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DAILY DRY–WET BEHAVIOUR IN CATALONIA (NE SPAIN) FROM THE VIEWPOINT OF MARKOV CHAINS

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ABSTRACT

The pluviometric regime of Catalonia (NE Spain) is analyzed from the point of view of empirical dry period lengths by considering 78 rain stations for an average of 34 years. Two possible statistical models are tested. The first model is the exponential distribution that offers reasonable results for moderate or long sequences of dry days. The other possibility comes from Markov chains of first or second order, with two or four states quantifying precipitation amounts, that are used to give better results for all the range of sequences. The Kolmogorov-Smirnov test has been applied with the aim of verifying the fit between empirical probabilities of the sequences and theoretical probabilities given by the exponential distribution and the Markov chains. It is noticeable that the Markov chain of second order is many times the distribution either satisfying the test criteria or the closest one to its fulfilment. The exponential distribution satisfies better the test criteria only for a few gauges and we have to keep in mind that for a remarkable number of pluviometric stations, none of the proposed models accomplish the test. In spite of these limited results, the Markov chains are employed to quantify important aspects which can not be studied by the exponential distribution. These aspects include return periods for a new dry or wet episode and the stationary probabilities for different precipitation amounts quantifying the states of the chains, among other aspects. As a global achievement and remembering the limited success of the test, the results depicted by Markov chains of first and second order are employed to show a differentiated behaviour among the Pyrenees and Pre-Pyrenees areas, the Central Basin, the Littoral and Pre-Littoral areas, the Transversal chain and the southern Mediterranean coast. © 1998 Royal Meteorological Society.

KEY WORDS: Dry–wet length; statistical distribution; Markov chains; spatial distribution; orographic and Mediterranean influences; NE Spain

1. INTRODUCTION

The pluviometric regime corresponding to Catalonia (NE Spain) has been recently studied from several different viewpoints. One of them (Fernández Mills and Lana, 1991) is the correlation of orography, vicinity to the Mediterranean sea and to the most important mountainous ranges with annual and seasonal amounts of precipitation. Another important aspect is the variability of the regime, as corresponds to Mediterranean behaviour (Periago *et al*., 1991; Burgueño, 1991). With respect to extreme episodes, Lana *et al*. (1995) have studied the spatial and temporal distribution of extreme rainfall episodes and, more recently, a similar methodology has depicted the behaviour corresponding to extreme episodes of consecutive dry days (Lana and Burgueño, 1998).

From the viewpoint of water resources management policy, a detailed study of drought periods is absolutely necessary, as only forecasting extreme episodes of consecutive dry days is insufficient. Generally, a period of hydric deficit could be the result of dry sequences (not necessarily of extreme

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length) following periods with a very moderate amount of precipitation. Consequently, we need a set of statistical tools that allow us to model aspects as important as the expected duration of a dry episode or the probability of a consecutive number of dry days, among other aspects.

For the present study, our attention will be focused on establishing a reasonable statistical model to describe the properties of the dry–wet behaviour of each gauge, based on a database consisting of 24-h records from 78 pluviometric gauges belonging to the *Instituto Nacional de Meteorología*, with an average of 1390 dry episodes and 34 years of records.

Conditions imposed about the temporal continuity of the records are not so strict as in the recent study (Lana and Burgueño, 1998) of extreme drought episodes, where we need complete annual series without temporal discontinuities to be sure that the detected extreme episode was true. Consequently, the number of gauges is now slightly increased and the area studied is more densely covered. The pluviometric network and the main orographic features that can condition the rainfall behaviour are shown in Figure 1.

Empiric dry spell lengths are a discrete variable due to data of 24-h rainfall amounts. Discrete models such as the geometric distribution or the truncated negative binomial (De Arruda and Pinto, 1980) could have been used. Nevertheless, the use of a continuous distribution such as the exponential is not unusual (see for instance Burgueño *et al*., 1994). For this distribution, we define as a dry day, a 24-h period recording less than 0.1 mm. We will assess whether the fit between such a distribution and the empirical probability of a fixed number of consecutive dry days is good enough for 3 or more days. However, the discrepancy for one or two dry days is remarkable, making it necessary to use another statistical model such as Markov chains of first or second order.

The Markov chains give us a more complete description of the dry behaviour related to a rain gauge. First of all, the probabilities computed with the Markov chains are compared with those deduced from the exponential distribution. A Kolmogorov-Smirnov test (Benjamin and Cornell, 1970) will be convenient to decide if the Markov chains offer more confident results. Secondly, numerical values quantifying the number of expected days to begin a new dry or wet period or the expected length of a dry cycle can be obtained. Thirdly, if we define four states of the Markov chains, with one corresponding to a lack of precipitation and three more defining three rain amount levels, we will be able to compute the stationary probabilities for the transitions from one state to another, as well as the number of days (steps of the Markov chain) to obtain this stationary probability. The first state corresponds to 24-h periods recording less than 0.1 mm. The other three states of the Markov chains correspond to 24-h episodes recording from 0.1 to less than 10 mm, from 10 to less than 50 mm and, finally, more than or equal to 50 mm, respectively. The first threshold value of 0.1 mm is commonly accepted to classify a day as dry or wet (see e.g. Moon *et al*., 1994). The other three threshold values, trying to characterize small, moderate and abundant amounts, have been empirically established according to the authors' knowledge of the pluviometric regime of the country. Additional examples, where the empirical knowledge about the pluviometric regime of the target area is a relevant factor, can be found in Haan *et al*. (1976) and Gregory *et al*. (1993) who use different numbers of states and threshold lengths studying Kentucky (US) and Great Britain, respectively.

Although the results concerning the exponential distribution and the Markov chains will be obtained for each pluviometric gauge of the network, the incidence of the orography and the vicinity to the Atlantic and Mediterranean seas can be satisfactorily analyzed by mapping these results for all the gauges.

2. STATISTICAL APPROACH

2.1. Exponential distribution

Although the exponential distribution (Benjamin and Cornell, 1970) is a very simple statistical distribution, it gives good results when studying the distribution of rainfall durations (Burgueño *et al*., 1994) from hourly precipitation series. The distribution of dry lengths, taking as 24-h database records, could also be statistically modeled by assuming the same distribution. Nevertheless, discrepancies between empirical and theoretical probabilities have to be expected for the shortest dry lengths.

From a mathematical point of view, the probability of having n consecutive dry days in a pluviometric gauge can be expressed as:

$$P_e(n) = \lambda \exp(-n\lambda) \quad (1)$$

$$\lambda = 1/L_0 \quad (2)$$

where L_0 is the empirical mean length of all the dry episodes measured in a gauge that, at the same time, has to be coincident with the standard deviation of the lengths. From a theoretical point of view, Equation (2) is the solution to the maximization of the likelihood function in the case of an exponential

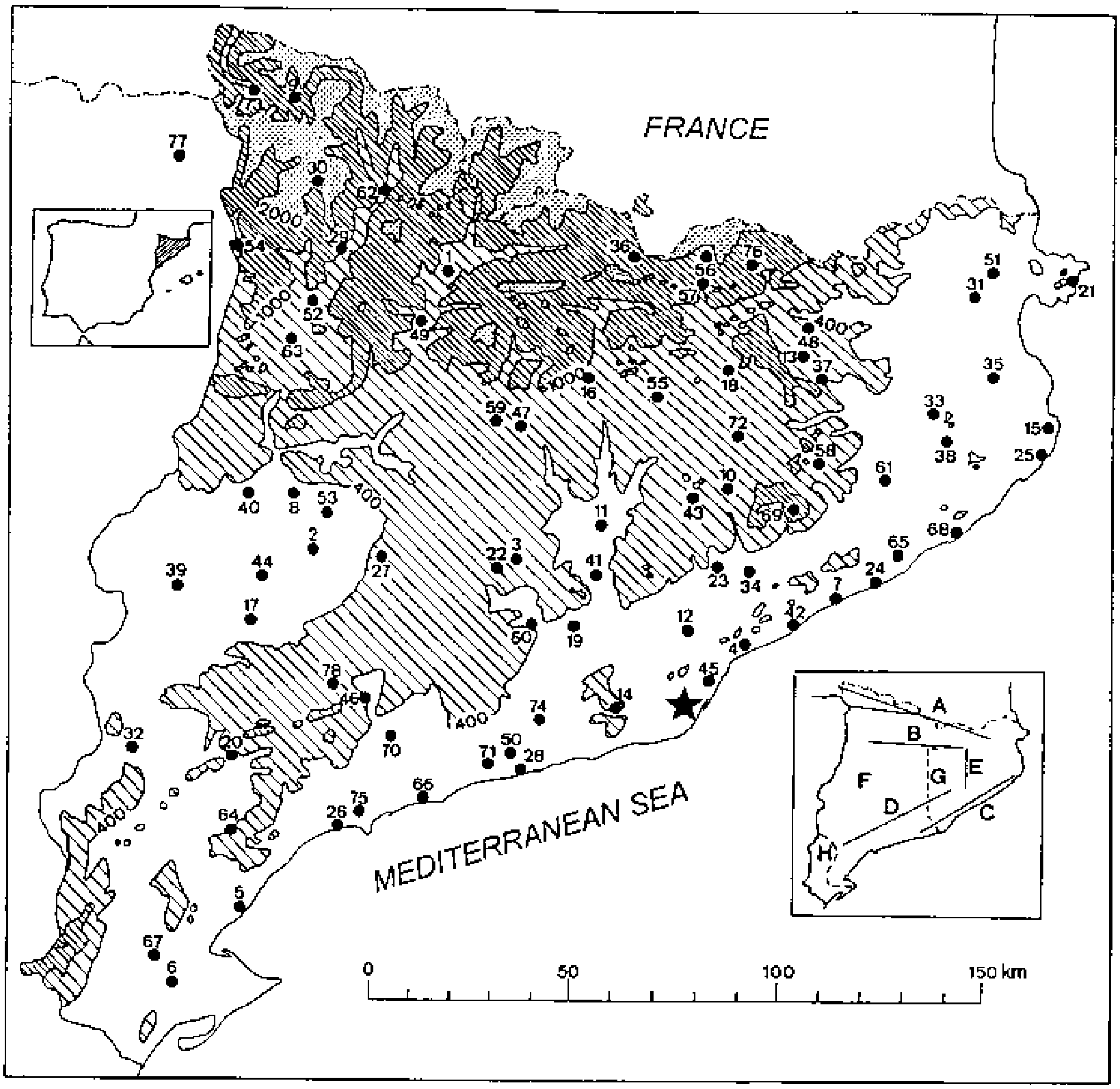

Figure 1. Location of the 78 rain gauges belonging to the *Instituto Nacional de Meteorología*. A and B, Pyrenees and Pre-Pyrenees chains; C and D, Littoral and pre-Littoral chains; E, the Transversal chain; and F, the Central Basin. G and H, the Llobregrat and Ebre rivers. The star indicates the location of Barcelona city

