Estimation of the tensile elastic modulus using Brazilian disc by applying diametrically opposed concentrated loads

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A B S T R A C T

The tensile elastic modulus $E_t$ of a rock is different from the compressive elastic modulus $E_c$, due to inhomogeneity and microcracks. There is no convenient method to obtain $E_t$ except using direct tension tests. However, the direct tension test for rock materials is difficult to perform, because of stress concentrations, and the difficulty of preparing specimens. We have developed a new method to determine $E_t$ of rock materials easily and conveniently. Two strain gauges are pasted at the center part of a Brazilian disc's two side faces along the direction perpendicular to the line load to record tensile strain, and a force sensor is used to record the force applied; then the stress–strain curve can be obtained; finally the $E_t$ can be calculated according to those related formulas which are derived on the basis of elasticity theory. Our experimental results for marble, sandstone, limestone and granite indicate that $E_t$ is less than $E_c$, and their ratio is generally between 0.6 and 0.9.

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1. Introduction

In general, rocks are inhomogeneous, and contain numerous microcracks. Consequently, rocks show different behavior under tensile and compressive conditions. Accordingly, there are two kinds of elastic modulus: the compressive elastic modulus $E_c$ and the tensile elastic modulus $E_t$. Parameters such as Young's modulus and Poisson's ratio are expected to be different under compressive or tensile stress [1,2]. The data reported by Krech et al. [3] and Liao et al. [4] indicate such differences for Young's modulus and Poisson's ratio in some rock types (granite, quartzite, sandstone, limestone, argilite).

The modulus $E_c$ and the compressive strength $\sigma_c$ are easy to measure in the laboratory by uniaxial compression tests. But the parameter $E_t$ and the tensile strength $\sigma_t$ are difficult to obtain by direct tension tests, because it is very difficult and complicated to prepare test specimens, and it is easy to generate stress concentrations at the ends of the specimen. In order to solve this problem, the International Society for Rock Mechanics (ISRM) officially proposed the Brazilian test as a suggested method for determining the tensile strength $\sigma_t$ of rock materials [5]; however, there is no indirect test proposed by the ISRM to determine the tensile elastic modulus $E_t$. In this study, a convenient and maneuverable method for determining $E_t$ of rock materials with the Brazilian disc approximately is to be developed.

In the Brazilian test, a disc specimen is compressed with diametrically opposite and symmetric line loads [6,7]. The theoretical basis for the Brazilian test is the analytical solutions that have been obtained by many researches for isotropic or transverse isotropic materials under concentrated loads, loads that are distributed over a small arc of the disc's circumference [8–11]. Fairhurst [12] discussed the validity of the Brazilian test, and concluded that failure is expected to initiate at the center of disc, but actually the failure sometimes initiates at the loading points. Hudson [13] verified this conclusion with experiments. Guo et al. [14] developed a simple method to measure the opening mode (mode-I) fracture toughness $K_{IC}$ with the Brazilian disc. Wang et al. [15] later made some improvement to Guo's method for determining $K_{IC}$.

Much attention has been placed on the elastic modulus by researchers. Hondros [8] developed an approach to measure the elastic modulus $E$ and Poisson’s ratio $\nu$ with the Brazilian disc. He also gave a complete stress analytic solution for the case of a radial load distributed over a finite circular arc of the disc. However, this kind of loading is very difficult to obtain in the laboratory. Consequently, there will be some differences between the actual stress field and the ideal analytical solution. And there is another problem for the method: the theory of the method is based on the strain of the center of the disc, but the strain measured by the strain gauge is the contribution of a line segment...
near the center of the disc. Yu et al. [16] invented a method for determining $E$ with the Brazilian test proposed by ISRM. They recorded the force applied and the displacement of loading point in experiment, and then a force-displacement curve could be obtained. The slope of the line section of force-displacement curve was defined as $E_0$. The elastic modulus $E$ could be determined by $E_0$ multiplying a correction coefficient $k$. According to finite-element analysis (ANSYS) and experiment, using Three Gorges granite, they concluded that $k$ is about 19.2. This method was an improvement, but it was just based on linear elastic, isotropic finite-element analysis and regressing, fitting test data. Its credibility and reliability may be low for other rock types.

