generally expands the set of design variables from the original (purely geometric and technical) design space to an expanded set that additionally includes economic variables such as composition of financing, amortization-depreciation terms, timing of individual terms, management of capital, etc.

In this paper, a simplified total cost formulation (TCO) formulation is proposed for the optimum design decision-support problem, which takes into account only the costs and disregards some of the additional freedom provided by the economic variables. The expansion of the engineering set of design variables can in this case be avoided while still preserving the above value-based formulation of the best compromise criterion.

In the proposed simplified case (TCO), Eqs. (7) and (8) can be relaxed and combined, resulting in the objective function:

$$\min\{NPV_{TCO}\} = \min\left\{ IN(0) + \sum_{i=1}^{n} \frac{M_C(i) + W_C(i)}{(1+D)^i} \right\} \tag{10}$$

The assumptions in Eq. (10) are as follows:

- no tax and amortization terms
- $I(i)=0$, insignificant for the TCO objective function formulation
- $R(i)=0$, no residual value within the lifespan considered
- $IN(0)$=investment, typically cost of material and manufacturing
- $M_C(i)$=periodic material cost such as energy, maintenance material
- $W_C(i)$=periodic labor cost, for example maintenance

As a simple illustrative example for the decision criterion in Eq. (10), one can consider the problem of the optimum design of a simple beam under bending. One simplified approach to the objective function formulation would be that of minimizing the cross-sectional area (equivalent to minimum mass of material) and therefore investment cost. Another simplification would lead to the objective function formulation minimizing the contour of the cross-section. The latter is based on the argument that the contour is linked to the maintenance costs such as regular periodic surface maintenance and protection, which belong to the category of operational expenses. Without loss of generality, the production cost is here assumed independent of the cross-sectional shape (Fig. 3).

Constraints of maximum permissible stresses and dimensional ratio are applied:

$$\max\left\{\frac{M}{W}\right\} \le \sigma_{perm}, \quad \frac{h}{b} \le 3 \Rightarrow x_2 - 3 \cdot x_1 \le 0 \tag{11}$$

where W is the section modulus in bending. The objective function for this simple example includes the criteria of minimum mass and minimum (squared) length of contour

$$f_1 = 2x_1x_3 + (x_2 - 2x_3)x_3$$
$$f_2 = (2x_1 + 2(x_1 - x_3) + 4x_3 + 2(x_2 - 2x_3))^2 \tag{12}$$

with corresponding normalization and scaling. In this particular case, using:

n=life-span in years
A=cross-sectional area, mm^2
C=length of contour, mm
c_A=investment (non-recurrent) cost of material per unit cross-sectional area
c_{CW}=periodic cost of work per unit length of contour of cross-section
c_{CM}=periodic cost of material (maintenance) per unit length of contour

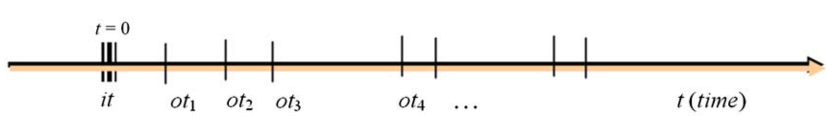

Fig. 2. Time-line for TCO evaluation with investment and operational costs.

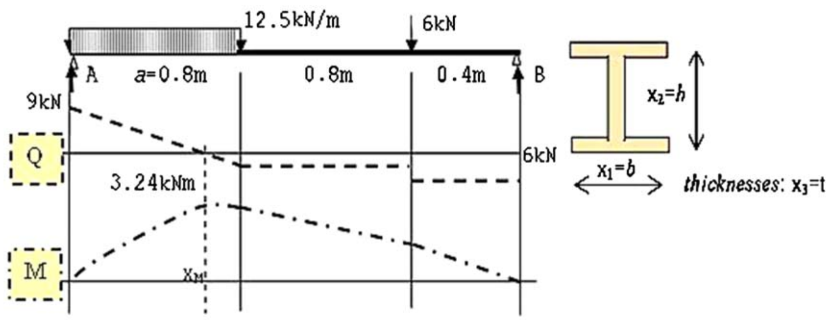

Fig. 3. Simple beam with shear forces Q and bending moments M.

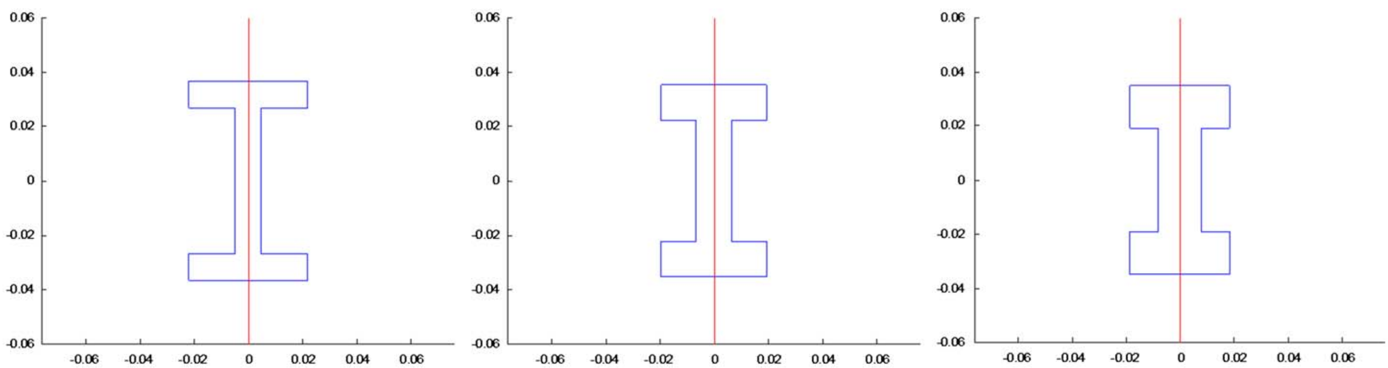

Fig. 4. Optimized designs for beam in Fig. 3, TCO formulation, variation of discounting rate D.