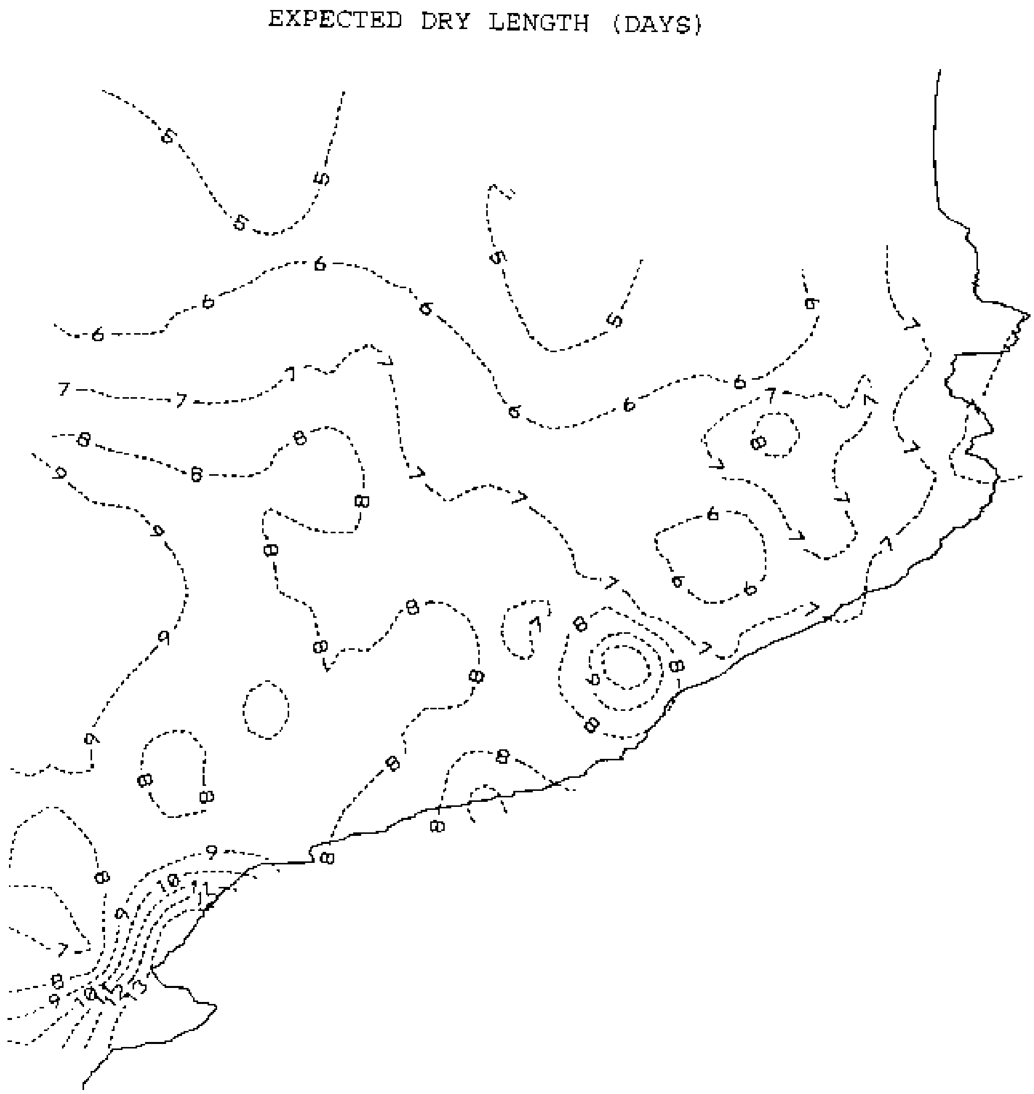

Figure 2. Spatial distribution of the averaged dry lengths (in days) derived from the 78 pluviometric stations

distribution (Benjamin and Cornell, 1970). Sometimes, the estimation made according to Equation (2) could produce some mistakes due to the statistical population of empirical dry lengths not being sufficiently representative. Bearing in mind this fact, λ will also be estimated for each gauge by searching for a minimum of the misfit function:

$$m = [P_e(i) - Q(i)]^T [P_e(i) - Q(i)] \quad i = 1, \ldots n \tag{3}$$

where $Q(i)$ the empirical probability of detecting a dry period with a length of i days. Discrepancies between parameters λ computed from Equation (2) or (3) will not be very relevant and the same conclusions about their geographical distribution can be obtained from anyone of both sets.

2.2. Markov chain formulation

Markov chains and their properties are used in many scientific fields. Classical textbooks where they are completely reviewed are, among others, Kemeny and Snell (1960), Cox and Miller (1965) and Benjamin

Table I. Number of years (Y) and number of dry sequences considered (N) for every rain gauge together with empirical (E) and the three theoretical expected lengths, in days, of the dry events. MS is the expected length corresponding to a minimum of the Equation (3) and M1 and M2 are the expected lengths deduced from Markov chains of two states and first and second order, respectively

Gauge	Y	N	E	MS	M1	M2
1	45	2071	6.02	5.29	6.05	6.06
2	27	1043	7.81	6.80	7.86	7.87
3	24	1049	7.25	6.24	7.40	7.40
4	53	2113	7.53	6.63	7.42	7.42
5	42	899	15.61	13.19	15.47	15.47
6	34	858	12.78	10.69	12.95	12.95
7	40	1519	7.77	6.94	7.52	7.52
8	19	710	8.05	6.97	7.94	7.94
9	13	772	3.82	3.27	3.91	3.90
10	51	2378	5.97	5.18	5.83	5.82
11	40	1695	6.86	5.91	6.86	6.85
12	32	883	11.55	9.29	11.86	11.86
13	23	1094	6.50	5.34	6.72	6.72
14	42	1723	7.17	6.23	7.32	7.31
15	34	1540	6.32	5.47	6.25	6.25
16	29	1410	5.59	4.82	5.41	5.41
17	16	616	7.92	6.37	8.24	8.23
18	40	1837	6.20	5.38	6.25	6.25
19	41	1802	6.44	5.67	6.23	6.23
20	22	877	7.30	6.10	7.25	7.26
21	41	1736	6.94	5.96	6.78	6.79
22	26	1026	7.56	6.43	7.63	7.62
23	47	2015	6.83	5.93	6.79	6.79
24	44	1781	7.19	6.29	7.38	7.37
25	34	1268	8.18	6.78	8.18	8.18
26	43	1497	8.74	7.64	8.79	8.78
27	32	1307	7.36	6.38	7.12	7.12
28	28	914	9.49	8.09	9.57	9.58
29	40	1767	6.54	5.67	6.58	6.58
30	34	1861	4.36	3.70	4.35	4.35
31	35	1571	6.32	5.46	6.33	6.33
32	43	1494	8.91	7.36	8.90	8.90
33	16	736	6.12	5.33	6.19	6.19
34	29	1265	6.54	5.47	6.35	6.35
35	46	1196	7.71	6.68	7.74	7.74
36	19	1103	4.23	3.77	4.23	4.24
37	25	873	8.96	7.28	9.25	9.25
38	30	1411	6.01	5.27	5.77	5.76
39	36	1156	9.77	7.78	9.68	9.68
40	33	1199	8.29	6.94	8.33	8.33
41	37	1446	7.66	6.72	7.71	7.71
42	50	2126	6.86	6.01	6.79	6.79
43	44	2027	6.06	5.18	6.07	6.07
44	21	763	8.58	6.93	8.55	8.54
45	37	1381	8.11	6.94	8.20	8.19
46	31	1200	7.85	6.70	7.62	7.62
47	45	1864	7.03	6.05	6.97	6.97
48	38	1912	5.36	4.63	5.15	5.15
49	27	1101	7.16	5.94	7.48	7.48
50	57	2029	8.68	7.49	8.72	8.72
51	20	756	7.98	6.72	8.04	8.03
52	29	1315	6.13	5.14	6.45	6.45
53	50	1705	9.02	7.36	9.16	9.17

Table I. (*Continued*)

Gauge	Y	N	E	MS	M1	M2
54	17	788	5.78	5.15	5.67	5.67
55	47	2104	6.32	5.48	6.16	6.16
56	25	1236	5.37	4.48	5.37	5.37
57	40	1960	5.47	4.76	5.48	5.48
58	20	884	6.30	5.79	6.64	6.65
59	39	1719	6.36	5.51	6.28	6.28
60	33	1151	8.75	7.29	8.77	8.76
61	20	740	8.05	6.81	7.88	7.88
62	23	1101	5.52	4.87	5.63	5.62
63	16	699	6.39	5.50	6.76	6.76
64	52	1834	8.79	7.55	8.74	8.74
65	29	1305	6.39	5.58	6.32	6.32
66	23	981	7.08	6.13	6.92	6.93
67	48	2012	6.75	5.85	6.47	6.47
68	53	2161	7.40	6.41	7.24	7.24
69	22	1173	4.55	3.90	4.41	4.41
70	30	1060	8.67	7.40	8.65	8.65
71	25	1009	7.47	6.45	7.41	7.41
72	13	539	7.18	6.27	7.28	7.26
73	37	2115	4.13	3.61	4.18	4.18
74	35	1392	7.65	6.50	7.73	7.73
75	58	2126	8.33	7.07	8.29	8.29
76	38	1851	5.50	4.69	5.62	5.62
77	27	1289	5.45	4.76	5.64	5.64
78	30	971	9.73	8.01	9.91	9.91

and Cornell (1970). As an example, two specific applications, where the Markov chain concepts are very useful, are the estimation of seismic risk (Patwardhan *et al*., 1980) and the parameter estimation process known as the annealing algorithm (Rothman, 1986).

The Markov chains, specially those of second order, are used in climatology to model rainy and drought behaviour or transitions from wet to dry episodes. Some examples can be found in Wiser (1965), Feyerherm and Bark (1967), Katz (1974), Haan *et al*. (1976), Edwin (1977), Pérez Manrique *et al*. (1984), Douguedroit (1987), Conesa and Martín Vide (1993), Gregory *et al*. (1993), Moon *et al*. (1994), among many others, with either transitions from wet to dry episodes, based on daily records, or the behaviour of the drought events, based on monthly amounts being analyzed. In our case, we take into consideration Markov chains of first and second order, representing the transition matrix between two possible states (precipitation and no-precipitation) or the transition matrix among four possible states defined above. The probability for each transition defining these matrices will be estimated by computing the quotient N_{ij}/N_i for the first order Markov chain, with N_{ij} as the number of transitions from a starting state i to another state j in 24 h and N_i as the whole number of days with rainfall amounts belonging to state i. Similarly, for the Markov chain of second order, we compute the quotient N_{ijk}/N_i, with N_{ijk} as the number of transitions from state i to state j in 24 h and from state j to state k in the next 24 h.