Wang et al. [17–19] developed a similar method for determining $E$ with flattened Brazilian disc. They gave out an approximate formula to calculate the relative displacement between the two ends according to Cauwellaert’s result for a uniformly and parallelly distributed load applied on a section of a circular arc [20]. However, You and Su [21] disputed Wang’s method. Because the loading and geometry between flattened Brazilian disc and Cauwellaert’s complete disc were completely different, they concluded that it was incorrect to use Cauwellaert’s results.

The elastic modulus $E$ mentioned above is the compressive elastic modulus. Some attention has also been placed on the tensile elastic modulus. Li and Yin [22] used pure bending beam to measure the $E_c$ and $E_t$. Two strain gauges were pasted on the upper surface (compressive zone) and lower surface (tensile zone), to record the compressive and tensile strain, respectively. The strongpoint of this method is that it can obtain $E_c$ and $E_t$ at the same time. Zhang et al. [23] devised a method to calculate the tensile elastic modulus $E_t$ of rock with a cracked Brazilian disc. A small vertical, straight and through notch was required at the center of disc. However, the theory of this method is immature at present, and it is difficult to make the notch required in Brazilian disc.

As we know, some rock types show non-homogeneity to some extent, and contain many microcracks. These two factors lead to some anisotropy for rock materials. Furthermore, the level of microcracking and the orientation of microcracks also have great effect on the mechanical properties of rock materials. However, it is very difficult to study the effect quantitatively at present. Therefore, for the sake of simplicity, and also adopting the same model proposed by ISRM for the Brazilian test who treat the rock material as an equivalent isotropic continuum medium, we also consider the disc to be an equivalent isotropic continuum medium. Correspondingly, in fact the elastic modulus and Poisson’s ratio are the equivalent ones. Of cause, the model adopted here is not suitable for those rock types that show anisotropy or transverse isotropy. For example, Fig. 1a is a white marble, which seems to be homogenous and isotropic. Fig. 1b is a banded marble, which shows transverse isotropy. Certainly, the white marble is suitable for the model proposed here, whereas the banding marble is not suitable.

In this study, a simple and convenient test method is proposed for determining $E_t$ for isotropic rock materials with the Brazilian disc. The configuration of the test is shown in Fig. 2. The core idea of the proposed method that two strain gauges are pasted, respectively, at the center of disc on the both side faces along the direction perpendicular to the line load $P$ (Fig. 3) to record the tensile deformation of the center part. Then, according to the stress obtained through elasticity theory and the recorded strain, the equivalent tensile elastic modulus $E_t$ can be calculated.

Obviously, the loading manner of the test method proposed here is completely different from that proposed by ISRM. The ISRM suggests that two concave loading plates can be used to apply load in order to distribute the load along an arc of the disc. The reasons that the test configuration shown in Fig. 2 is adopted in this study are as following. On the one hand, the stress analytic solution obtained by Hondros [8] for a pair of distributed loads applied over an arc of the disc oppositely and diametrically (Fig. 4) based on isotropy is

$$
\sigma_r = -\frac{2p}{\pi} \left\{ \alpha + \sum_{n=1}^{\infty} \left\{ 1 - \left( \frac{1}{n} \right) \left( \frac{r}{R} \right)^2 \right\} \sin 2nx \cos 2n\theta \right\}
$$

$$
\sigma_\theta = -\frac{2p}{\pi} \left\{ \alpha - \sum_{n=1}^{\infty} \left\{ 1 + \left( \frac{1}{n} \right) \left( \frac{r}{R} \right)^2 \right\} \sin 2nx \cos 2n\theta \right\}
$$

$$
\tau_{r\theta} = -\frac{2p}{\pi} \sum_{n=1}^{\infty} \left\{ 1 - \left( \frac{1}{n} \right) \left( \frac{r}{R} \right)^2 \right\} \sin 2nx \cos 2n\theta \right\}
$$

(1)

where $p$ is the applied pressure, $R$ is the radius of disc, $r$ and $\theta$ are the polar coordinates of a point in disc, and $\alpha$ is the half central angle related to the distributed load applied. From the equation above, we know that the stress field of the disc subjected to a pair of distributed load is the function of $\alpha$. Obviously, the magnitude of $\alpha$ affects the stress distribution in the disc directly. According to ISRM’s suggestion [5], if the standard concave loading plates are used in the Brazilian test, the $2\alpha$ is about $10^\circ$ at failure. As far as we know, the disc would show some plastic properties, rather than complete elasticity when approaching failure. However, the parameter of tensile elastic modulus $E_t$ is a physical quantity of the elastic stage in tension of rock materials. Therefore, $2\alpha$ is certainly not $10^\circ$, and there is no way to exactly know the specific value of $2\alpha$ when the disc is in the elastic stage. Furthermore, $2\alpha$ is a variable in the processing of loading. Additionally, the value of