the following is used with Eq. (10):

$$IN(0) = c_A A, M_C(i) = c_{CM} C, W_C(i) = c_{CW} C \tag{13}$$

which yields the NPV-optimized shapes of the beam cross-section. The results for three different values of the rate of discounting are shown in Fig. 4 and numerically given in the table below:

D	0.5	0.1	0.01
n	10	10	10
c_A	1	1	1
c_{CM}	0.1	0.1	0.1
c_{CW}	0.1	0.1	0.1
$A \times 10^3$	1.37	1.60	1.75
C	0.302	0.271	0.256
f_1	0.013	0.015	0.017
f_2	0.028	0.022	0.020
x_1	0.043	0.039	0.037
x_2	0.073	0.070	0.069
x_3	0.009	0.013	0.015
$f_{(\mathrm{Eq.(10)})}$	0.024	0.043	0.055

A similar result as in Fig. 4 is obtained if the lifespan is varied, or if the cost coefficients are varied. In the extreme case where the maintenance cost (c_{CW}, c_{CM}) dominates the material cost, the optimized shape of the I-profile degrades to the rectangular cross-section (Fig. 5).

Beyond the elements largely determined by the values of design (engineering) variables, the total value formulation would in the general case (Fig. 6) also include the impact of non-deterministic parameters external to the project (market conditions, etc.).

The non-recurrent investment costs typically include product development and production, the recurrent operational expenses typically measure expenses related to the operation of the product in generating revenue (labor, maintenance, etc.) within the life-span. As shown in Eq. (5), depreciation, costs of financing and similar elements which have an impact on the economic potential (value) of the project also belong here. Decision-support procedures based on elements in Fig. 6, formulated as objective functions, yield engineering designs optimized for respective total business performance.

The coupled technical–financial integrated excellence metrics as proposed here is the natural criterion in decision-making for educated decision-makers aware of the economic consequences of technical parameters of the product. This applies for both possible roles of the decision maker: (1) potential buyer or commercial operator of the product, and (2) manufacturer of the product.

In the former case, the educated buyer will evaluate the product by including it into the respective business plan or feasibility study and comparing. A product with a lower price but causing higher running costs or lower income might turn out to be the less beneficial option. In the latter case, (2), manufacturers are well aware that the price of the product is not the single criterion in the customer's decision process. Moreover, the manufacturer is also directly responsible for the product's safety, failures, down-time, etc., with financial consequences for the direct potential damage but also indirect financial liability in terms of lost profits for the buyer/operator of the product.

In both of their potential roles, decision-makers make their decisions under the implicit objective of maximizing profits. Hence, the short-term superficial illusion of the lower price limits the decision criteria to the respective acquisition/manufacturing price, while the thorough long-term consideration includes the aggregated financial impacts altogether. In the latter case,

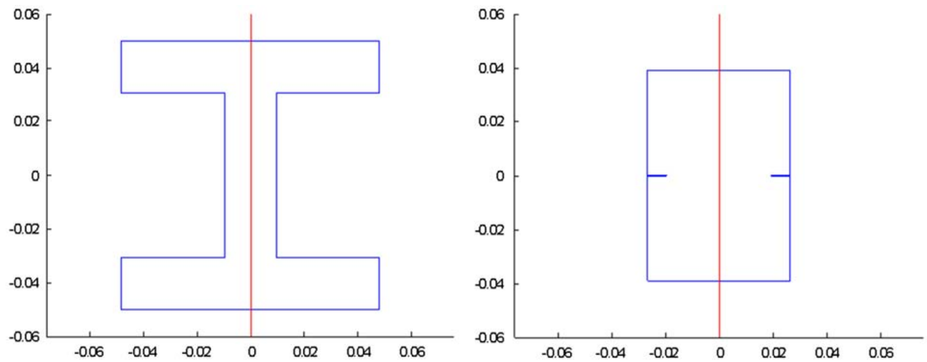

Fig. 5. Optimized designs for beam in Fig. 3, TCO formulation, increasing maintenance cost.

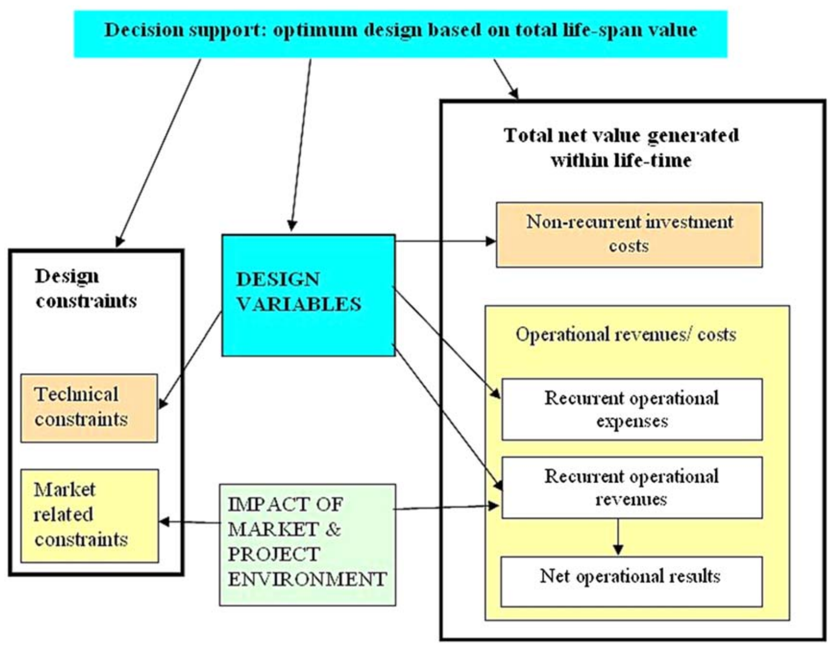

Fig. 6. Elements for total life-span value/cost modeling of aggregate design excellence for the NPV/IRR-based decision support.

technical product parameters need to be optimized for the total product performance measured in overall costs or profits metrics.

The Eqs. (5)–(10) are generic expressions that measure the integral commercial impact of a product measured in value-based terms as a function of the technical design variables-product properties. These expressions take case-specific forms depending on the particular product and its role in the corresponding business process and environment, which is termed coupled technical–economic modeling of product excellence in this paper. These expressions are functions that map design variables of a technical product into a value-based objective function that measures excellence.