The case of first order and two states will be quantified by a 2×2 transition matrix:

$$P1_2(i, j) = P_{ij} \quad i = 0, 1; \; j = 0, 1 \tag{4}$$

where 0 and 1 represent the states of no-precipitation and precipitation, respectively, and P_{ij} the transition probability in one step (day) from state i to state j. On the other hand, it is evident that $P_{i0} + P_{i1}$, is equal to 1 for i equal to 0 and 1.

The case of first order and four states will be quantified by a 4×4 transition matrix:

$$P1_4(i, j) = P_{ij} \quad i = 0, 1, 2, 3; \; j = 0, 1, 2, 3 \tag{5}$$

where 0, 1, 2 and 3 are the states defined by rainfall amounts less than 0.1 mm, from 0.1 to less than 10 mm, from 10 to less than 50 mm and greater than or equal to 50 mm, respectively. In this case, the constraint on P_{ij} is given by $\Sigma_j P_{ij} = 1$ for i equal to 0, 1, 2 and 3.

The second order Markov chain, with two states, will be represented by a transition tensor:

$$P2_2(i, j, k) = P_{ijk} \quad i = 0, 1;\ j = 0, 1;\ k = 0, 1 \tag{6}$$

In this case, the constraints applied to the elements are given by $\Sigma_{j,k} P_{ijk} = 1$ for all pairs (j, k), being P_{ijk} the transition probability, in two steps (days), from state i to state j and from state j to state k.

The definition of transition matrices (4), (5) and (6) permits computation of several aspects of the dry–wet behaviour of each gauge in terms of different orders and states, such as the following.

(i) The probabilities of detecting n consecutive dry days in a rain gauge, according to the Markov chain of first and second order, are:

$$Q1(n) = (1 - P_{01})^{n-1} P_{01} \quad n \geq 1 \tag{7}$$

$$Q2(n) = P_{100} P_{000}^{n-2} P_{001} \quad n \geq 2 \tag{8}$$

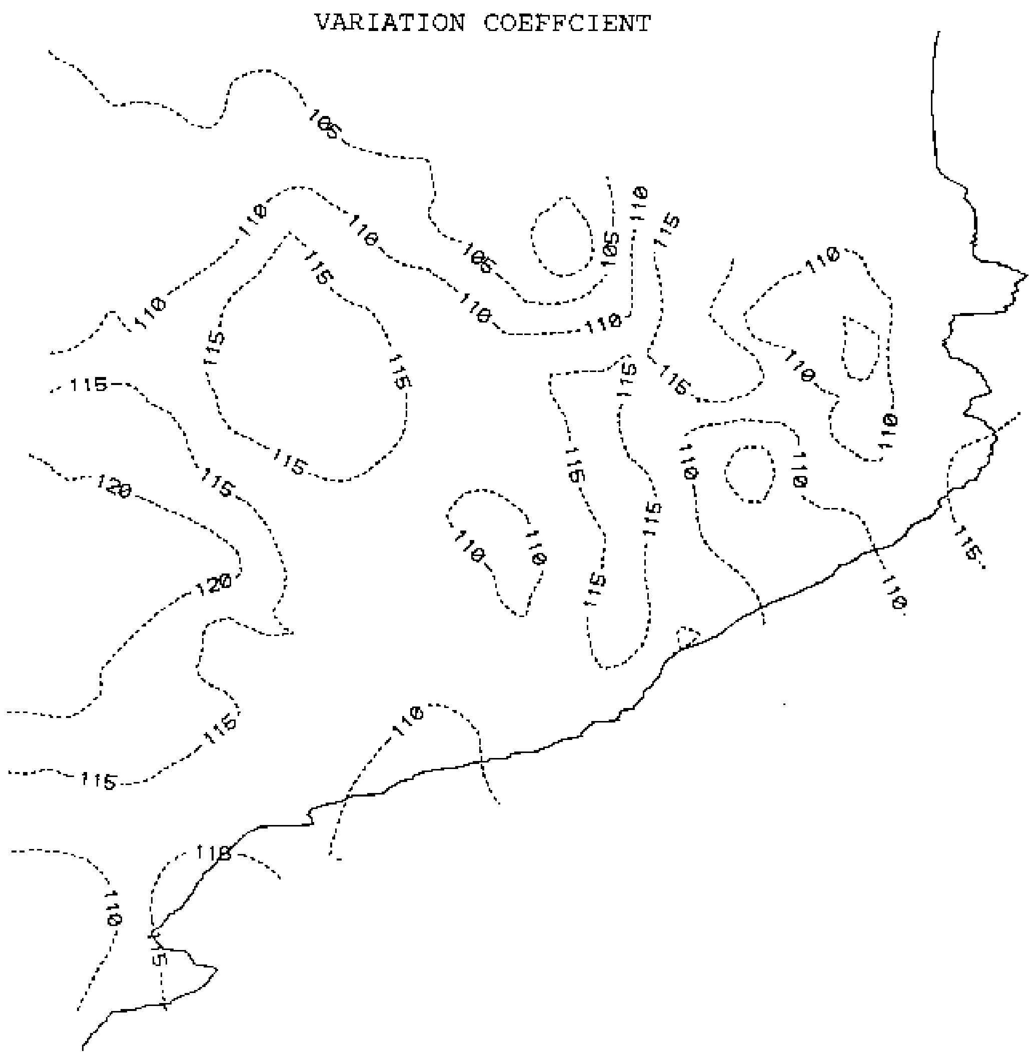

Figure 3. Spatial distribution of the variation coefficient (%), computed as the quotient between standard deviation and expected length

$$Q2(1) = P_{101} \tag{9}$$

(ii) The probability of a state, n days after a fixed starting state, according the concept of transition matrix of first order and two states, $\mathbf{P1}_2$, can be computed as follows:

$$\mathbf{S}1_2(n) = \mathbf{V} \cdot \mathbf{P}1_2^n \tag{10}$$

where $\mathbf{V}$ is a vector of components (0, 1), if the starting state corresponds to a rainy day and components (1, 0), if the starting day is dry. An easy generalization can be made for the case of first order and four states, $\mathbf{P1}_4$, obtaining:

$$\mathbf{S}1_4(n) = \mathbf{V} \cdot \mathbf{P}1_4^n \tag{11}$$

where $\mathbf{V}$ is now a vector of four components, similarly defined as in the previous case. For example, the components of $\mathbf{V}$ will be (0, 0, 0, 1), if the starting state corresponds to a daily record with precipitation greater than or equal to 50 mm.

(iii) A more interesting aspect is the detection of the number of days required to obtain a stationary value for the elements of matrices $\mathbf{P1}_2^n$ and $\mathbf{P1}_4^n$, that will represent the stationary transition probabilities for the states. We can develop the case for two states and then to extrapolate for four states. According to elementary matrix algebra, we decompose $\mathbf{P1}_2$ and $\mathbf{P1}_2^n$ matrices in terms of eigenvector matrix, $\mathbf{R}$, and diagonal eigenvalues matrix, $\mathbf{L}$, of $\mathbf{P1}_2$:

$$\mathbf{P}1_2 = \mathbf{R} \cdot \mathbf{L} \cdot \mathbf{R}^{-1} \tag{12}$$

$$\mathbf{P}1_2^n = \mathbf{R} \cdot \mathbf{L}^n \cdot \mathbf{R}^{-1} \tag{13}$$

being easy to prove that, with n tending to ∞, the elements of $\mathbf{P1}_2^n$ tend to:

$$P1_2^n(0, 0) = P1_2^n(1, 0) = P_{10}/(P_{01} + P_{10}) \tag{14}$$

$$P1_2^n(0, 1) = P1_2^n(1, 1) = P_{01}/(P_{01} + P_{10}) \tag{15}$$

representing these equations, respectively, the first and second stationary columns of $\mathbf{P1}_2^n$. The stationary 2×2 matrix obtained is then formed by two columns, the first quantifying the stationary probability of recording a dry day in a gauge and the second a day with precipitation.

Computations for four states are equivalent for calculating eigenvalues and eigenvectors of a 4×4 matrix and the same steps are applied when considering the 2×2 matrix. In this last case, the elements of the first stationary column, designed by $\{P1_4^n(i, 0);\ i = 0, 1, 2, 3\}$, have to be coincident with values given by Equation (14), that quantify again the stationary probability of recording a dry day in a gauge. Now we can distinguish between stationary probabilities of recording low amounts ranging from 0.1 to less than 10 mm $\{P1_4^n(i, 1);\ i = 0, 1, 2, 3\}$, moderate precipitation from 10 to less than 50 mm $\{P1_4^n(i, 2);\ i = 0, 1, 2, 3\}$, and, finally, very high amounts greater than or equal to 50 mm $\{P1_4^n(i, 3);\ i = 0, 1, 2, 3\}$. Moreover, addition of the second, third and fourth columns give us a column which has to be coincident with stationary values given by Equation (15), bearing in mind that state 1 of the Markov chain of two states summarizes states designated by 1, 2 and 3 of the Markov chain of four states.

(iv) The transition matrix elements also provide a way to quantify the expected length of a wet or dry period as well as the expected length of a dry–wet cycle. Computations can be made according to Markov chains of two states, provided that we are only interested now in the possibility of wet or dry episodes, but considering a first or second order chain. First of all, we have to quantify the probability of detecting n consecutive rainy days or n consecutive dry days.