Fig. 1. (a) is white marble which seem to be homogenous and isotropic and (b) banded marble shows obvious transverse isotropy.
$2\alpha$ would vary despite using the same concave loading plates and under the same loading, due to the type of rock, component and size of mineral, brittleness and stiffness. In short, these factors make it very difficult to know and calculate the value of $2\alpha$ when the disc is in its elastic stage. Accordingly, it is also very difficult for us to calculate exactly the stress field in the disc subjected to distributed loads over an arc when the disc is in elastic stage. This brings difficulty to determine the tensile elastic modulus $E_t$ through the strain measurement due to the uncertainty of the stress field in disc of elastic stage. On the other hand, in the test configuration proposed here showed in Fig. 2, two steel bars are used to provide loads. Because the diameter of the steel bars is much smaller than the diameter of the disc, the area of contact between the steel bars and the disc is very small. Correspondingly, the loads applied by the two steel bars can be considered as two concentrated loads applied to the disc oppositely and diametrically along the diameter. This type of loading can effectively avoid the problem of the uncertainty of the stress field in disc subjected to distributed load over an arc due to the difficulty in knowing $2\alpha$.

The analytical solution for a pair of diametrically opposite, symmetric and compressive line loads applying on a disc of isotropic rock materials [Fig. 5] has been given by Muskhelishvili [9]. Therefore, the distribution of the stress at the center part of disc is completely known. As long as the tensile strain is recorded by strain gauges in the experiment, the tensile elastic

![Fig. 2. The configuration of the test adopted in this study: (a) the experiment box, (b) the loading plate, (c) the Brazilian disc, (d) steel bar, (e) force sensor and (f) the press machine.](image)

![Fig. 3. Two extra strain gauges are pasted, respectively, at the center part of Brazilian disc on both side faces.](image)

![Fig. 4. A pair of distributed load applies over an arc of the disc oppositely and diametrically on the disc.](image)
modulus $E_t$ of isotropic rock materials can be calculated from the slope of the line section of stress–strain curve, the Poisson ratio $v$, the diameter of disc $D$ and the half length of strain gauge $l$ (see formula (12)).

Some experiments have been performed on four rock types to support this method and related theory, including the uniaxial compression tests and the Brazilian tests. The experimental results indicate that the ratio between $E_t$ and $E_c$ for the same rock type is generally about 0.6–0.9, which is consistent with the data reported in Ref. [22]. It proves to some extent that the method proposed is flexible and reliable.

2. Analytic solution for a pair of diametrically opposed, symmetric and compressive line loads applied on an isotropic Brazilian disc (Fig. 5)

The solution for an isotropic Brazilian disc subjected to concentrated loads is [9]

$$\sigma_x = \frac{2p}{\pi l} \left( \cos \theta_1 \sin^2 \theta_1 + \frac{\cos \theta_2 \sin^2 \theta_2}{r_2} \right) - \frac{2p}{\pi Dl}$$

$$\sigma_y = \frac{2p}{\pi l} \left( \cos^3 \theta_1 + \frac{\cos^3 \theta_2}{r_2} \right) - \frac{2p}{\pi Dl}$$

$$\tau_{xy} = \frac{2p}{\pi l} \left( \cos^2 \theta_1 \sin \theta_1 + \frac{\cos^2 \theta_2 \sin \theta_2}{r_2} \right)$$

(2)

where $P$ is the line load applied, whose units are (N/m). $l$ is the thickness of the disc, $D$ is the diameter of the disc. $\theta_1$, $\theta_2$ are positive when the point $E$ is at the right of load $P$, and they are negative when $E$ is at the left of load $P$. $r_1$ and $r_2$ are the distances from the point $E$ to the loading points $C$ and $F$ (Fig. 5). There are following relations in the triangle CEF:

$$r_2^2 = r_1^2 + D^2 - 2r_1D \cos \theta_1$$

$$\cos \theta_2 = \frac{D^2 + r_2^2 - r_1^2}{2r_2D} = \frac{D - r_1 \cos \theta_1}{r_2}$$

$$\sin \theta_2 = \sqrt{1 - \cos^2 \theta_2} = \frac{r_1 \sin \theta_1}{r_2}$$

(3)