For example, if the decision maker is in the role of the commercial operator of the product, the value of the term E_N in Eq. (6) is directly decreased due to product failures and downtime as the term I decreases and IN possibly increases. Lower adaptivity of the product to variable operating conditions has a similar impact on E_N as I is decreased due to lower average product efficiency (beyond nominal operating regimes) and likely higher values of M_C and W_C terms. All these terms, modeled quantitatively for the particular case, enter the terms in Eqs. (8)–(10). If the decision maker is in the role of the commercial manufacturer of the product, failing in designing products that are optimal in terms of value-based excellence criteria in Eqs. (5)–(10) will lead to reduced sales since potential customers (operators of the product) will in their own evaluations learn that the product is performing worse than the competitors' products within the framework of their business processes.

4. Optimum design of sandwich plates with corrugated core

Multi-layer metal sandwich plates with corrugated core are in this paper subjected to the NPV/IRR aggregate optimality criterion instead of the classical 'minimum weight' approach to design optimization. The term multi-layer sandwich plates refers to metal plates with corrugated core, intended for use as structural elements in applications ranging from construction to naval and interior architecture (Fig. 7).

Metal sandwich plates as high-performance structural elements are considered in detail in Vinson (2005). Different design topologies of periodic cellular metal sandwich structures, their respective manufacturing methods and design issues such as core and face failure modes are discussed in Wadley et al. (2003). Various topologies of sandwich plates are considered with several load cases and buckling modes using analytic and approximate

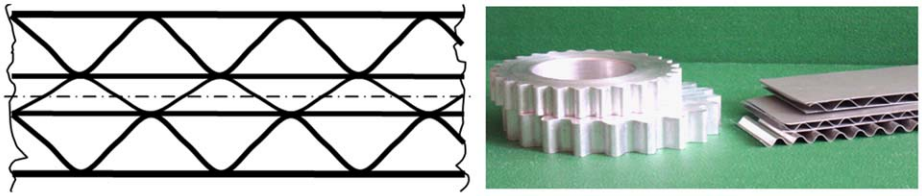

Fig. 7. Multi-layer composite metal sandwich plate with corrugated cores and production of corrugated core by shaped rolling.

expressions (truss cores) and compared in terms of minimum weight for given loading (Evans et al., 2001; Rathbun et al., 2005; Wicks and Hutchinson, 2004).

In this paper, a different approach to the optimum design of sandwich plates that allows for a general shape of the corrugated core (shape parameterization) is considered with approximate expressions for critical loads. The development of the decision-support algorithm for designing custom-optimized metal sandwich plates for particular applications is expected to result in substantial savings since typical implementations involve large quantities. The decision-support process is based on design optimization for the specific requirements of the particular application with constraint equations derived from given geometric, mechanical and technological requirements. The goal is to design optimized plates whereby the objective function realistically captures the corresponding total life-time cost of operation (TCO) of the plates as the excellence criterion.

In this paper, a multi-layer metal sandwich is modeled generally as consisting of a number of plane metal sheets and a number of corrugated cores that are glued together to constitute a composite structure (Fig. 7). It is assumed that the corrugated cores will be produced by shaped rolling.

The design variables include the topology of the sandwich structure, i.e. the number of layers, with their respective composition and thicknesses (possibly with introduction of symmetries). Further design variables include the thicknesses of metal sheets, both for plane sheets and for corrugated cores. Additional design variables are needed to define the continuous shapes (curves) of the corrugated cores for each layer (Fig. 7).

In order to include the shapes of the curves as a design degree of freedom, they are here discretized (parameterized) into sets of parameters by performing piecewise interpolation of the curves. The interpolation parameters are then used as optimization variables which replace (represent) the continuous shape in the process of shape parameterization. The particular implementation of piecewise interpolation needs to provide for sufficient generality in the representation of curves and sufficient overall degree of continuity.

Investment costs and operational expenses for the plates are combined to yield an NPV-based decision criterion. The constraints include structural integrity under operational loads, strength of bonds, minimum radii of curvature, technological feasibility, etc. The constraints impose the conditions necessary to make the local loadings sustainable for the elements of the structure and to prevent local buckling of plane sheets and corrugated cores from taking place. Further constraints are imposed to ensure sufficient length of glued joints (as they depend on the geometric shapes of the respective gaps between plane sheets and corrugated cores) and their respective load capacities. Other constraints are of technological nature, such as minimum possible radii of curvature of corrugated cores for the material used (e.g. hardened aluminum). Generally, the shapes of the cores also need to be bound by constraints that will prevent locking from taking place during production by the technological process of shaped rolling. Optimization using genetic algorithms with penalized constraints is used as shown in Fig. 8.

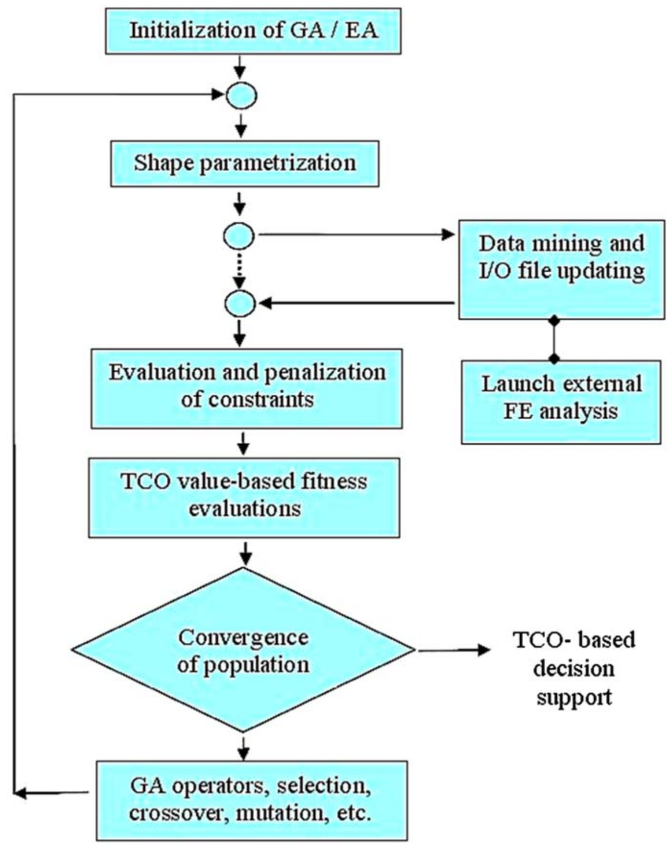

Fig. 8. TCO-based decision support in multi-objective design using genetic algorithms.