Considering for the moment the first order case:

$$R1_r(n) = P_{11}^{n-1} P_{10} \tag{16}$$

$$R1_d(n) = P_{00}^{n-1} P_{01} \tag{17}$$

where $R1_r(n)$ and $R1_d(n)$ are the probabilities for rainy and dry sequences of n days, respectively, then, the expected length of a sequence of rainy or dry days can be expressed as:

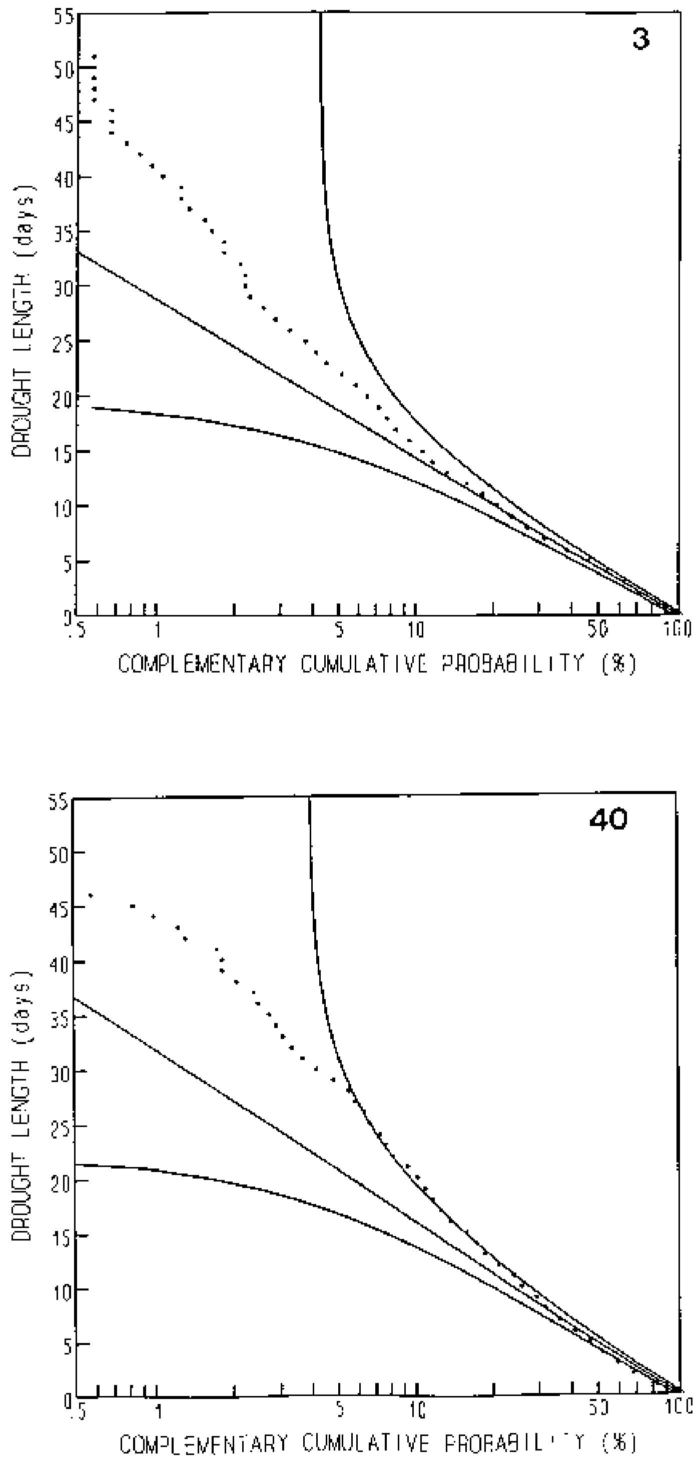

Figure 4. (a) A comparison between exponential (solid line) and empirical distribution (points) for the dry lengths. The curved lines represent the control band given by the parameter C of the Kolmogorov-Smirnov test. This example corresponds to gauge 3. (b) As for (a). This example corresponds to gauge 40

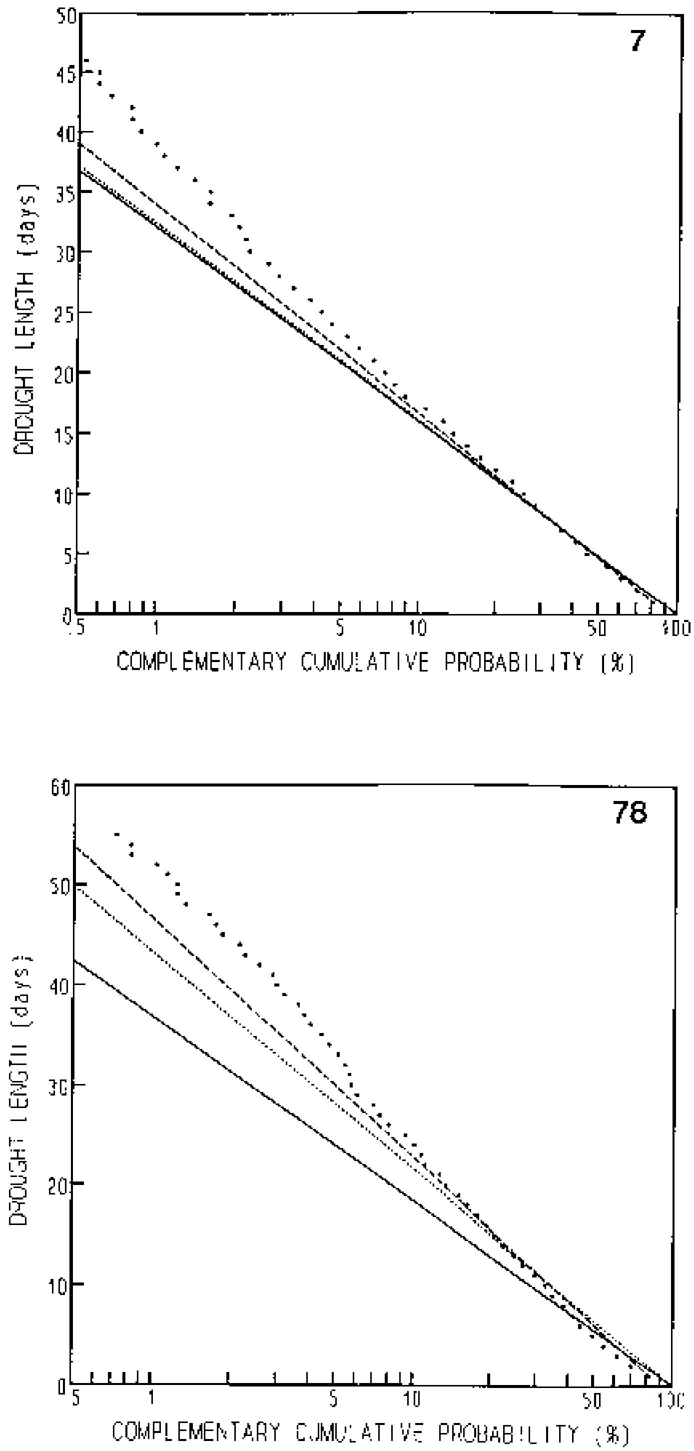

Figure 5. (a) A comparison between the exponential model (solid line), Markov chains of first order (line of points), second order (dashed line) and empirical distribution (points) for the dry lengths. This example corresponds to gauge 7. (b) As for (a). This example corresponds to gauge 78

$$E1(\mathrm{r}) = \Sigma_n n R1_{\mathrm{r}}(n) \quad n = 1, \ldots, \infty \tag{18}$$

$$E1(\mathrm{d}) = \Sigma_n R1_{\mathrm{d}}(n) \quad n = 1, \ldots, \infty \tag{19}$$

Both expressions can be rewritten as arithmetic-geometric series, being straightforward to prove that:

$$E1(\mathrm{r}) = 1/P_{10} \tag{20}$$

$$E1(\mathrm{d}) = 1/P_{01} \tag{21}$$

Then, the expected length of a dry–wet cycle can be expressed as:

$$E1(\mathrm{c}) = E1(\mathrm{r}) + E1(\mathrm{d}) = (P_{10} + P_{01})/(P_{10}P_{01}) \tag{22}$$

(v) The return period could also be interesting to evaluate. The probabilities to return to a dry day after n consecutive days, PRD, or to a rainy day after n consecutive days, PRR, can be expressed as:

$$\mathrm{PRD}(n) = P_{01}P_{11}^{n-2}P_{10} \quad n \geq 2 \tag{23}$$

$$\mathrm{PRR}(n) = P_{10}P_{00}^{n-2}P_{01} \quad n \geq 2 \tag{24}$$

Then, the expected return period for wet or dry episodes can be computed and the arithmetic-geometric series appear again:

$$\mathrm{ER}(\mathrm{r}) = \Sigma_n n \mathrm{PRR}(n) \quad n = 1, \ldots, \infty \tag{25}$$

$$\mathrm{ER}(\mathrm{d}) = \Sigma_n n \mathrm{PRD}(n) \quad n = 1, \ldots, \infty \tag{26}$$

Finally, the expected return period for a rainy and for a dry day can be expressed, respectively, by:

$$\mathrm{ER}(\mathrm{r}) = (P_{01} + P_{10})/P_{01} \tag{27}$$

$$\mathrm{ER}(\mathrm{d}) = (P_{01} + P_{10})/P_{10} \tag{28}$$

(vi) With respect to the expected lengths of dry and wet cycles, the same computations can be made by considering a second order Markov chain. Now, the probability of detecting n days of consecutive rainy and dry days can be quantified as:

$$R2_r(n) = (1 - P_{010})(1 - P_{110})^{n-2}P_{110} \quad n \geq 2 \tag{29}$$

$$R2_{\mathrm{r}}(1) = P_{010} \tag{30}$$

$$R2_{\mathrm{d}}(n) = (1 - P_{101})(1 - P_{001})^{n-2}P_{001} \quad n \geq 2 \tag{31}$$

$$R2_{\mathrm{d}}(1) = P_{101} \tag{32}$$

If we evaluate the expected length of each possible sequence, we obtain again two arithmetic-geometric series, finally obtaining:

$$E2(\mathrm{r}) = (1 - P_{010} + P_{110})/P_{110} \tag{33}$$

$$E2(\mathrm{d}) = (1 - P_{101} + P_{001})/P_{001} \tag{34}$$

and the expected length of a whole cycle of wet–dry days will be:

$$E2(\mathrm{c}) = E2(\mathrm{r}) + E2(\mathrm{d}) = 2 + (1 - P_{101})/P_{001} + (1 - P_{010})/P_{110} \tag{35}$$

The definition of return periods for a second order Markov chain cannot be properly derived due to the fact that these transition probabilities do not include the case $n = 1$ and expressions similar to Equations (25) and (26) can not be entirely reproduced.

2.3. Test of significance

Taking into account that we have to decide for each gauge which is the best statistical model describing consecutive dry days probabilities, the Kolmogorov-Smirnov test (Benjamin and Cornell, 1970) has been applied to theoretical and empirical distributions obtained for each recording station. The test is based on the computation of the parameter:

$$D = \max\{Q^*(i) - P^*(i)\} \quad i = 1, \ldots, n_0 \tag{36}$$

where $Q^*(i)$ is the cumulative empirical distribution of consecutive dry days and $P^*(i)$ is the cumulative theoretical distribution, derived either from the exponential distribution or from the Markov chains of first and second order and two states. n_0 is the number of intervals conforming to the experimental distribution $Q(i)$, introduced for the computation of the misfit function (Equation (3)). After the calculation of the parameter D, we have to decide if the hypothesis that a theoretical model, representing the empirical distribution, must be discarded or not. The applied rule is simple. If D is equal or less than a quantity C, the hypothesis is accepted. The hypothesis is rejected if the opposite is found. C is computed according to the asymptotic expression (Lindgren, 1962):

$$C = 1.36 n_0^{1/2} \quad n_0 \geq 40 \tag{37}$$

with a confidence level of the 5%.