Using the relation formula above, expression (2) can be written as

$$\sigma_x = \frac{2p}{\pi l} \left( \cos \theta_1 \sin^2 \theta_1 + \frac{(D - r_1 \cos \theta_1)r_2^2 \sin^2 \theta_1}{(r_2^2 + D^2 - 2r_1D \cos \theta_1)^2} \right) - \frac{2p}{\pi Dl}$$

$$\sigma_y = \frac{2p}{\pi l} \left( \cos^3 \theta_1 + \frac{(D - r_1 \cos \theta_1)^3}{(r_2^2 + D^2 - 2r_1D \cos \theta_1)^2} \right) - \frac{2p}{\pi Dl}$$

$$\tau_{xy} = \frac{2p}{\pi l} \left( \sin \theta_1 \cos \theta_1 + \frac{(D - r_1 \cos \theta_1)r_2 \sin \theta_1}{(r_2^2 + D^2 - 2r_1D \cos \theta_1)^2} \right)$$

(4)

Creating a rectangular coordinate system O–x–y at the center of disc by taking the point O for the origin (Fig. 5), the relationship between the rectangular coordinates $(x, y)$ and the polar coordinates $(r_1, \theta_1)$ of point $E$ is

$$x = r_1 \sin \theta_1$$

$$y = \frac{D}{2} - r_1 \cos \theta_1$$

$$r_1 = \sqrt{\left(\frac{D}{2} - y\right)^2 + x^2}$$

$$\sin \theta_1 = \frac{(D/2 - y)}{r_1}$$

(5)

Substituting (5) into Eq. (4), we get

$$\sigma_x = \frac{2p}{\pi l} \left( \frac{(D/2 - y)x^2}{((D/2 - y)^2 + x^2)^2 + (D/2 + y)^2 + x^2} \right) - \frac{2p}{\pi Dl}$$

$$\sigma_y = \frac{2p}{\pi l} \left( \frac{(D/2 - y)^2x^2}{((D/2 - y)^2 + x^2)^2 + (D/2 + y)^2 + x^2} \right) - \frac{2p}{\pi Dl}$$

$$\tau_{xy} = \frac{2p}{\pi l} \left( \frac{(D/2 - y)x^2}{((D/2 - y)^2 + x^2)^2 + (D/2 + y)^2 + x^2} \right) - \frac{2p}{\pi Dl}$$

(6)

Fig. 6 shows the distributions of $\sigma_x$, $\sigma_y$, $\tau_{xy}$ in the disc. From formula (6) and Fig. 6, it is obvious that the points $C$ and $F$ are singular points in the stress field, and there is stress concentration phenomenon at the region near $C$ and $F$. However, the phenomenon disappear at the region far away the force acting point. The sections on which the strain gauges are pasted are the center part of the disc, which is far away from the points $C$ and $F$. Therefore, the validity of the method presented in this paper is certainly not affected.

3. Estimation of tensile elastic modulus of rock

3.1. The method and theory

The stress field of the Brazilian disc based on elasticity mechanics for isotropic rock materials has been known completely according to formula (6). It is the theoretical foundation for measuring the tensile elastic modulus of rock through the strain measurement.

The constitutive equation expresses the relation between the stress and the strain through Young’s modulus and Poisson’s ratio. Accordingly, as long as the strain and the force applied are recorded in experiment by strain gauge and force sensor, the tensile elastic modulus can be determined with the slope of the line section of stress–strain curve according to the constitutive equation. Therefore, the most principal problem needed to be solved is to record the stress–strain curve of elastic stage when rock is in tensile condition. The method adopted in this paper is that two strain gauges are pasted, respectively, at the center on the both side faces of disc along the direction perpendicular to the line load $P$ to record tensile deformation of the center part, and a force sensor is used to record the line load $P$ applied. The stress and strain data are picked by stress–strain acquisition system in experiment. At last, the tensile elastic modulus can be determined by using the computer processing system in which the strain used
in calculating and plotting should be the average value of the two strain gauges (Fig. 7).