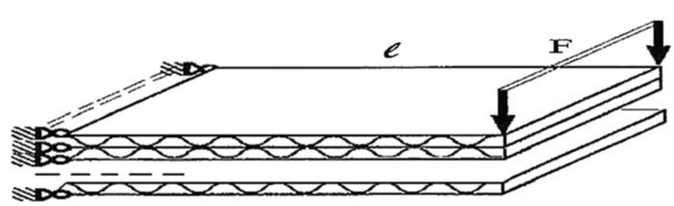

Fig. 9. 'Numerical specimen' of a multi-layer sandwich for TCO-based decision-making.

The optimization model is tested on a simple 'numerical specimen' as shown in Fig. 9:

As one of the cases considered here, Fig. 10 shows the idealized geometric model of the symmetric multiple-layer metal sandwich. Based on the geometric model presented in Figs. 9 and 10, the following optimization (decision) variables are defined for the particular case:

- thickness of layer 1, H_1
- the wavelength of the corrugated core: d

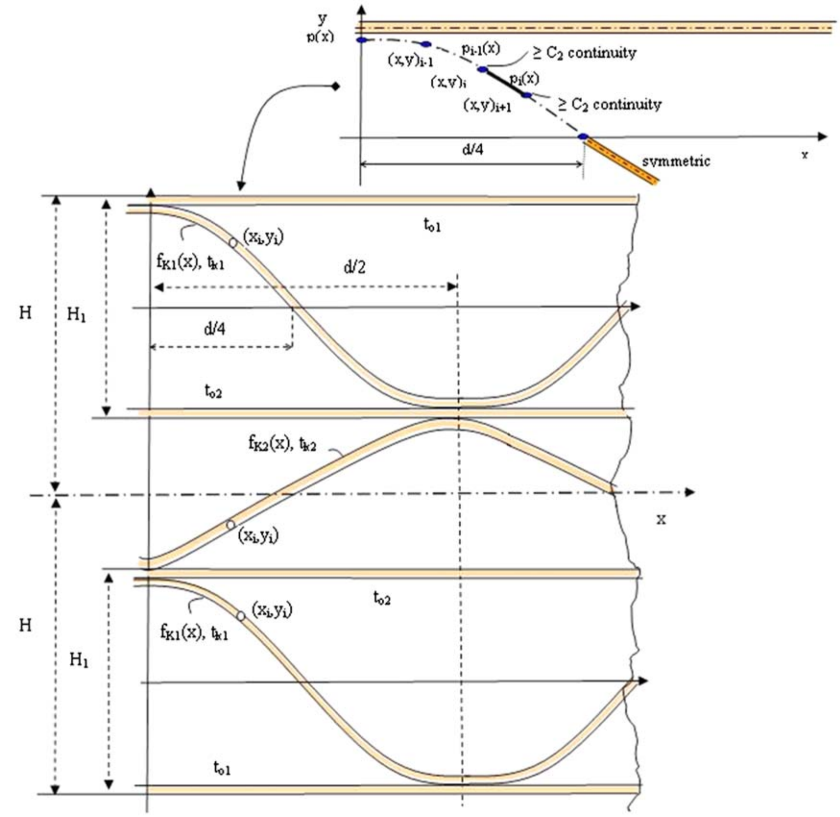

Fig. 10. Geometric model of the three-layer metal sandwich.

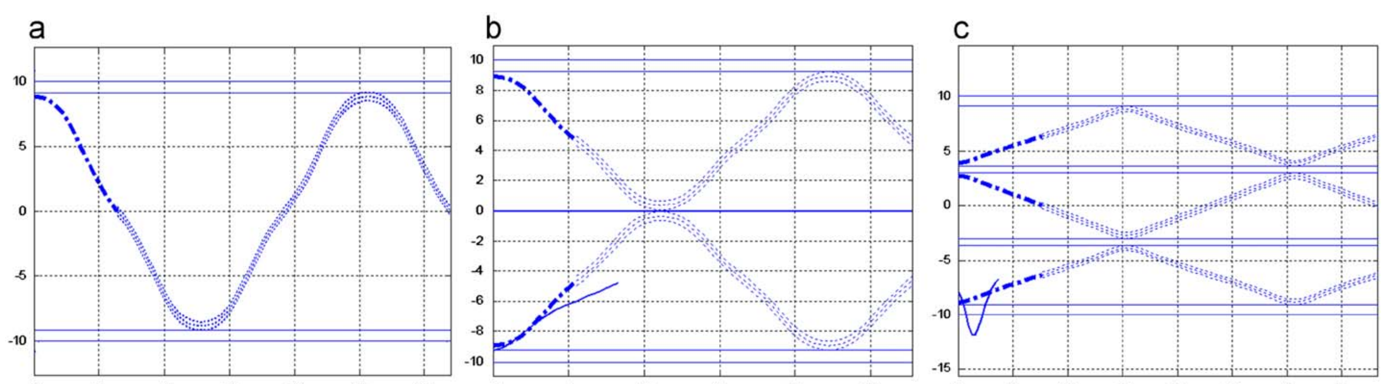

Fig. 11. Optimized shapes of the sandwich for the selected 'numerical specimen', (a) single layer, (b) two layers and (c) three layers (all dimensions in mm).