3. APPLICATION

3.1. Comparison of statistical models

The exponential distribution and the Markov chains of first and second order and two states have been tested for each one of the 78 rain gauges depicted in Figure 1. Figure 2 shows a distribution of the average dry length (in days) for all the studied area. As a general feature, we can observe that the expected length becomes larger from north to south, with remarkably low values, in relative terms, of less than or equal to 6 days, assigned to most of the Pyrenees and Pre-Pyrenees areas. Conversely, the rest of the country is linked to expected lengths of 8, 9 and 10 days, with a noticeably strong positive gradient evident in the south of the country, reaching up to 16 days. This gradient is due to a local effect detected in gauge number 5. We have to remember that this outstanding behaviour was also detected for this gauge in terms of extreme dry episodes (Lana and Burgueño, 1998). This local phenomenon is a consequence of the foehn effect that originates from the NW winds that, channelized by the Ebre valley, overcome the littoral mountains and arrive at the Mediterranean sea. The inverse of quantities depicted in Figure 2 should be a good approximation of the λ parameter of the exponential distribution represented by Equation (1). The λ parameters for each rain gauge are also determined by minimization of the misfit function designated by Equation (3) and both estimations. These, together with the expected values given by Markov chains, are listed in Table I.

Figure 3 shows the variation coefficient, defined as the quotient between the standard deviation of the dry lengths and their expected values depicted in Figure 2. The geographical distribution of this coefficient is not exactly coincident with the expected values of Figure 2.

The largest variation, with values above 115%, is detected for a large part of the Central Basin and extends across the Eastern Pyrenees, the west face of the Transversal chain and arrives in the vicinity of Barcelona city. Less irregular behaviour is obtained for the rest of the Catalan Pyrenees and Pre-Pyrenees, with the average value and the standard deviation almost equal for some small areas. Provided that the expected lengths and standard deviation differ from each other for most of the country, we can assume that the exponential distribution is not the best model describing the dry behaviour.

Two opposite examples of fit between empirical and exponential distributions are shown in the probability plots of Figure 4(a) and (b). Points represent empirical data, the straight line the exponential distribution and the curved lines tolerances given by Equation (37), according to the Kolmogorov-

Table II. Results of the Kolmogorov-Smirnov test for the whole set of gauges G. E_{max}, $M1_{max}$ and $M2_{max}$ design, respectively, statistic D given by Equation (36) for the exponential distribution and the Markov chains of first and second order. C is the quantity given by Equation (37) for a significance level of 5%

G	E_{max}	$M1_{max}$	$M2_{max}$	C	G	E_{max}	$M1_{max}$	$M2_{max}$	C
1	0.0608	0.0723	0.0448	0.0299	40	0.0706	0.0956	0.0621	0.0393
2	0.0502	0.0714	0.0504	0.0421	41	0.0454	0.0638	0.0362	0.0358
3	0.0232	0.0477	0.0469	0.0420	42	0.0541	0.0632	0.0334	0.0295
4	0.0634	0.0726	0.0324	0.0296	43	0.0666	0.0846	0.0580	0.0300
5	0.0518	0.0673	0.0389	0.0454	44	0.0573	0.0975	0.0712	0.0492
6	0.0607	0.0912	0.0586	0.0464	45	0.0502	0.0745	0.0474	0.0366
7	0.0489	0.0501	0.0252	0.0349	46	0.0538	0.0659	0.0289	0.0393
8	0.0649	0.0751	0.0343	0.0510	47	0.0736	0.0889	0.0396	0.0315
9	0.0682	0.0788	0.0448	0.0489	48	0.0579	0.0588	0.0229	0.0311
10	0.0606	0.0675	0.0383	0.0279	49	0.0652	0.0911	0.0606	0.0410
11	0.0539	0.0754	0.0521	0.0330	50	0.0457	0.0658	0.0376	0.0302
12	0.0582	0.1028	0.0704	0.0458	51	0.0487	0.0802	0.0554	0.0495
13	0.0592	0.0969	0.0651	0.0421	52	0.0542	0.0923	0.0576	0.0375
14	0.0532	0.0761	0.0484	0.0328	53	0.0529	0.0949	0.0650	0.0329
15	0.0514	0.0637	0.0355	0.0347	54	0.0672	0.0675	0.0268	0.0484
16	0.0712	0.0755	0.0277	0.0362	55	0.0694	0.0770	0.0413	0.0296
17	0.0579	0.1107	0.0804	0.0548	56	0.0569	0.0790	0.0482	0.0387
18	0.0656	0.0853	0.0566	0.0317	57	0.0565	0.0679	0.0428	0.0307
19	0.0551	0.0573	0.0261	0.0320	58	0.0452	0.0589	0.0425	0.0457
20	0.0745	0.0971	0.0559	0.0459	59	0.0622	0.0720	0.0391	0.0328
21	0.0422	0.0575	0.0358	0.0326	60	0.0569	0.0899	0.0557	0.0401
22	0.0437	0.0661	0.0556	0.0425	61	0.0545	0.0767	0.0379	0.0500
23	0.0674	0.0813	0.0463	0.0303	62	0.0305	0.0387	0.0302	0.0410
24	0.0484	0.0680	0.0451	0.0322	63	0.0799	0.1111	0.0559	0.0514
25	0.0375	0.0676	0.0477	0.0382	64	0.0726	0.0899	0.0539	0.0318
26	0.0543	0.0748	0.0443	0.0352	65	0.0423	0.0498	0.0215	0.0376
27	0.0608	0.0687	0.0283	0.0376	66	0.0452	0.0584	0.0396	0.0434
28	0.0653	0.0866	0.0377	0.0450	67	0.0544	0.0569	0.0139	0.0302
29	0.0674	0.0881	0.0508	0.0324	68	0.0576	0.0718	0.0426	0.0293
30	0.0559	0.0667	0.0366	0.0315	69	0.0652	0.0643	0.0288	0.0397
31	0.0400	0.0562	0.0364	0.0343	70	0.0650	0.0824	0.0492	0.0418
32	0.0812	0.1074	0.0505	0.0352	71	0.0471	0.0686	0.0336	0.0428
33	0.0511	0.0672	0.0365	0.0501	72	0.1770	0.0828	0.0639	0.0586
34	0.0504	0.0664	0.0378	0.0382	73	0.2322	0.0575	0.0285	0.0296
35	0.0485	0.0660	0.0403	0.0321	74	0.0711	0.0759	0.0520	0.0365
36	0.0691	0.0644	0.0381	0.0409	75	0.1481	0.0787	0.0396	0.0295
37	0.0584	0.0980	0.0625	0.0460	76	0.0566	0.0759	0.0560	0.0316
38	0.0485	0.0478	0.0233	0.0362	77	0.2206	0.0884	0.0648	0.0379
39	0.0640	0.1000	0.0622	0.0400	78	0.2040	0.1081	0.0675	0.0436

Smirnov test. The discrepancies between exponential and empirical models for gauge 3 are within the control bands of the tolerance test. Consequently, for this particular gauge, the exponential function is a good distribution. Opposite to that behaviour, gauge 40 shows empirical data outside the range given by Equation (37) for a wide range of lengths. This is a clear example of an exponential distribution incorrectly reproducing the empirical data.

Figure 5(a) and (b) compares the exponential distribution, the Markov chains of first and second order and empirical data for two other gauges. We can easily detect in both cases that the best fit with empirical data is obtained for the Markov chain of second order (dashed line), whereas the model derived from a Markov chain of first order (line of points) is very similar to the exponential function for gauge 7 and closer to the second order chain for gauge 78. A common feature for both examples is the noticeable discrepancy between empirical data and the exponential model, specially for gauge 78.

Table III. Return period, computed according to Markov chains of first order and given in days, to a new dry episode (Dry) and to a new wet episode (Wet)

Gauge	Dry	Wet
1	1.3084	4.2431
2	1.2059	5.8566
3	1.1655	7.0436
4	1.2156	5.6387
5	1.0921	11.860
6	1.1164	9.5938
7	1.2530	4.9523
8	1.2171	5.6063
9	1.5876	2.7019
10	1.3240	4.086
11	1.2520	4.968
12	1.1225	9.1657
13	1.2447	5.0872
14	1.2324	5.3033
15	1.2813	4.5543
16	1.3687	3.7120
17	1.1812	6.5187
18	1.2824	4.5406
19	1.3080	4.2470
20	1.2383	5.1958
21	1.2484	5.0263
22	1.2256	5.4331
23	1.2521	4.9664
24	1.2417	5.1376
25	1.1872	6.3422
26	1.1848	6.4126
27	1.2411	5.1485
28	1.1564	7.3923
29	1.2474	5.0412
30	1.5307	2.8842
31	1.2715	4.6826
32	1.1724	6.8007
33	1.2890	4.4600
34	1.2862	4.4941
35	1.2055	5.8653
36	1.4957	3.0172
37	1.1480	7.7562
38	1.3119	4.2060
39	1.1628	7.1434
40	1.1906	6.2475
41	1.2156	5.6383
42	1.2517	4.9737
43	1.2987	4.3477
44	1.1958	6.1068
45	1.1987	6.0320
46	1.2111	5.7374
47	1.2595	4.8540
48	1.3782	3.6439
49	1.2279	5.3887
50	1.1764	6.6683
51	1.1984	6.0396
52	1.2845	4.5148
53	1.1818	6.5007
54	1.3480	3.8734
55	1.2980	4.3554
56	1.3729	3.6819

Table III. (*Continued*)

Gauge	Dry	Wet
57	1.3541	3.8242
58	1.2777	4.6011
59	1.2995	4.3388
60	1.1930	6.1804
61	1.2222	5.4995
62	1.3609	3.7711
63	1.2696	4.7099
64	1.1888	6.2962
65	1.2701	4.7028
66	1.2264	5.4173
67	1.2825	4.5397
68	1.2149	5.6536
69	1.5403	2.8509
70	1.1755	6.6980
71	1.2117	5.7233
72	1.2163	5.6221
73	1.5322	2.8790
74	1.1927	6.1901
75	1.1977	6.0584
76	1.3410	3.9324
77	1.3739	3.6744
78	1.1524	7.5607

Figures 4 and 5 are only some examples showing general patterns when comparing the exponential distribution and Markov chains. A more general picture of these comparisons is obtained from Tables I and II. Table I summarizes expected lengths deduced empirically and computed by means of expressions (3), (21) and (34). We can find that the differences between the exponential and the Markov formulation are small. When trying to decide which is the best model, provided that the results from Table I are not conclusive , the Kolmogorov-Smirnov test has been applied to each distribution and each rain gauge. Table II, where we can compare parameters D for the different distributions and limiting values given by C, summarizes the obtained results. We can observe that for 30 rain gauges the Kolmogorov-Smirnov test criteria are met (five according to the exponential distribution, one related to the Markov chain of first order and 24 considering the Markov chain of second order). In addition, another 17 gauges almost satisfy the test (three according to the exponential distribution and 14 considering the Markov chain of second order). In contrast, the discrepancies between parameters D and C for gauges 72, 73, 75, 77 and 78 (case of exponential distribution) and gauges 17 and 63 (Markov chain of first order) are especially remarkable. Although the Markov chain of second order does not always fulfil the test, as a general feature, its parameter D is less than those corresponding to the first order chain and the exponential model. As a matter of fact, the maximum number of gauges satisfying the test corresponds to the second order chain.