The measuring theory is illustrated in detail in the following. Assuming \( y = 0 \) in formula (6), we obtain the following expression for the stress state on the diameter \( AB \):

\[
\begin{align*}
\sigma_x &= \frac{2P}{\pi DL} \left\{ \frac{16D^2 x^2}{(4x^2 + D^2)^2} - 1 \right\} \\
\sigma_y &= \frac{2P}{\pi DL} \left\{ \frac{4D^2}{(4x^2 + D^2)^2} - 1 \right\} \\
\tau_{xy} &= 0
\end{align*}
\]  

(7)

The distribution of the stress on the diameter \( AB \) is showed in Fig. 8. From Fig. 8 we can know that the compressive stress \( \sigma_y \) is two times than the absolute value of the tensile stress \( \sigma_x \) at the center part of Brazilian disc. Consequently, the tensile strain generated by compressive stress \( \sigma_y \) due to Poisson's effect cannot be ignored in estimation of tensile elastic modulus.

The tensile strain at the center part of disc along the diameter \( AB \) in the range the strain gauges are pasted can be calculated with the following expression:

\[
\varepsilon_t = \frac{1}{2L} \int_{-L}^{L} \frac{1}{E_t} \left( -\sigma_x + \nu \sigma_y \right) dx - \frac{1}{E_t L} \int_{0}^{L} \left( -\sigma_x + \nu \sigma_y \right) dx
\]  

(8)

where \( L \) is the half-length of the strain gauges pasted at the center part of disc, \( E_t \) is the tensile elastic modulus of rock, and \( \nu \) is Poisson's ratio. The tensile stress is intended to be negative in this paper, so an extra minus sign is appended to \( \sigma_x \) in formula (8). Substituting formula (7) into (8), we obtain

\[
\varepsilon_t = -\frac{2P D}{\pi E_t L L} \int_{0}^{L} \frac{16y^2}{(4y^2 + D^2)^2} dy + \frac{2P}{\pi E_t L} \int_{0}^{L} \frac{4}{(4y^2 + D^2)^2} dy + \frac{2P}{\pi E_t L} (1 - \nu)
\]  

(9)
Fig. 9. The typical stress–strain curves recorded in the Brazilian test and the uniaxial compressive test.
The two definite integrals in above expression (9) can be worked out analytically as follows:

\[
\int_{0}^{1} \frac{16x^2}{(4x^2 + D^2)^2} \, dx = \frac{1}{D} \arctan \left( \frac{2L}{D} \right) - \frac{2L}{4L^2 + D^2} \\
\int_{0}^{1} \frac{4}{(4x^2 + D^2)} \, dx = \frac{1}{D^2} \arctan \left( \frac{2L}{D} \right) + \frac{2L}{D^2(4L^2 + D^2)}
\]

(10)

If the formula (10) is substituted into (9), the following expression is obtained:

\[
e_t = \frac{2P}{\pi D l E} \left\{ \left( 1 - \frac{D}{L} \right) \arctan \left( \frac{2L}{D} \right) (1 - v) + \frac{2D^2(1 + v)}{4L^2 + D^2} \right\}
\]

(11)

Exchanging the \(e_t\) and \(E_t\) in formula (11), and taking \(E_t = 2P/\pi D l e_t\), we obtain

\[
E_t = E_t \left\{ \left( 1 - \frac{D}{L} \right) \arctan \left( \frac{2L}{D} \right) (1 - v) + \frac{2D^2(1 + v)}{4L^2 + D^2} \right\} = A' e_t
\]

(12)

\[
A = \left( 1 - \frac{D}{L} \right) \arctan \left( \frac{2L}{D} \right) (1 - v) + \frac{2D^2(1 + v)}{4L^2 + D^2}
\]

(13)

where \(E_t\) is the tensile elastic modulus of rock, \(E_s\) is defined as splitting elastic modulus which can be determined from the stress–strain curve recorded in the Brazilian test, \(v\) is Poisson’s ratio. \(A\) is a correction coefficient which is related to \(D\), \(L\) and \(v\). From formula (12), we see that after the diameter of Brazilian disc \(D\) and the half-length of strain gauge \(L\) are known; Poisson’s ratio \(v\) has been determined with the uniaxial compression test; and the splitting elastic modulus \(E_s\) is obtained which is equal to the slope of line section of the stress–strain curve recorded in the Brazilian test. Then the tensile elastic modulus \(E_t\) can be determined easily and conveniently.