- shape piecewise interpolation points for corrugated core, curve k_1: (x_i,y_i), $i=1, mt_1$
- the thicknesses of the plates: t_{o1}, t_{k1}
- shape piecewise interpolation points for corrugated core, curve k_2: (x_i,y_i), $i=1, mt_2$
- the thicknesses of the plates: t_{o2}, t_{k2}

For the case displayed in Fig. 10 and $mt_1=mt_2=4$ (simplest case without loss of generality), full description of the geometry yields 10 decision variables; other options increase the number of variables further. This simplest case applies to three-segment piecewise interpolation and C_2 inter-segment continuity (Fig. 11).

Full description of the geometry of the corrugated cores is needed for several elements of the optimum design model, such as the calculation of the mass of material, state of stresses, geometric properties of the glued bonds, etc. In this paper, the shape parameterization is accomplished by piecewise interpolation. Within each segment, a polynomial with a sufficiently high number of degrees of freedom (coefficients) is interpolated based on a set of geometric conditions that include: (1) interpolation points, and (2) continuity (C_1 or C_2 or higher) between segments. These interpolation points become design variables that represent the shapes of the corrugated cores in the overall set of optimization variables of the problem. One of the particular methods applied in this paper are cubic splines.

The total cost generally includes the investment cost such as expenditure of material and production cost, but also the operational costs such as maintenance costs. In this paper, the cumulative impact is modeled based on Eqs. (8) and (9). The simplest formulation of the objective function which includes only the cost of material in a $2H$ segment from Fig. 10:

$$f_1 = 2H\sum_{i=1}^{n_o} t_{oi} + \frac{8H}{d}\sum_{k=1}^{n_o-1} t_k \int_0^{d/4} \sqrt{1+(y_k'(x))^2}\,dx \tag{14}$$

where (n_o-1) is the number of layers, i denotes summation over plane sheets and k over corrugated cores. The differentiation of the shape functions and integration of the lengths of the corrugated cores is implemented numerically for the current shapes of the cores based on the current values of the piecewise interpolation coefficients. In the total cost formulation of the objective function, in addition to the investment cost according to Eq. (14), a regular annual maintenance cost term is assumed. This term accounts for possible inspection costs and repair, surface protection, etc., depending on the particular application of the sandwich plates. This element can be modeled approximately as annual costs proportional to the total area of the corrugated plates and total number of glued bonds multiplied by their lengths as

$$f_2 = 2H + \frac{8H}{d}\sum_{k=1}^{n_o-1} \int_0^{d/4} \sqrt{1+(y_k'(x))^2}\,dx \tag{15}$$

$$f_3 = \frac{3H}{d}(n_o-1)l_G \tag{16}$$

Some of the constraints (Bezuhov et al., 1973; Hufnagel, 1988) are as follows:

- Dimensional constraints: bounds on thicknesses of all sheets (plane and corrugated):

$$\begin{aligned} &t_{oi,\min} \le t_{oi} \le t_{oi,\max}, \quad i = 1, n_o \\ &t_{ki,\min} \le t_{ki} \le t_{ki,\max}, \quad i = 1, n_o - 1 \end{aligned} \tag{17}$$

  where t_{oi} denotes the thickness of the i-th plane sheet and t_{ki} denotes the thickness of the i-th corrugated core.
- Bounds on thicknesses of individual layers of the sandwich:

$$H_i \le H_{i,\max},\; i = 1,\; n_o - 1 \tag{18}$$

  where H_i denotes the thickness of the i-th layer of the multi-layer sandwich. Non-negativity constraints for the dimensions are implemented, and symmetry of the layers is also introduced in the cases presented.
- Permissible local stresses due to axial (membrane) forces in all plane plate segments

$$\sigma_{ax} \le \sigma_{perm} \tag{19}$$

  where σ_{perm} denotes the permissible stress.
- Allowable local stresses due to combined axial (membrane) forces and bending in all corrugated core segments (simplified beam/plate models are used)

$$\sigma_{(ax+bend)} \le \sigma_{perm} \tag{20}$$

  In the more general case, finite element analysis yields the values for σ in Eq. (20).
- Local buckling forces in all plane plate segments less than critical

$$p_o \le p_{o,crit} \tag{21}$$

  where p_o and $p_{o,crit}$ denote the buckling and critical load, respectively.
- Local buckling forces in all corrugated core segments less than critical

$$p_k \le p_{k,crit} \tag{22}$$

- Sufficient contact length and adequate shape of all glued joints between plane sheets and corrugated cores. The shape of the corrugated cores must be such that the gap lengths between the plane sheets and corrugated cores (with gap widths less than δ_G) at points of glued joints are at least l_G:

$$p_{k1}(x) : \frac{1}{2}\left(H_1 - t_{o1} - t_{o2} - 2\frac{1}{2}t_{k1}\right) - p_{k1}(x) \le \delta_G, \quad x \in \left(0, \frac{1}{2}l_{G1}\right) \tag{23}$$

$$p_{k2}(x) : \frac{1}{2}\left(2H - 2H_1 - 2\frac{1}{2}t_{k2}\right) - p_{k2}(x) \le \delta_G, \quad x \in \left(0, \frac{1}{2}l_{G2}\right)$$

  where $p_{k1}(x)$ and $p_{k2}(x)$ denote the interpolated curves representing the corrugated cores, δ_G the maximum effective thickness of the glue and l_G the resulting length of the glued bond.
- Allowable local stresses in all glued joints

$$\sigma_{gi} \le \sigma_{g,perm} \tag{24}$$

$$\tau_{gi} \le \tau_{g,perm}$$

  where $\sigma_{g,perm}$ and $\tau_{g,perm}$ denote the permissible normal and tangential stresses in the glued joints. In this paper, the assumption is made that the geometric characteristics of the gap have direct impact on the bond strength. More realistic modeling of the glued joints is beyond the scope of this paper.
- Radii of curvature of all corrugated core curves at any point need to be larger than minimum acceptable values for the material given (hardened aluminum):

$$\frac{(1+(d/dx)\{p_{ki}(x)\}^2)^{3/2}}{(d^2/dx^2)\{p_{ki}(x)\}} \ge R_{adm} \tag{25}$$

  where R_{adm} denotes the admissible radius of curvature of the material of the corrugated cores (Fig. 10) for the hardened condition.
- Technological requirements that need to be imposed on the geometry of the sandwich. For example geometric consequences on the design of cores, derived from the fact that they are produced by shaped rolling and that the corresponding shaped rolls are to be produced like gears. This imposes such a shape of the corrugated cores that does not cause locking of 'gears' during the rolling of the corrugated cores by shaped rolls.