3.2. Return periods and stationary probabilities

The Markov chain of second order, gives an interesting complement to the dry behaviour as it provides an evaluation of the return periods for dry or rainy episodes. On the other hand, in order to obtain a better statistical description of the pluviometric behaviour linked to a rain gauge, the stationary transition probabilities for the four states of precipitation defined previously can be computed. In spite of the poor results of the Kolmogorov-Smirnov test for the chains of first order, two reasons have been considered to choose this chain to evaluate the stationary probabilities instead of the second order chain. First, a Markov chain of second order and four states implies the computation of $4 \times 4 \times 4$ different probabilities. This fact is a very important shortcoming bearing in mind the limited database, possibly leading to very erroneous computation of probabilities linked to the third and fourth states which are characterized, especially the latter, by outstanding amounts of precipitation. And second, at the end of the section it is shown that the number of steps (days) of the Markov chain to achieve the stationary probability is less than or equal to 10 days for all the gauges. A close review of the whole set of figures similar to Figure 5 shows that the theoretical probabilities for the Markov chain of first order begin to underestimate empirical probabilities for lengths greater than 20 days, approximately. Consequently, the stationary transition probabilities derived from the first order formulation should not be submitted to similar mistakes.

Table III lists the return periods obtained for a new dry or wet episode. We can observe that expected return periods for dry episodes are almost constant, ranging from 1.1 to 1.6 days. According to that, we should conclude that no significant differences could be established among the different domains of the country. However, the number of days to return to a new rainy episode, ranging from 2.7 to 11.9 days, emphasizes the diversity of the country. In addition, the geographical distribution of the return periods (Figure 6) is in agreement with previous deductions made according to Figures 2 and 3. As an example, the shortest expected return period (2.9 days) is obtained for gauge 9, located on the north face of the Pyrenees, and the longest (11.9 days) is associated with gauge 5, on the southern Mediterranean coast.

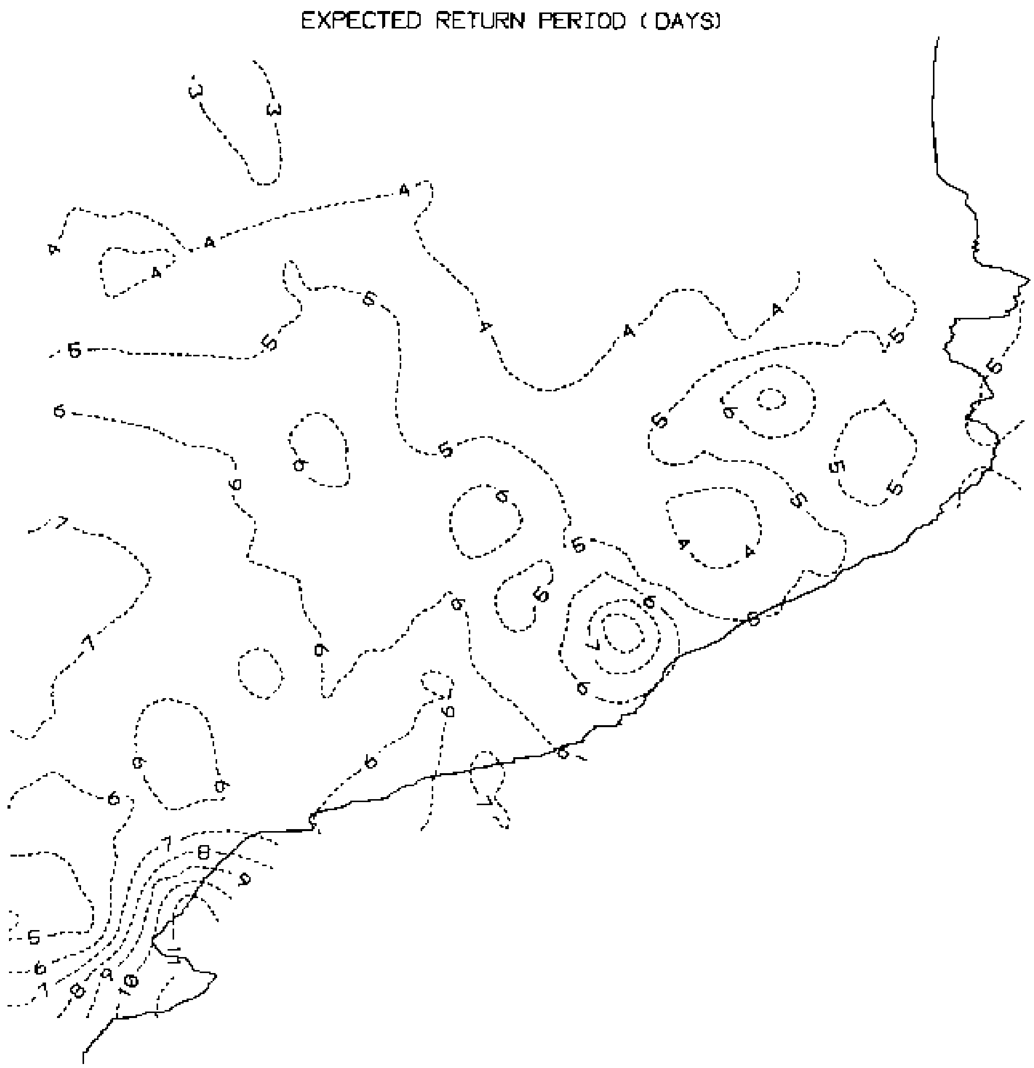

Figure 6. Expected return periods (days) deduced from the Markov chain of two states and first order for Catalonia

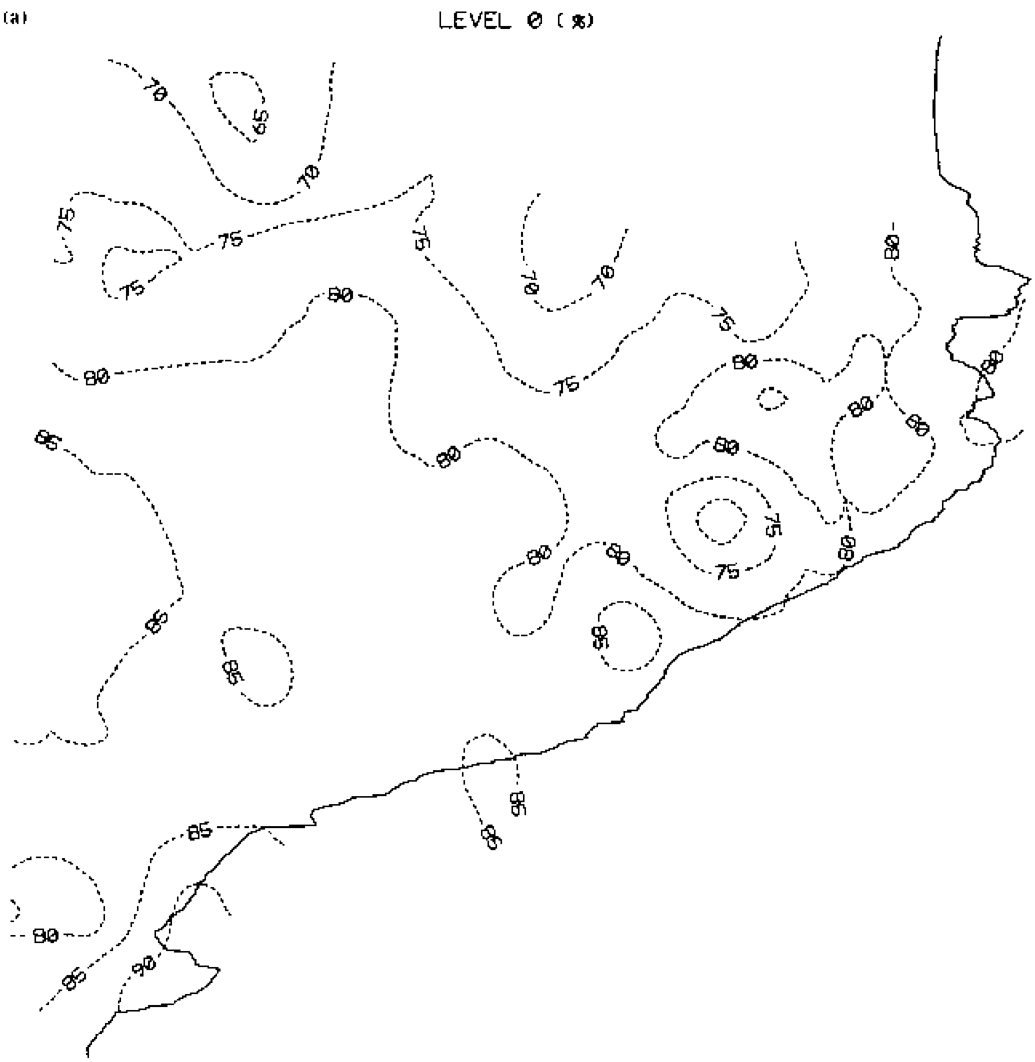

Figure 7. Stationary transition probability for the four pluviometric levels defined in the text and designed by (a) state 0; (b) state 1; (c) state 2; and (d) state 3

Figure 7(a–d) describes the geographical evolution of stationary probabilities for pluviometric states defined by levels 0, 1, 2 and 3 and quantified, respectively, by $P1_4^n(i, 0)$, $P1_4^n(i, 1)$, $P1_4^n(i, 2)$ and $P1_4^n(i, 3)$ for an arbitrary value of the index i and n tending to ∞. As we can expect, the highest stationary probabilities are linked to the state defined by amounts of daily precipitation less than 0.1 mm (Figure 7(a)). The lowest percentages are obtained for the Pyrenees and Pre-Pyrenees areas ($\leq 75\%$), reaching a minimum at the north face of the Pyrenees, clearly subjected to Atlantic influences. Another relevant minimum corresponds to gauge 69, with 70% probability, at 1700 m height, localized on the southern end of the Transversal chain and commonly affected by Mediterranean advection. The rest of the country reaches probabilities greater than 75%, with a remarkable 85% located on the western extreme of the Central basin and 90% related to gauge number 5, with the pluviometric regime influenced by the foehn effect, as previously discussed. This first geographic description could be an approximation scheme representing the distribution of the wet and dry parts of the country.