In fact, the stress–strain curve recorded in test sometime shows the properties of non-linearity. There is no obvious linear section in the stress–strain curve in the elastic stage. Accordingly, if there is no obvious linear section in the stress–strain curve due to a high degree of non-linearity, the parameter of splitting elastic modulus \(E_s\) would be determined by the following formula:

\[
E_s = \frac{(1/2)\sigma_t}{\epsilon_t}
\]

(14)

where \(\sigma_t\) is the maximum stress in test, namely tensile strength. \(\sigma_t/2\) is the stress in stress–strain curve which is half of the tensile strength. \(\epsilon_t\) is the strain related to \(\sigma_t/2\) in the stress–strain curve (Fig. 9—S01). Actually, this suggestion of determining elastic modulus is also proposed by some specification in China.

3.2. The experimental results for the measuring method and theory

Four kinds of rock materials are used in the experiment: marble, limestone, sandstone and granite. These specimens are comparatively homogenous and fine grained. Therefore, they can be treated as approximately isotropic. Six specimens are prepared for each kind of rock and they are all circular cylinders. Three specimens of the six are used for uniaxial compression test; their height and diameter are about 100 and 50 mm, respectively. The other three are used for the Brazilian tests; their height and diameter are about 25 and 50 mm, respectively. The compressive strength \(\sigma_c\), Poisson’s ratio \(v\), and compressive elastic modulus \(E_c\) can be determined by the uniaxial compression test. The tensile strength \(\sigma_t\) and splitting elastic modulus \(E_s\) can be determined by the Brazilian test. The half-length \(L\) of strain gauge used in experiment is 5 mm. The loading rate for the Brazilian test and the uniaxial compressive test is 200 and 2000 N/s, respectively. Those stress–strain curves have been recorded by applying the measuring method stated formerly. Fig. 9 shows the typical stress–strain curves of the Brazilian test and the uniaxial compressive test in the laboratory. The experimental results are shown in Tables 1 and 2.

We can see from Tables 1 and 2 that the tensile elastic modulus \(E_t\) of the four kinds of rock materials are all less than their compressive elastic modulus \(E_c\). The average value of the \(E_t\), \(E_c\) and their ratio of the four kinds of rock materials are listed in Table 3.

The experimental results show that the ratio between average tensile elastic modulus \(E_t\) and average compressive elastic modulus \(E_c\) of the four kinds of rock materials are 87%, 71%, 75%, and 69%, respectively. The scope of these ratios is about 60–90%. This result is roughly consistent with the conclusion of Ref. [22].

---

### Table 1

<table>
<thead>
<tr>
<th>Rock materials</th>
<th>Serial number</th>
<th>(\sigma_t) (MPa)</th>
<th>(v)</th>
<th>(E_c) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marble</td>
<td>D04</td>
<td>70.32</td>
<td>0.353</td>
<td>77.54</td>
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<tr>
<td>D05</td>
<td>109.34</td>
<td>0.320</td>
<td>75.31</td>
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<td>D06</td>
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<td>0.317</td>
<td>77.72</td>
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<tr>
<td>Average value</td>
<td></td>
<td>97.70</td>
<td>0.330</td>
<td>76.86</td>
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### Table 2

<table>
<thead>
<tr>
<th>Rock materials</th>
<th>Serial number</th>
<th>(v)</th>
<th>(D) (mm)</th>
<th>(A)</th>
<th>(E_t) (GPa)</th>
<th>(E_s) (GPa)</th>
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<tbody>
<tr>
<td>Marble</td>
<td>D01</td>
<td>0.50</td>
<td>1.905</td>
<td>33.4</td>
<td>63.6</td>
<td></td>
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<tr>
<td>D02</td>
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<td>62.1</td>
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<tr>
<td>D03</td>
<td>0.50</td>
<td>39.2</td>
<td>74.7</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Average value</td>
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<td>35.1</td>
<td>66.8</td>
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### Table 3

<table>
<thead>
<tr>
<th>Rock materials</th>
<th>(\sigma_t) (MPa)</th>
<th>(v)</th>
<th>(\sigma_t/2)</th>
<th>(E_t) (GPa)</th>
<th>(E_s) (GPa)</th>
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<tr>
<td>Granite</td>
<td>Hu01</td>
<td>0.50</td>
<td>3.02</td>
<td>9.31</td>
<td>14.0</td>
</tr>
</tbody>
</table>
Table 3
The average value of $E_t$, $E_c$, and their ratio of the four kind of rock materials