$$conditions\{p_{ki}(x)\} \tag{26}$$

  Since the geometric shapes of the corrugated cores are numerically represented by sets of interpolation coefficients using the process of piecewise interpolation with C_2 (or higher) continuity, above constraints transform into implicit constraint functions defined on the design optimization variables. These constraint equations are not expressed explicitly due to a number of numerical operations (Pedersen and Nielsen 2003; Press et al., 1992) that are necessary in their evaluation. Instead, the values of all the constraint conditions are evaluated numerically at each step of the optimization process and penalized accordingly. Additional numerical procedures used in the evaluation of constraints include line search for the identification of critical sections along the interpolated corrugated core curves in each design iteration.

5. Optimum design process and results

The design optimization is performed by genetic algorithms (GA) with (SUMT) penalization of constraints (overall concept in Michalewicz, 2005), applying scaled constraints and relative balancing of objective functions versus the penalty terms (Fig. 8). The numerical implementation was developed using MATLAB scripts and in some cases ADINA software for basic structural analysis. In terms of the constraint functions, approximate formulas based on simplified beam or plate models were used, or alternatively external finite-element-based analysis was launched from optimization loops within MATLAB scripts. In the latter cases, bi-directional communication of data between MATLAB and ADINA was arranged by text files with position-based data-mining. The optimization loops were also in charge of updating the input files for FE analysis based on the current values of the design variables, while the resulting FE analysis output files were searched for corresponding stresses needed for constraint evaluation. In both cases custom-made data-mining procedures were used.

The size of the GA populations was varied between 20 and 200, the number of generations was in the range between 50 and 1000. Standard values of the process parameters (Fig. 8) for selection, reproduction, cross-over and mutation were used along with uniform population creation, variable number of elite members and crossover share in population, ranked fitness scaling, stochastic uniform selection, scattered crossover and Gaussian mutation. The initial, randomly generated shapes are also shown (partly) on the same figures for visual comparison.

In the optimization process itself, standard effects could be observed: too small penalties led to insufficiencies in satisfying constraints, too large values led to dominance of penalty terms with respect to the objective function. In some cases, divergence of the optimization process has occurred. The maximum number of generations had significant impact on the final shape of the corrugated cores, since the 'straightening' of the corrugated core curves due to the minimization of the length of corrugated core is a slow process with low values of respective slopes ('sensitivity').

The test data for the numerical specimen applied in the first case were:

- Thickness of sandwich: $2H=20$ mm, width of sandwich-specimen: $b=20$ mm, length of numerical specimen: $l=200$ mm,
- Load: $F=100$ N,
- Maximum thickness of glue at the joint: 0.2 mm
- Minimum length of glued joint: 1.0 mm
- Minimum possible radius of curvature of corrugated core: 2.5 mm

Some of the respective solutions for the simple minimum mass criterion are presented below:

H	8830	4286	2490
D	12,824	11,111	15,352
t_0	0847	0764	0915
t_k	0644	0601	0465
x_1	2137	1851	2558
y_1	7599	3647	1703
x_2	4274	3703	5117
y_2	3457	1727	0787
L_1	141,062	103,688	84,317
M ($f_{\min.M}$)	226,396	256,627	375,922
t_{02}		0114	0646
H_1			7027
h_2			2713
t_{k2}			0517
x_1			2558
z_1			1977
x_2			5117
z_2			1003
L_2			85,112

where h denotes the amplitudes of the centerline of the corrugated cores, d the half-wavelength of the corrugated cores, t_0 and t_k the respective sheet thicknesses of the face plates and cores, (x,y) the interpolation points for the outer cores, (x,z) the interpolation points for the inner core, H_1 the overall thickness of the outer layer, L the length of core and M the mass of the sandwich.

The following figures present the impact of changing some of the constraint bounds on the optimized shape of the sandwich. Fig. 12 demonstrates the impact of changing the minimum prescribed radius of curvature of the hardened aluminum sheets used for (shaped) rolling of the corrugated core on the optimal sandwich geometry.

Fig. 13 shows the impact of changing the minimum length of the glued joint between the plane sheets and the corrugated cores. The required length of glued joints influences the optimized shape of the corrugated cores by virtue of constraints imposed on the sandwich design. It is here assumed that the maximum glue

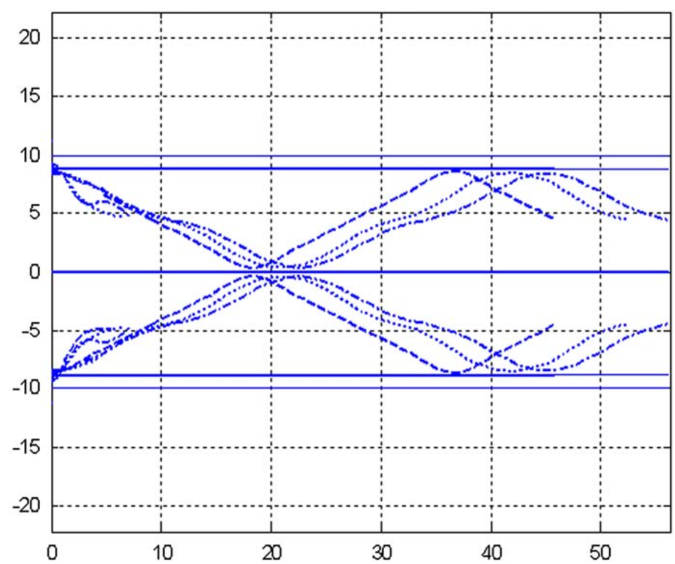

Fig. 12. GA optimization of metal sandwich plates: minimum radius of curvature constrained to (- - -) 2 mm, (…) 5 mm and (-.-) 8 mm.