The picture linked to the state designated by state 1, with daily amounts ranging from 0.1 to 10 mm, corresponds to Figure 7(b). Now the highest percentages are obtained for the main mountainous ranges of the country. Only the southern extreme of the Transversal chain, gauge 69, reaches a high percentage. Low percentages ($\leq 9\%$) are detected for the southeastern coast, due to the same reasons outlined above, and for the western face of the Transversal chain, possibly due to the pluviometric screen effect. As a general picture, Figure 7(b) is a bit more complex than Figure 7(a), with probabilities for a larger part of the country less than or equal to 12% differences disappearing between the Central Basin and the Littoral and Pre-Littoral ranges. In short the percentage distribution seems to be controlled by orography both at a local and regional scale.

For the next level, corresponding to daily precipitations from 10 to 50 mm, we can observe in (Figure 7(c)) a clear increasing tendency from west to east and from south to north. This distribution suggests the role of the main mountainous ranges namely, the Pyrenees and Pre-Pyrenees, where the highest percentages are detected, and the Transversal chain. Whereas the preceding Figure 7(b) is linked to very moderate daily amounts, now we correlate Figure 7(c) with moderate and relevant amounts which could be the effects of Eastern advection, enhanced by orography (Eastern Pyrenees and Transversal chain) or Atlantic perturbations affecting the west part of the Catalan Pyrenees and, specially, the north face of the chain.

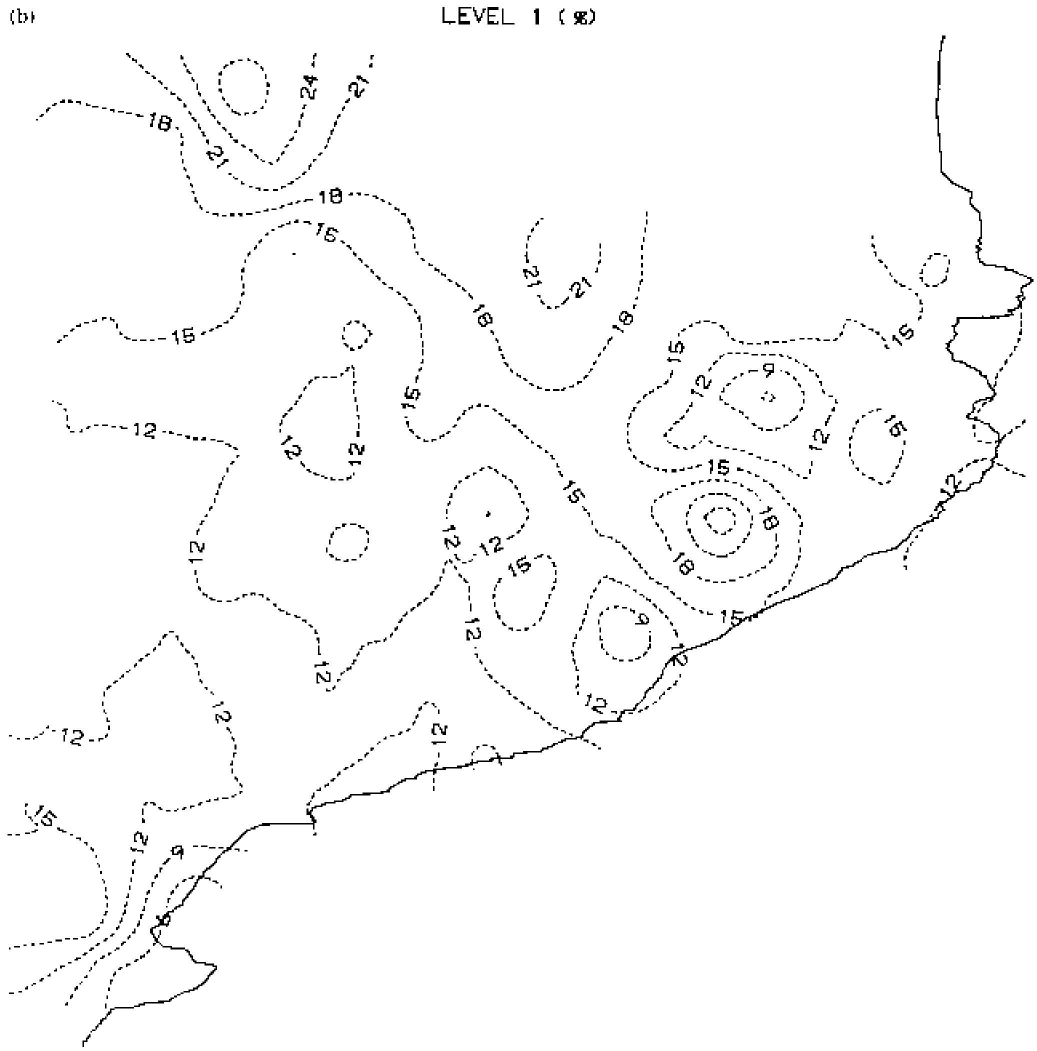

Figure 7 (*Continued*)

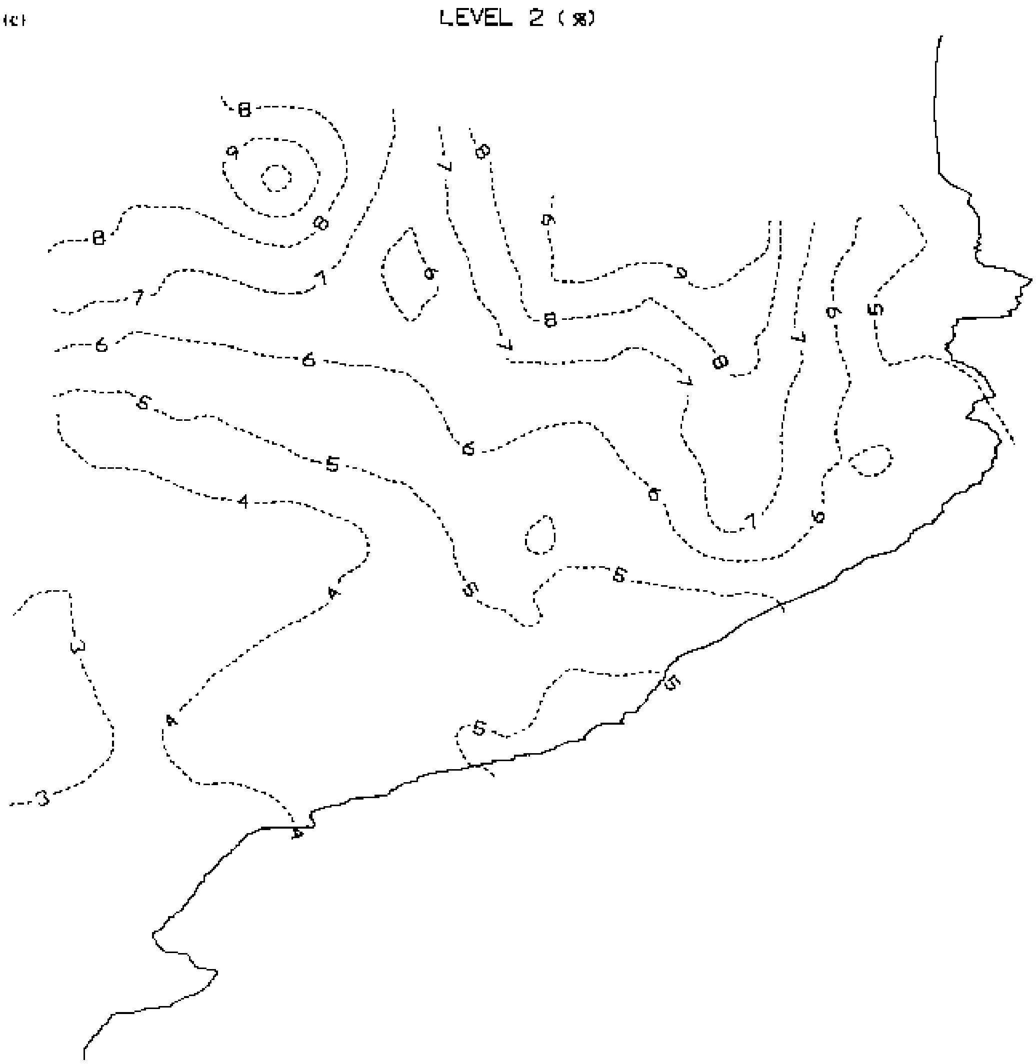

Figure 7 (*Continued*)

Figure 7(d) depicts the stationary percentages for daily amounts greater than 50 mm. Although the percentages are the lowest and the discrepancies among the different areas are small, we can observe some interesting peculiarities. Firstly, the lowest percentage is obtained for the western part of the Central Basin. Secondly, the western part of the Catalan Pyrenees does not appear as the highest percentage, similar to the behaviour shown by most of the Mediterranean coast. The highest ratios are obtained for the Eastern Pyrenees and the Transversal chain, suggesting that the remarkable amounts for 24-h records could be mainly attributed to north-eastern Mediterranean advection with effects reinforced by the orography.

Finally, Figure 8 summarizes the number of steps (days) of the Markov chain of first order and four states to obtain the stationary state on every gauge. Provided that the required difference, established as 1.0×10^{-7}, between elements of matrix $\mathbf{P1}_4^n$ and $\mathbf{P1}_4^{n+1}$, with increasing n, to detect the stationary situation has been the same for each gauge, we can conclude that no significant differences can be established among the pluviometric stations, as the required number of steps is very similar and scattered throughout the country.