<table>
<thead>
<tr>
<th>Rock</th>
<th>Marble</th>
<th>Sandstone</th>
<th>Limestone</th>
<th>Granite</th>
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</thead>
<tbody>
<tr>
<td>$E_t$</td>
<td>66.8</td>
<td>7.1</td>
<td>43.3</td>
<td>14.0</td>
</tr>
<tr>
<td>$E_c$</td>
<td>76.86</td>
<td>10.02</td>
<td>57.73</td>
<td>20.29</td>
</tr>
<tr>
<td>Ratio (%)</td>
<td>86.9</td>
<td>70.9</td>
<td>75.0</td>
<td>69.0</td>
</tr>
</tbody>
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Table 4
The value of $A$ corresponding to different Poisson’s ratio $v$ ($D = 50$ mm, $L = 5$ mm)

<table>
<thead>
<tr>
<th>v</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>1.954</td>
<td>1.239</td>
<td>1.384</td>
<td>1.528</td>
<td>1.671</td>
<td>1.818</td>
<td>1.963</td>
</tr>
</tbody>
</table>

4. Discussion

From formula (6), the compressive stress $\sigma_y$ is about two times than the absolute value of the tensile stress $\sigma_x$ at the center part of the Brazilian disc. Therefore, the tensile strain generated by compressive stress $\sigma_y$ due to Poisson’s effect must be considered, unless the measuring result is inaccurate. However, the tensile strain contributed by $\sigma_x$ usually accounts for a small proportion of the total tensile strain, because Poisson’s ratio $v$ usually is a small value of 0.1–0.3. That is to say, the tensile stress $\sigma_x$ contributes the majority in the total tensile strain.

From formula (12), we know that the determination of the tensile elastic modulus $E_t$ is not only affected by the diameter of disc $D$ and the half-length of strain gauge $L$, but also by Poisson’s ratio $v$. Furthermore, Poisson’s ratio $v$ is the most important effective factor whose accuracy directly determines the reliability of the $E_t$. Table 4 lists the different values of $A$ when Poisson’s ratio $v$ is 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, and 0.35 ($D = 50$ mm, $L = 5$ mm). Table 4 indicates that the $A$ is very sensitive to Poisson’s ratio. However, frequently, there is uncertainty when measuring Poisson’s ratio $v$ in the laboratory with uniaxial compressive tests. Therefore, in order to obtain reliable value of $E_t$, the uniaxial compressive test must be performed carefully and at least three specimens are used for each type of rock.

As stated above, it is difficult for us to perform the direct tension test in laboratory. Consequently, the experimental data of the direct tension test for the four kinds of rock have not been obtained by applying the measuring method proposed in this paper with the results obtained from direct tension test. It is a deficiency, and also a research direction later.

Lots of rock types show anisotropy or transverse isotropy. How to measure or estimate their tensile elastic modulus at all directions with the Brazilian disc is also worthy of research further later.

5. Conclusions

The stress analytic solution of rectangular coordinates form for the Brazilian disc (formula (6)) are given based on the results obtained by Muskhelishvili for isotropic rock materials. The origin of coordinates is the center of Brazilian disc. It is the theoretical foundation for determining the tensile elastic modulus with Brazilian disc.

The tensile elastic modulus $E_t$ is an important parameter, which characterizes the tensile property of rock materials. But it is difficult to obtain attributing to the fact that the direct tensile test is difficult to perform in laboratory. A new measuring method with the Brazilian disc is proposed in this paper that can determine the $E_t$ easily and conveniently (Fig. 7). The method is that two strain gauges are pasted, respectively, at the center on the both side faces of disc along the direction perpendicular to the line load $P$ (Fig. 3) which are used to record tensile strain of the center part, and a force sensor is used to record the force applied; then the splitting elastic modulus $E_s$ can be obtained from the slope of the line section of the stress–strain curve recorded or according to formula (14); the $v$ are determined by uniaxial compression test; finally, the $E_c$ can be determined according to formula (12). Additionally, a new experimental set-up is invented which is used to provide the line concentration loads (Fig. 2).

Our experimental results show that the $E_t$ of rock materials are all less than their $E_c$, and the ratio between $E_t$ and $E_c$ of rock materials, including marble, sandstone, limestone, and granite is about 60–90%. These results are basically consistent with the conclusion of Ref. [22]. Consequently, the flexibility, maneuverability, and the reliability of the measuring method proposed here are shown to be good.

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References