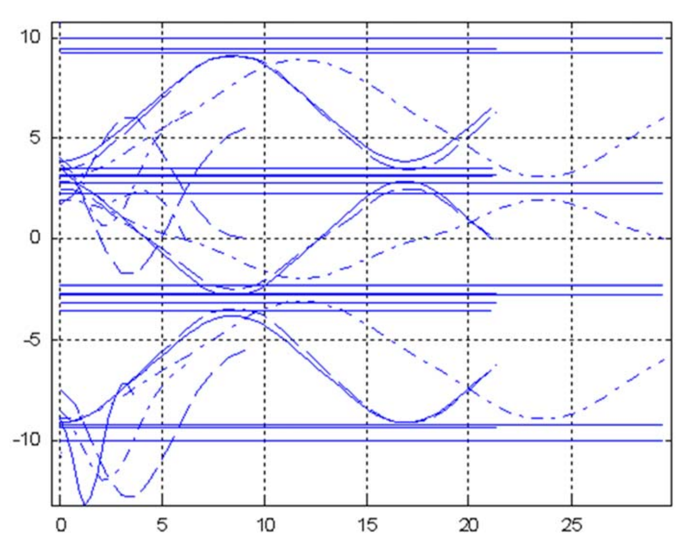

Fig. 13. GA optimization of metal sandwich plates: minimum half-length of glued joints constrained to (___) 0.7 mm, (- - -) 1 mm and (-.-) 1.3 mm.

thickness is 0.1 mm, and therefore the core curves at the glued joints need to 'adhere' to the plane sheets such that the gap lengths (for gap widths less than 0.1 mm) are at least equal to the respective required lengths of the glued joints. Other simplified models of glued joints lead to different optimized shapes by imposing the glued joint conditions as functions of core shapes.

In the following cases, the TCO formulation of the objective function is applied based on Eqs. (5)–(9). Instead of the total life-span value metrics, the total-lifetime-cost aggregate measure is used as defined in Eq. (10). The simplified model includes the mass of material to define the investment costs while the periodic operational expenses are made proportional to the overall surface area of all plates. Of course, more realistic and more detailed modeling of non-recurrent and recurrent costs can be used with Eq. (10). For example, the complexity of the cross-sectional shape described by some function of design variables can be used to construct the criterion of the production cost or recurrent maintenance time needed.

The following parameters are held fixed in the following examples:

- Life-span=10 years
- Mass cost coefficient=1 (price per unit mass)
- Surface cost coefficient=0.1 (annual cost per unit surface)
- Investment factor=0.1
- Maintenance costs factor=0.9

The first case in Fig. 14 is with the two-layer sandwich where the rate is discounting *D* is varied, which results in different optimal designs.

D	0.2	0.05	0.01
h	4072	4045	3967
d	20,527	21,463	26,500
t_0	1164	1194	1382
t_k	0637	0667	0611
x_1	3421	3572	4416
y_1	3085	3124	2771
x_2	6842	7145	8833
y_2	1255	1582	1382
L	86,701	85,923	83,685
f_{NPV}	95,955	150,772	176,168
t_{02}	0103	0095	0098
M	305,269	313,468	334,976
S	173,403	171,846	167,371

where S is the surface area and f_{NPV} the decision objective according to Eqs. (10) and (14)–(16) with penalized constraints (Eqs. (17)–(25)). Based on the proposed decision-support procedure, it can be observed that with all the engineering parameters held fixed, a decrease in the rate of discounting has the impact of reducing the surface and increasing the mass of the respective optimized shape of the sandwich. This is a consequence of the increased relative impact of recurrent costs as opposed to non-recurrent in the decision criterion-TCO objective function. This happens by an increase in wavelength of the core (which reduces the surface area) accompanied by a necessary increase of sheet thicknesses due to imposed constraints in strength of material and buckling (Eqs. (19)–(22)).

The same decision-support algorithm with NPV objectives provides similar results if the cost coefficients are varied as parameters. For the three-layer sandwich, with a fixed rate of discounting *D* set to 0.05 the variation of the lifespan results in the following:

n	3	5	12
H	2643	2477	2015
d	12,004	16,991	17,320
t_0	0791	0999	0997
t_k	0846	0761	0636
x_1	2000	2831	2858
y_1	2258	1967	1556
x_2	4001	5663	5716
y_2	1172	0921	0769
L	88,670	83,698	82,338
f_{NPV}	173,425	240,420	448,506
t_{02}	0531	0667	0996
H_1	7418	7383	6724
h_2	2054	2277	2882
t_{k2}	0982	0677	0835
x_1	2000	2831	2858
z_1	1789	1738	2396
x_2	4001	5663	5716
z_2	0950	0634	1325
L_2	85,577	83,287	85,049
M	445,471	450,611	500,982
S	262,918	250,683	249,725

Again, as in Fig. 14, the mass of the respective optimum shapes is increased while reducing the total surface area, which is due to the increased impact of periodic costs proportional to surface area in the total cost metric measured by NPV and IRR indicators.

The graphs also include 1/4 segment of the wavelength for the initial 'randomly generated' solution, and a full segment of the optimized shape ('final'). The resulting optimized plate thicknesses are visible at the x=0 position.