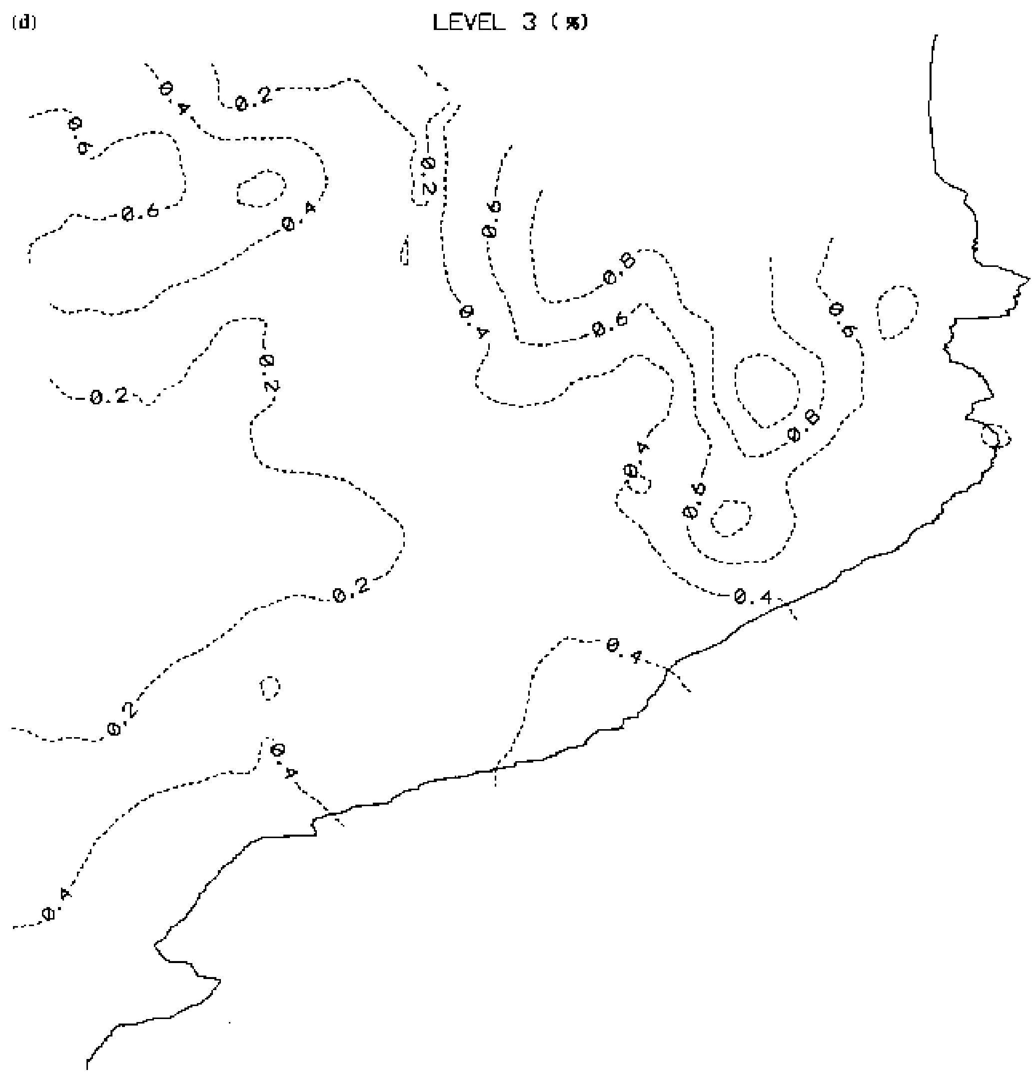

Figure 7 (*Continued*)

4. CONCLUSIONS

NE Spain, as many parts of the Iberian Peninsula (Pérez Manrique *et al*., 1984; Conesa and Martín Vide, 1993), is frequently submitted to drought episodes. In spite of a global drought pattern, a set of orographic and geographical factors lead us to consider relevant differences among parts of the country and to distinguish between humid and dry parts of the studied area. Similar division was recently investigated by Lana and Burgueño (1998) who studied extreme dry episodes based on 24-h records and a preliminary regionalization in terms of expected extreme lengths for different return periods. Nevertheless, if we want a more detailed description of the drought phenomena and their relations with orographic and geographic factors conditioning them, we need to study extreme, moderate and short dry episodes. Consequently, the search for the best statistical model reproducing the empirical distribution of dry events, based on 24-h records, is needed.

From a methodological point of view, no large differences have been found between the exponential distribution and Markov chains with respect to the expected lengths of the dry periods. Therefore the concordance between expected lengths deduced from Markov chains and those derived from Equation (2),

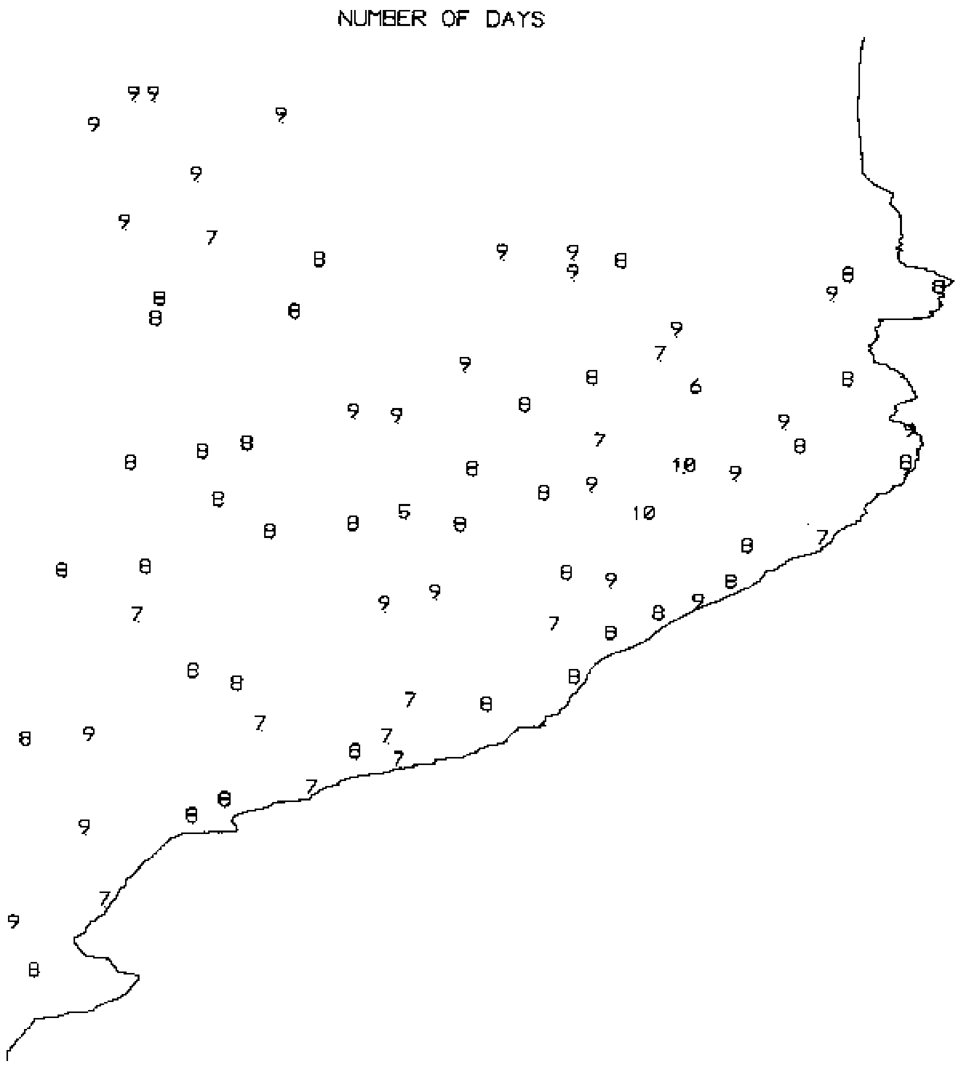

Figure 8. Number of steps (days) required to reach the stationary transition probability, according to the Markov chain of first order and two states, on each gauge

which is derived from the concept of maximum likelihood, is especially relevant. However, the differences are enhanced by the Kolmogorov-Smirnov test, appearing as a relatively complex pattern. A few gauges satisfy the test by assuming the exponential distribution or the chain of first order. A more numerous set of gauges fulfil the test according to the second order chain formulation and a great number of the remaining gauges is close to passing the test provided that this chain is considered. The goodness of the second order chain with respect to the first order is in agreement with results obtained with the Akaike and Bayesian Information Criteria to estimate the appropriate order of the chain. According to Gregory *et al.* (1993), quite often, a chain of order greater than 1 is preferred by the two mentioned AIC and BIC tests.

With respect to the return periods, they have to be derived from chains of first order, in spite of the Kolmogorov-Smirnov test results, due to the specific characteristics of the theory involved for the second order chains. Another relevant aspect of the study is the definition of four precipitation ranges, that lead us to establish a Markov chain of first order and four states and the corresponding stationary transition probabilities. The Markov chain of second order and four states had to be discarded due to the limited size of the database which does not assure reliable transition probabilities affecting the third and fourth states.

An alternative to many ranges of precipitation and high order Markov chains is the so called chain-dependent process (Gregory *et al*., 1993), where conditional distribution of precipitation is fitted by a continuous distribution. This more complete analysis is not applied here, as the interest of the present research is not the generation of daily rainfall series. Also, to reproduce the empirical distribution of dry periods and compute expected dry–wet sequences and return periods, two states (precipitation and no precipitation) and first and second order Markov chains seem to be sufficient. And, finally, the agreement between the spatial characteristics of the stationary matrices of first order and four states, with the orography of the country, together with the main features of the atmospheric circulation over Catalonia and previously published results is quite good. Consequently, higher order chains or more states of precipitation, leading to a chain-dependent process, are at present unnecessary.

From an applied point of view, the results deployed for the present study establish important differences between a domain formed by the Pyrenees, Pre-Pyrenees, Transversal Chain and Northern Littoral and the rest of the country (Central and Southern Coast and Central Basin). In addition, phenomena such as the pluviometric screen, foehn effect, vicinity to the Mediterranean sea, Atlantic influences on the North face of the Pyrenees or reinforcing of precipitation due to orographic barriers are underlined by the different results, especially those concerning the geographical distribution of stationary probabilities. All these aspects are in agreement with conclusions obtained in previous studies of the same area, where spatial and temporal distribution of annual extreme rainfall episodes and annual extreme drought periods (Lana *et al*., 1995; Lana and Burgueño, 1998) have been recently investigated.

The present study also contributes to a better understanding of the drought behaviour of NE Spain, establishing clear differences among drought and wet areas. A first attempt, not completely successful, has been recently presented by Lana and Burgueño (1998), who tried to classify the Catalonian domain in terms of expected annual extreme dry lengths for return periods of 2, 5, 10, 25 and 50 years. In addition, new valuable information concerning return periods, expected lengths for dry–wet cycles and evaluation of probabilities for consecutive dry days has been now established for each gauge analyzed. In spite of all these studies, problems concerning the wet–dry behaviour of NE Spain are not completed and further studies would be necessary. These further researches would include three topics. Firstly, the previous regionalization of the country achieved by Lana and Burgueño (1998) in terms of expected extreme lengths could be improved by characterizing each gauge with more parameters, as the ones introduced here, including a standard principal component analysis. Secondly, it would be interesting to apply the same Markov chain formulation to different levels of monthly amounts of precipitation. And, finally, bearing in mind that the usual statistical tools for extreme episodes do not include probabilities concerning repeated phenomena of moderate or long dry sequences that could generate drought events, some statistical tools for modeling the behaviour of these phenomena would prove useful.

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