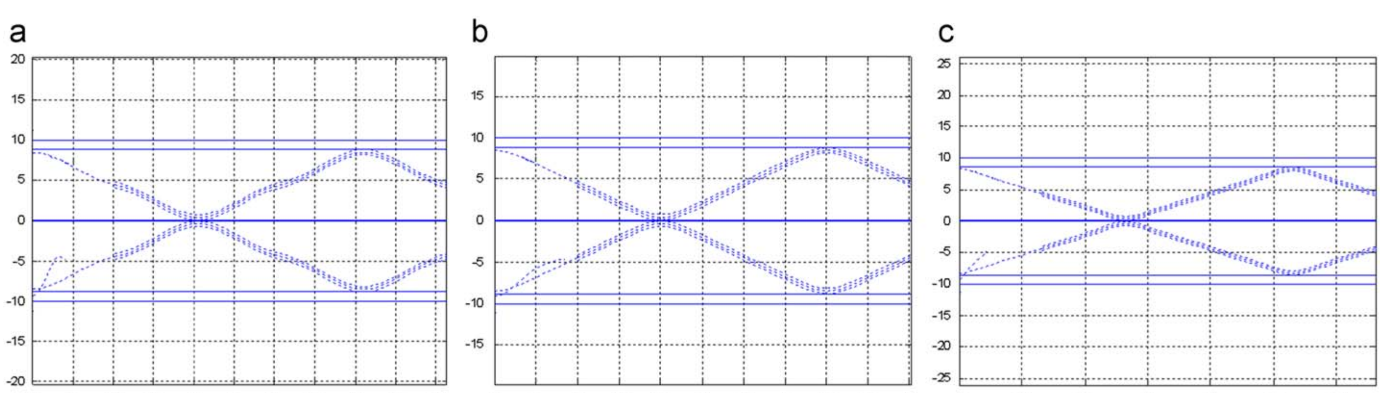

Fig. 14. a–c. GA optimization of two-layer sandwich plates based on the minimum total cost criterion, variation of the rate of discounting (all dimensions in mm).

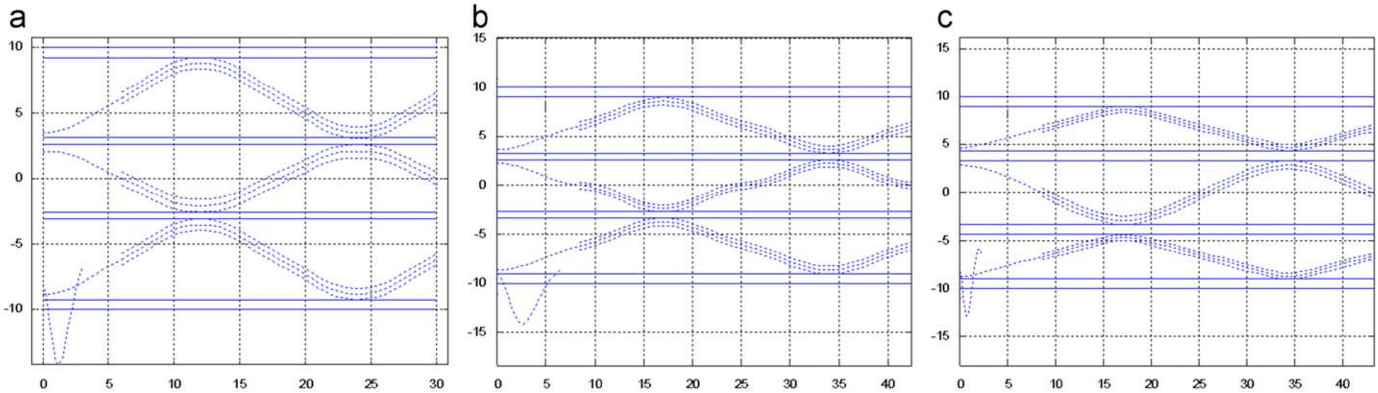

Fig. 15. a–c. GA optimization of three-layer sandwich plates based on minimum total cost criterion, variation of life-span (all dimensions in mm).

Both products selected to demonstrate the proposed objective function are very convenient in visually presenting the difference in the 'optimum shape' of the product, depending on the definition of 'excellence'. If the traditional 'minimum mass' type of excellence criteria are used, then the shapes close to those in Figs. 14a, or 15a are optimal. However, as elaborated in this paper, these shapes are not optimal in the sense of value they deliver to the operator or owner during their respective life-times. Excellence in commercial–economic performance of those products in their respective business environments is provided by the shapes presented in Fig. 14b and c and 15b and c.

Therefore, although the traditional 'minimum mass' excellence and NPV-based excellence as developed here operate on the same sets of technical design variables, they result in different optimum shapes due to different definitions of excellence. Since the fundamental objective of economic activity is creating new value, we propose that the NPV or TCO excellence criteria should be used instead of the traditional 'minimum mass' approach in optimizing the technical design variables of a product. In fact, these examples demonstrate that the optimum shape of a product is a consequence of both technical and commercial parameters.

The proposed TCO- or NPV-based objective function for multicriteria optimization is not necessarily a substitute for methods that generate the full Pareto fronts. The method developed here can be used with two different approaches. The first one is using the Pareto-optimality-based methods in generating the non-dominated set and then using the proposed excellence objective function in the decision-making process. This is done in order to select that particular Pareto point, which is optimal in the NPV or TCO sense, which for a commercial product can be justified much better than Eq. (1) or (4), since it generates more value. The second approach is numerically more affordable and uses the proposed NPV or TCO criterion from the beginning as a compromise function, eliminating the search for the Pareto front. This approach can essentially be seen as a special form of Eq. (1) where the weight factors are not chosen arbitrarily and subjectively, but instead are assigned values that result in highest possible commercial value that the product can deliver.

6. Conclusions

It is shown that the equivalent aggregate economic value or cumulative total cost of operation can lead to reasonable a priori best-compromise formulations and decision-making in some multi-objective design optimization problems. Different technical and cost/revenue-based objectives need to be valuated, discounted and aggregated over the life-span of a product to arrive at the NPV or IRR aggregate measures of total value-based design excellence. Of course, this requires that the specification includes modeling of the economic impact of the technical design variables in the aggregate objective function. This concept leads to coupled engineering–financial evaluation and decision-making even though the design variables space in simple cases includes engineering terms only.

This is also compatible with the 'decision support' interpretation of optimization since the decisions are made based on integral economic judgment of excellence and optimality, therefore the NPV- and IRR-based best-compromise formulations can be utilized as realistic measures in this sense.

The GA-based optimization of multi-layer metal sandwich plates based on the above approach provides an 'expert system' for the customized optimum design of sandwich plates. It yields optimized designs for particular application cases derived from the total value they generate or consume in both production and exploitation, which is well justified from the decision-support perspective.

With the proposed decision-making approach, a full specification for the general optimum design problem has to include the loading and geometry conditions, but beyond engineering terms also the cost basis and exploitation conditions. Two examples are presented to demonstrate that the optimum shape of a product is a consequence of both technical and commercial parameters.

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